Capacitors

A capacitor also known as a condenser, is a device which is used to store electric charge.

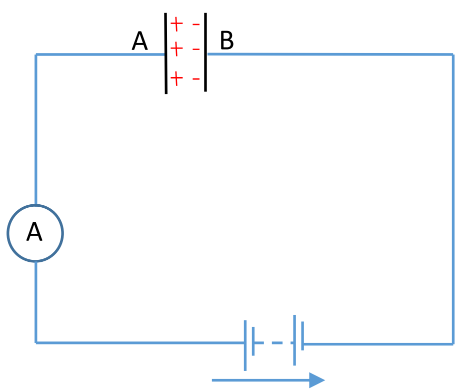
A capacitor consists of two metal plates separated by an insulator called dielectric usually air or oil.

A dielectric is a nonconductor of electric charge in which an applied electric field causes a displacement of charge but not flow of charge. Electrons within the atom of a dielectric are on average, displaced by an electric field with respect to the nucleus, giving rise to a dipole that has an electric moment in the direction of the field. In this case, we say the molecules have been polarized.

A dielectric does not only keep the two metal plates apart, but also increase the capacitance of a capacitor.

Charging and discharging a capacitor

A capacitor is charged when a battery or e.m.f source is connected across it.



Direction of electron flowing during charging

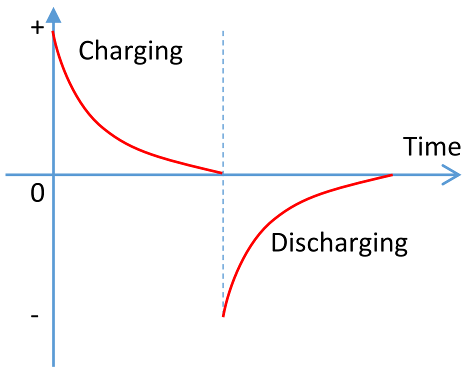
On connecting the battery across the capacitor, electrons are drawn from metal plate A by the positive terminal of the battery, whilst at the same time they are deposited on metal B by the action of negative terminal. Thus there is a momentary flow of current as indicated by the ammeter.

After some time, the number of electrons deposited on plate B keeps on reducing due to electrostatic force of repulsion between already deposited electrons and those that are flowing to plate B. At one point, no further deposition occurs. At this point, the potentials at A and B become equal to that across the battery and oppositely directed to it. The current drops to zero. The capacitor is said to be fully charged.

The charging process leaves A with a positive charge and B with an equal but negative charge.

When the battery is disconnected and the plates are joined by a wire, electrons flow from plate B to A until the positive charge on A is completely neutralized. A current thus flows for a time in the wire and at the end of time, the charges on the plates become zero. The capacitor is said to be discharged.

The graph below shows how the currents vary when the capacitor is being charged and when being discharged.



Capacitance

Capacitance of a capacitor is the ratio of the magnitude of charge on either plate of the capacitor to the potential difference between the plates.

Or

It is charge required to cause a potential difference of 1V between plates of the capacitor

$C = \frac{Q}{V}$

where C is capacitance, Q is the magnitude of charge on either plate and V is the potential difference between the plates.

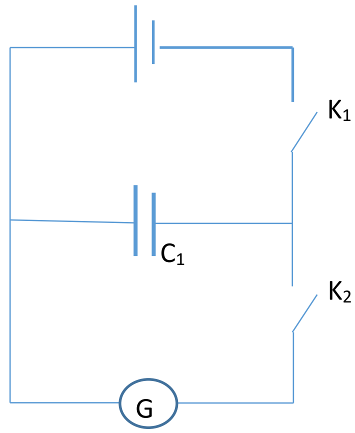
The units of C is Coulomb per volt (CV^{-1}) or Farad (F). The farad is a very large unit, 1μF(micro) = 10^{-6}F , nF (nanofarad) = 10^{-9}F,1mF (picofarad) = 10^{-12}F.

The Farad

The farad is the capacitance of a capacitor such that 1C of charge is stored when the p.d of 1Vis applied.

i.e. 1F = CV^{-1}

Comparison of capacitances or measurement of capacitance of a capacitor



Large capacitances, of order micro-farads, can be compared with the aid of a ballistic galvanometer. The circuit is shown above. The capacitor of capacitance C1 is charged by a battery of e.m.f V, and then discharged through ballistic galvanometer G. The corresponding deflection, θ1 is noted.

The capacitor is now replaced by another capacitor C2, charged again by the battery, and the new deflection θ2 is noted when the capacitor is discharged.

Now

$$\frac{Q_1}{Q_2} = \frac{\theta_1}{\theta_2}$$

$$\frac{C_1V}{C_2V} = \frac{\theta_1}{\theta_2}$$

$$\frac{C_1}{C_2} = \frac{\theta_1}{\theta_2}$$

If C_2 is a standard capacitor, whose capacitance is known, then the capacitance C_1 can be found

Factors that affect capacitance of a capacitor

(i) The plate separation
(ii) The area of overlap (or area common) between the plates
(iii) The dielectric or medium between the plates, and most specifically the permittivity of the dielectric material.

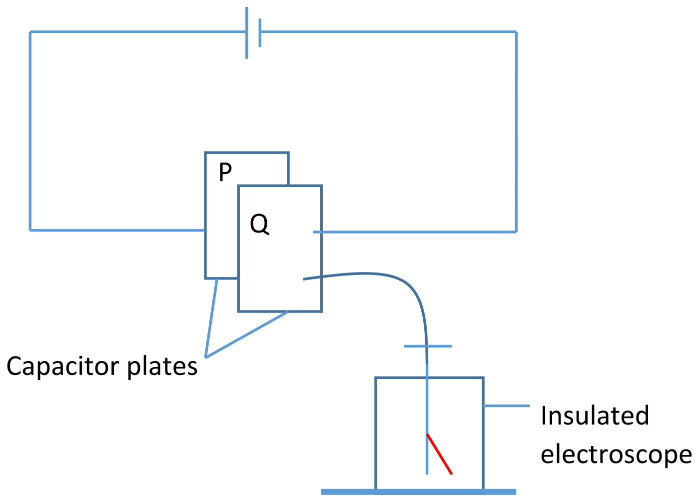
Experiments show the following

(i) $C \propto \frac{1}{d}$
(ii) $C \propto A$

(iii) $C \propto \varepsilon$

Experiment to verify that $C = \varepsilon A/d$ for parallel plate capacitor.

(a) To show that $C \propto A$



A p.d is set across the capacitor plates P and Q, by connecting a cell across it. One plate Q is connected to the cap of electroscope. With the p.d constant and the plates in the above position with air in between them, the divergence of the leaf of electroscope is noted.

Place P is then slid relative to Q keeping the separation constant, hence reducing the effective area of the plate. The divergence is seen to decrease.

Since divergence is proportional to charge on the plates, which in turn is proportional to the capacitance, C, then, C is reduced.

This shows that $C \propto A$

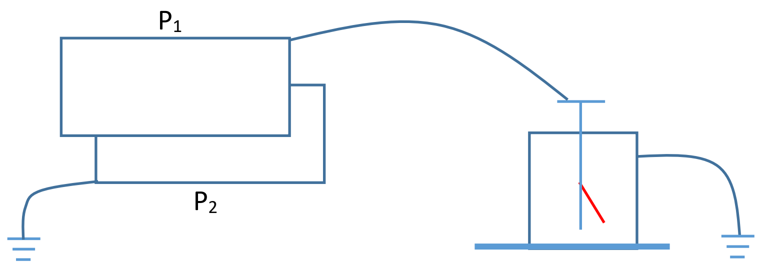
(b) To show that $C \propto 1/d$

The plate P is then slid back and separation between the plates is increased by moving P away from Q. The divergence of the leaf is observed to decrease. Since divergence of the leaf is proportional to charge on the plates, which in turn is proportional to capacitance C, then C is reduced. This shows that $C \propto 1/d$.

(c) To show that C ∝ε

The position of P and the separation between the two plates are restored. A dielectric is then put in the space between the plates. The divergence of the leaf is seen to increase. Since divergence is proportional to the charge on the plates, which in turn is proportional to capacitance C, then C has increased. This shows that **C ∝ε.**

Alternative experiments



P_1 and P_2 are capacitor plates initially with air between then. P_2 is earthed while P_1 is charged and connected to the cap of electroscope. Note that for this arrangement, the charges remain constant while the p.d between the plates varies.

(a) To show that C ∝ A

Plate P_2 is displaced sideways relative to P_1 to reduce the effective area A of the plates. The divergence of the leaf increase. This shows that the p.d. between the plates has increased since divergence is proportional to p.d between the plates. From C = Q/V, since Q is constant, c decreases. Thus **C ∝ A.**

(d) To show that C ∝ 1/d

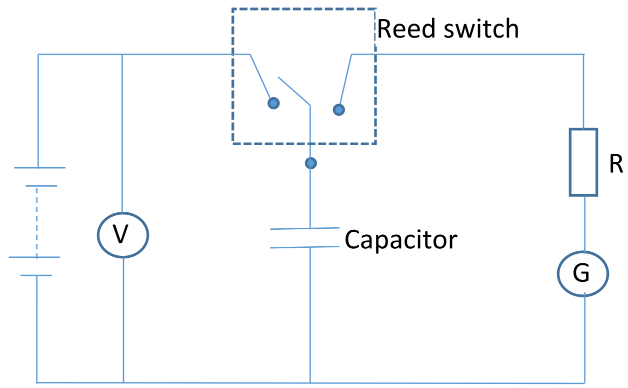
Plate P_2 is restored to its initial position, and it is then moved closer to P_1. The divergence of the leaf is seen to decrease. This shows the p.d between the plates has decreased. From C = Q/V, this show that C has increased. Thus C increases as d decreases. Thus **C ∝ 1/d**

(e) To show that C ∝ε

The position of P2 is again restored and dielectric (an insulator) like paper is inserted between the plates. The divergence of the leaf decreases. This shows that V has decreased. From C = Q/V, C has increased. Thus **C ∝ε.**

Experiment to describe the factors affecting capacitance of capacitor using a red switch

(Factors which determine the capacitance of a capacitor using reed switch)



The capacitor is alternatively charged and discharged trough a sensitive light beam galvanometer, G by the reed switch.

(a) Keeping the separation and the plate area constant, the capacitor is given a charge Q for different p.d. (V).
  - It is found that charge is proportional to V ($Q \propto V$)

(b) Keeping the p.d V and plate area constant, the deflection θ is measured for different separation, d.
  - Results show that charge is proportional to deflection. $C \propto 1/d$

(c) Keeping the p.d V and the separation constant, the deflection θ is measured for different overlapping plate area A. The results show that $C \propto A$

(d) Keeping the area, separation of the plates, and p.d constant; the charge is measured with a dielectric constant.
  - The results show that the charge increases, implying that C also increases. Thus **$C \propto \varepsilon$.**

Capacitance of a parallel plate capacitor

Consider a parallel-plate capacitor above, where the charge on either plate is Q and the p.d between them is V. the surface density σ is then $\frac{Q}{A}$

Wher A is the area of either plate, and the intensity between the plates, E = $\frac{\sigma}{\varepsilon} = \frac{Q}{\varepsilon A}$

The work done in taking a unit charge from one plate to another = force x distance

= Ed (d = the distance between the plates)

But the work per unit charge = V, the p.d between the plates.

$\therefore$ V = $\frac{\sigma}{\varepsilon}d = \frac{Q}{\varepsilon A}d$

$\therefore \frac{Q}{V} = \frac{\varepsilon A}{d}$

$\therefore C = \frac{\varepsilon A}{d}$

It should be note that the formula for C is approximate, as the field becomes non-uniform at the edges

Capacitance of isolated sphere

An isolated metal sphere acts as a capacitor. The sphere itself is one plare; the earth is the other plate.

Suppose a sphere of radius r meter situated in air is give a charge of Q coulombs. We assume that the charge on a sphere gives rise to potentials on and outside the sphere as if all the charge were concentrated at the center.

Then V = $\frac{Q}{4\pi\varepsilon_0 r}$

$\therefore \frac{Q}{V} = 4\pi\varepsilon_0 r$

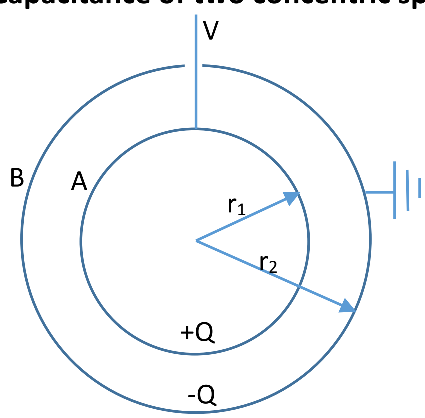
C = $4\pi\varepsilon_0$

Example 1

Suppose r = 10cm = 0.1m, then

C = $4\pi\varepsilon_0 r$ = 4π x 8.85 x 10^{-12} x 0.1= 11 x 10^{-12}F = 11pF

Capacitance of two concentric spheres



Faraday used two concentric spheres to investigate the dielectric constant of liquids. r_1 and r_2 are the respective radii of the inner and outer spheres as shown above and the outer sphere earthed, with air between them.

Let +Q be the charge on the inner sphere, the induced charge on outer sphere = -Q.

The potential of the inner sphere = potential due to +Q + potential due to –Q

i.e. $V_a = \frac{+Q}{4\pi\varepsilon_{0r_1}} + \frac{-Q}{4\pi\varepsilon_{0r_2}} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

sincw the outer sphere is earthed $V_b = 0$

therefore the potential differnce V = $V_a - V_b = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - 0 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

thus V = $\frac{Q(r_2 - r_1)}{4\pi\varepsilon_0 r_1 r_2}$

From C = $\frac{Q}{V}$, then capacitance, C = $\frac{4\pi\varepsilon_0 r_1 r_2}{(r_2 - r_1)}$

Example 2

Suppose r_2 = 10cm = 0.1m and r_1 = 9cm = 0.09m

C = $\frac{4\pi\varepsilon_0 r_1 r_2}{(r_2 - r_1)} = \frac{4\pi \; x \; 8.85 \; x \; 10^{-12} \; x \; 0.1 \; x \; 0.9}{(0.1 - 0.09)}$ = 100pF

Relative permitivity, ε_r or dielectric constant

It is defined as the ratio of permitivity of a substance to the permitivity of free space

$$\varepsilon_r = \frac{capacitance\ of\ a\ given\ capacitor, with\ space\ between\ plates\ filled\ with\ dielectric}{capacitance\ of\ the\ same\ capacitor\ in\ plates\ in\ vacuo}$$

Taking the case of a parallel plate capacitor as an example, then

$\varepsilon_r = \frac{\varepsilon A/d}{\varepsilon_0 A/d} = \frac{\varepsilon}{\varepsilon_0}$

Relative permitivity is a ratio without units.

Dielectric strength

It is the maximum potential gradient that can be applied to a dielectric material without causing its insulation to break down.

Or

It is the potential gradient at which insulation of dielectric breaks downn and spark passes through it.

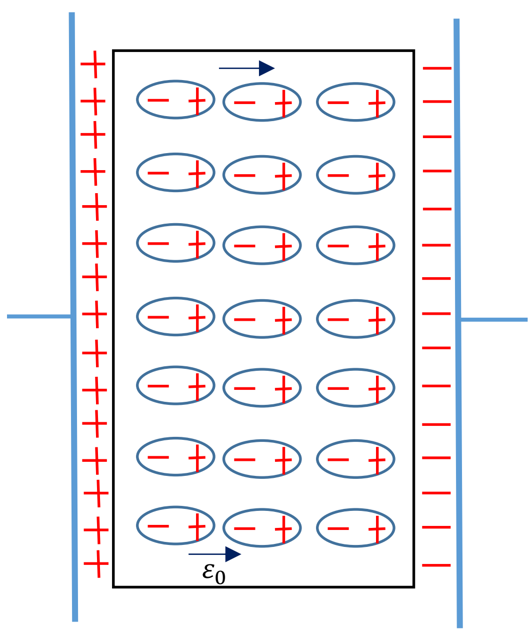
Note: once the insulation of dielectric breaks down, it becomes a conductor anf the capacitor becomes useless since it can long store charge.

Effect of dielectric between the plates of a charged capacotor

Introducing a **dielectric** into a capacitor decreases the electric field, which decreases the voltage, which increases the capacitance. A capacitor with a **dielectric** stores the same charge as one without a **dielectric**, but at a lower voltage. Therefore a capacitor with a **dielectric** in it is more effective.

Dielectric, insulating material or a very poor conductor of electric current. When dielectrics are placed in an electric field, practically no current flows in them because, unlike metals, they have no loosely bound, or free, electrons that may drift through the material.

Instead, electric polarization occurs. The positive charges within the dielectric are displaced minutely in the direction of the electric field, and the negative charges are displaced minutely in the direction opposite to the electric field as shown in the figure below. This slight separation of charge, or polarization, reduces the electric field within the dielectric.



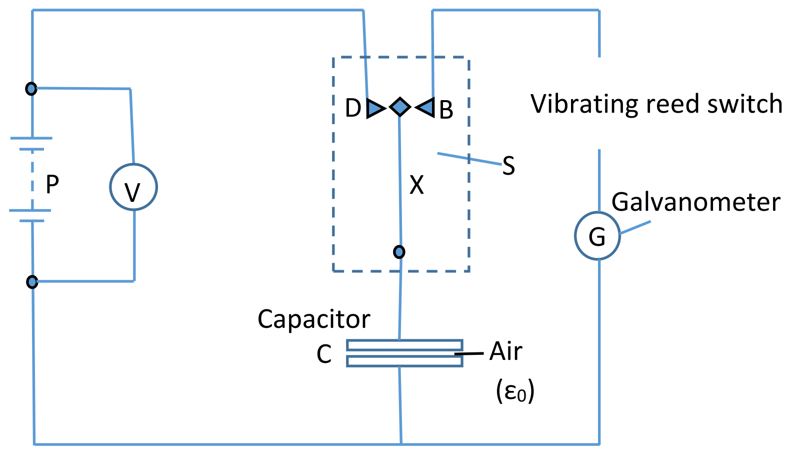
ε_0 and its measurement

Units of ε_0

From C = $\frac{\varepsilon_0 A}{d}$; $\varepsilon_0 = \frac{Cd}{A}$

Thus, the units of $\varepsilon_0 = \frac{farad\ x\ meter}{meter^2}$ = faradmeter^{-1}

Measurement of ε_0



The circuit above is used.

C is a parallel capacitor with are in between the plates of area A (m^2) and separation, (d m)

P is a high tension of about 200V,

G is a sensitive galvanometer,

S is a vibrating switch unit, energized by a low a.c. voltage from the mains.

When operating, the vibrating bar X touches D and then B, and the motion is repeated at the mains frequency, fifty times a second.

When the switch is in contact with D, the capacitor is charged from the supply P to a potential difference of V volts, measured on a voltmeter.

When the contact moves over to B, the capacitor discharges through the galvanometer. The galvanometer thus receives fifty pulse of charge per second.

This gives an average steady current I

From C = $\varepsilon_{0A/d}$

On charging the charge stored, Q = CV = $\frac{\varepsilon_0 VA}{d}$

The capacitor is discharged fifty times per second. And since current is the charge flowing per second

I = $\frac{\varepsilon_0 VA.50}{d}\ ampere$

$\varepsilon_0 = \frac{Id}{50VA}$ farad meter^{-1}

The following results were obtained in one experiment

A = 0.0317m^2, d = 1.0cm = 0.02m, V = 150V, I = 0.21 x 10^{-6}A

$\varepsilon_0 = \frac{Id}{50VA} = \frac{0.21 \, x \, 10^{-6} \, x \, 0.01}{50 \, x \, 150 \, x \, 0.0317} = 8.8 \text{ x } 10^{-12}$ farad meter^{-1}

This method can also be used to find the permittivity of various material. Thus if the experiment is repeated with a material of permittivity ε completely filling the space between the plates, then

$\varepsilon = \frac{I\prime d}{50VA}$ where I' is the new current

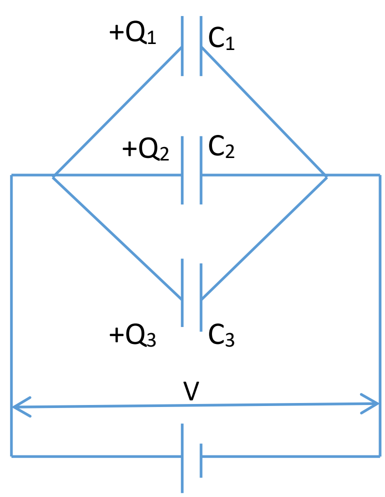
If only the relative permittivity or dielectric constant ε_r is required, there will be no need to know p.d or dimension of the capacitor. In this case

$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{I'd/50VA}{Id/50VA} = \frac{I\prime}{I}$

Thus ε_r is the ratio of respective currents in G with and without the dielectric between plates.

Arrangement of capacitors

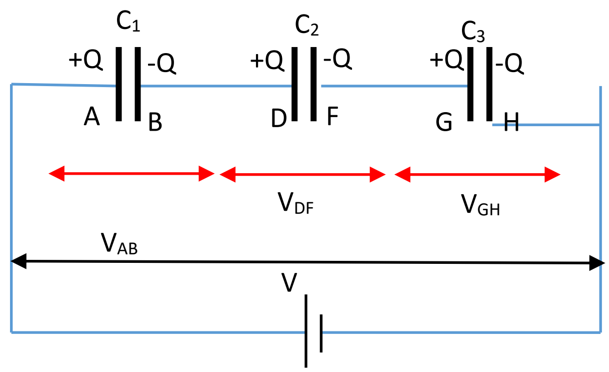
(a) Parallel connection



In parallel each capacitor experiences the same p.d.
For charged capacitor, Q = CV
Therefore, $Q_1 = C_1V$; $Q_2 = C_2V$; $Q_3 = C_3V$;
The total charge, $Q = Q_1 + Q_2 + Q_3 = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

If C is the capacitance of a single capacitor which has the same effect as these three capacitors store charge equal to Q then,
$Q = CV = V(C_1 + C_2 + C_3)$
$C = \frac{Q}{V} = (C_1 + C_2 + C_3)$
$C = (C_1 + C_2 + C_3)$
Therefore for capacitors in parallel, the total capacitance is the sum of individual capacitances of individual capacitor.

(b) Series connection



On connection, the battery draws electrons from plate A and leaves it with a positive charge +Q. this induces a charge –Q on B. A and D form an isolated conductor and the charge that appears on B results from electrons moving from D to B. D is left with charge +Q and this induces a charge –Q on F, and so on.

The p.d across each capacitor is given by

$V_{AB} = \frac{Q}{C_1}; V_{DF} = \frac{Q}{C_2};\ V_{GH} = \frac{Q}{C_2};$

Total voltage, V = V_{AB} + V_{DF} + V_{GH}

$\frac{Q}{C} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

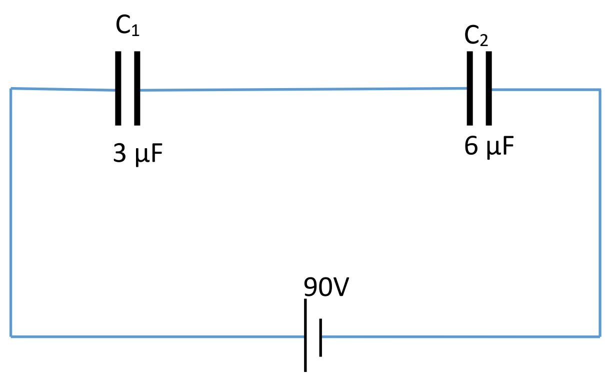
$From\ which\ \frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

Comparison of capacitors and resistor in network

	Capacitors in network	Resistors in network
Series connection	$\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$	$R = (R_1 + R_2 + R_3)$
Parallel connection	$C = (C_1 + C_2 + C_3)$	$\frac{1}{R} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

Example 3

In the figure below, two capacitors of capacitances, C_1 and C_2 of 3μF and 6μF respectively are connected across a 90V d.c. calculate the charge on C_1 and C_2, and the p.d across each.



Solution

The effective capacitance, c is obtained from

$\frac{1}{C} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{1}{3} + \frac{1}{6}$

C = 2μF

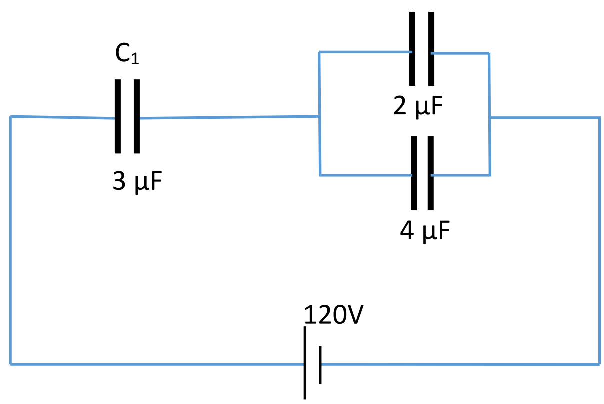
Q = CV =2 x 10^{-6} x 90 = 1.8x 10^{-4} C

For C_1; V_1 = $\frac{Q}{C_1} = \frac{1.8\,x10^{-4}}{3\,x\,10^{-6}}$ = 60V and.

For C_2; V_2 = $\frac{Q}{C_1} = \frac{1.8\,x10^{-4}}{6\,x\,10^{-6}}$ = 30V

Example 4

Calculate the charge on the capacitors below and the p.d across each.



Solution

The effective capacitance, C_1 for parallel capacitors 2μF and 4μF = 2 + 4 = 6μF.

Now 3μF and 6μF are in series and their effective capacitance C is given by

$\frac{1}{c} = \frac{1}{3} + \frac{1}{6}$

C = 2μF

For the whole circuit, Q = CV = 2 x 10^{-6} x 120 = 2.4 x 10^{-4}C

The charge 3μF capacitor = 2.4 x 10^{-4}C

The p.d across the 3μF capacitor = $\frac{2.4 \ x \ 10^{-4}}{3 \ x \ 10^{-6}}$ = 80V

The p.d across each either 2μF or 4μF = 120 – 80 = 40V

From Q = CV

The charge on 2μF capacitor = 40 x 2 x 10^{-6} = 8 x 10^{-5} C

Charge on 4μF capacitor = 40 x 4 x 0^{-6} = 1.6 x 10^{-4} C

Energy stored in a charged capacitor

A charged capacitor stores electrical energy. To find the energy stored, we assume that a capacitor of capacitance C is already charged to a potential difference V.

When the charge on the plates is increased from Q to Q + δQ where δQ is very small, then a charge δQ is transferred from the negative to the positive plate.

The increase in potential difference δV =$\frac{\delta Q}{C}$

Since δQ is very small compared with Q, δV will be very small compared to V and the potential difference will be almost constant at value V.

Then the work done, δW, in displacing the charge δQ is given by

δW = VδQ

Since V = $\frac{Q}{C}$; δW = $\frac{Q}{C}$δQ

For a fully discharged capacitor charged to a value Q1, work done is given by

$\int_{Q=0}^{Q=Q_1} \delta W \ = \int_0^{Q_1} \frac{Q}{C} \delta Q = \left[\frac{1}{2}\frac{Q^2}{C}\right]_o^{Q_1} = \frac{Q^2}{2C}$

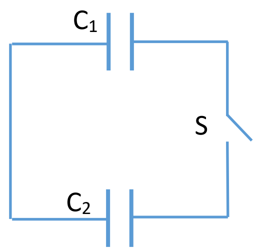
In general, the energy stored by a capacitor of capacitance C, carrying a charge Q at a potential difference V is

W = $\frac{Q^2}{2C}$ = $\frac{1}{2}QV$

  = $\frac{1}{2}CV^2$

If C is in farad, Q in coulomb and V in volt, then W is in joules.

Joining two charged capacitors



When two capacitors are joined together by means of switch S, charge flows from one capacitor with greater p.d to the other until the p.d across each is the same.

The final p.d and energy of each can be calculated by considering the following

- There is no change in the total charge
- Two capacitors acquire the same p.d
- Capacitors are considered to be in parallel and therefore the effective capacitance of the combination, $C = C_1 + C_2$

Note that the final energy stored by the capacitors is less that the initial energy before the capacitors are joined together. This is because some energy is dissipated as heat in the connecting wires when the charge flows from one capacitor to another.

Example 5

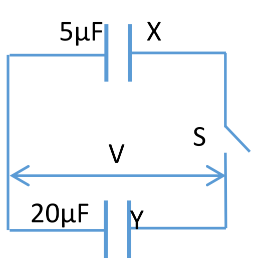
A 5μF capacitor X is charged by a 40V supply and then connected across an charged an uncharged 20μF capacitor Y. Calculate

(a) Final p.d across each
(b) Final charge on each, and
(c) Initial and final energies stored by the capacitors.

Solution

The initial charge, $Q_0 = CV = 5 \times 10^{-6} \times 40 = 2 \times 10^{-4}$C

After connection



Since the capacitors are in parallel, the effective capacitance, $C = C_1 + C_2 = 5 + 20 = 25$μF

Initial charge = final charge

Let V be the final voltage, then

$2.0 \times 10^{-4} = 25 \times 10^{-6}V$

V = 8V

(b) charge on 5μF capacitor = CV = $8 \times 5 \times 10^{-6} = 4.0 \times 10^{-5}$C

Charge of 20μF capacitor = CV = $20 \times 20^{-6} \times 8 = 1.6 \times 10^{-4}$C

(c) from W = $\frac{1}{2}CV^2$

Initial energy =$\frac{1}{2}\ x\ 5\ x\ 10^{-6}\ x\ 40^2$ = 0.004J

Final energy = $\frac{1}{2}\ x\ 25\ x\ 10^{-6}\ x\ 8^2$ = 0.0008J

The final energy is less than the initial energy because some energy is lost as heat due to flow of charge from 5μF to 20μF

Example 6

Two parallel capacitors, one with air as dielectric and the other with mica as dielectric, are identical in all other respect. The air capacitor is charged from 400V d.c supply, isolated and then connected across the mica capacitor which is initially uncharged.

The p.d across this parallel combination becomes 50V. Assuming the relative permittivity of air to be 1.00, calculate the relative permittivity of mica.

Solution

Let C_a = capacitance of the air capacitor, C_m = capacitance of mica capacitor

Charge stored by air capacitor, $Q_a = C_aV = 400C_a$ ………………….(i)

For air = $\frac{C_a}{C_0}$ =1, where C0= capacitance of free space or vacuum.

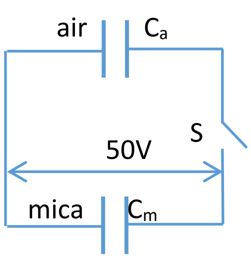
Then, $C_a = C_0$ and,

$Q_a = 400C_a$

Charge store in mica capacitor before connection = 0

Therefore total initial charge = $400C_a + 0 = 400C_a$…………….(ii)

After connection



Total charge before = total charge after

$400C_a = 50(Ca + C_m)$

Since $C_a = C_0$

$400C_0 = 50(C_0 + C_m)$

$7C_0 = C_m$

$\frac{C_m}{C_0} = 7$

Therefore relative permittivity of mica = 7

Example 7

(a) Define a capacitance of a capacitor.
(b) Explain how, using a capacitor in conjuction with a gold-leaf electroscope, the voltage sensitivity of electroscope may be increased.
(c) Two capacitors, of capacitance 4μF and 2μF respectively are joined in series with a battery of e.m.f 100volts. The connections are broken and the terminals of the capacitors are then joined. Find the final charge on each capacitor.

Solution

The combined capacitance, C, of capacitors is given by

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$C = \frac{4}{3}\ \mu F$

The charge on each capacitor = charge on the effective capacitance

$= CV = \frac{4}{3} x\ 10^{-6}\ x\ 100 = \frac{400}{3} x\ 10^{-6} coulomb$

When like terminals are joined together, the p.d across each capacitor, which is different at first, becomes equilized. Suppose it reaches a p.d V, as the total charge remains constant.

Initial toatal charge = $(\frac{400}{3} + \frac{400}{3})\ x\ 10^{-6}$ = final charge total chagre = $4 \times 10^{-6}V + 2 \times 10^{-6} V$

$4 \times 10^{-6}V + 2 \times 10^{-6} V = \frac{800}{3}\ x\ 10^{-6}$

$V = \frac{400}{9}$ volts,

Therefore, the charge on 4μF capacitor = CV = $4 \times 10^{-6} \times \frac{400}{9} = 1.78 \times 10^{-4}C$

The charge on 2μF capacitore = CV = $2 \times 10^{-6} \times \frac{400}{9} = 8.89 \times 10^{-5}C$

Example 8

(a) Define the electrostatic potential of an isolated conductor.
(b) Obtain an expression relating the enrgy of a charged conductor to the charge on it and its capacitance
(c) Two insulated spherical conductors of radii 5.00cm and 10.00cm are charged to potentials of 600volts and 300 volts respectively. Calculate
    (i) the total energy of the system.
    (ii) The energy after the spheres have been connected by a fine wire.
    (iii) Comment on the difference between the two results in C(i) and C(ii).

Solution

(a) The electrostatic potential of an isolated conductor is work done to move a unit positive charge from infinity to a point in an electric field of the conductor.
(b) Refer to the notes
(c) (i) Capacitance C of a sphere of radius r = $4\pi\varepsilon_0 r$

For 5cm = 0.05m radius, $C_1 = 4\pi\varepsilon_0\ x\ 0.05 = \frac{0.05}{9\ x\ 10^9} = \frac{5}{9}\ x\ 10^{-11}F$

Since $4\pi\varepsilon_0 = \frac{1}{9\ x\ 10^9}$

For 10cm = 0.1m radius, $C_2 = 4\pi\varepsilon_0\ x\ 0.05 = \frac{0.1}{9\ x\ 10^9} = \frac{10}{9}\ x\ 10^{-11}F$

But work = $\frac{1}{2}CV^2$

Total energy = $\frac{1}{2}\ x\ \frac{5}{9}\ x\ 10^{-11}\ x\ 600^2 + \frac{1}{2}\ x\ \frac{15}{9}\ x\ 10^{-11}\ x\ 300^2$ = 1.5 x 10^{-6}J

(ii) when the spheres are connected by a fine wire, the potentials become equalized

Let V be the common voltage, since Q = CV and the total charge is constant

Original total charge = total final charge

$4\pi\varepsilon_0$(5 x 10^{-2} x 600 + 10 x 10^{-2} x 300) = (5 x 10^{-2}V + 10 x 10^{-2}V)

V = 400volts

Total final energy = $\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$

$= \frac{1}{2}(C_1 + C_2)V^2$

$= \frac{1}{2}\ x \frac{1}{9\ x\ 10^9}\ (5\ x\ 10^{-2} + 10\ x\ 10^{-2})x\ 400^2$

= 13.33J

(iii) When spheres are connected stored energy is less because part of energy is converted into heat because current flows to equalize.

Example 9

(a) Define electrical capacitance.
(b) Describe experiments to demonstrate the factors which determine its value for parallel plate capacitance
(c) The plates of a parallel plate air capacitor consisting of two circular plates each of radius 10cm and laced 2mm apart, are connected to terminals of an electrostatic voltmeter. The system is charged to give a reading of 100 on the voltmeter scale. The space between the plates is then filled with oil of dielectric constant 4.7 and the voltmeter reading falls to 25. Calculate the

capacitance of the voltmeter. You may assume that the voltage recorded by voltmeter is proportional to the scale reading.

Solution

Let V be the initial p.d across the air capacitor, and C1 be the voltmeter capacitance.

Then total charge = CV + C_1V = (C + C_1)V ……………………(i)

When the plates are filled with oil the capacitance increases to 4.7C, and the p.d falls to V_1. But the total charge remains constant.

From (i)

4.7CV_1 + C_1V_1 = (C + C_1)V

(4.7C + C_1)V1 = (C + C_1)V

$\frac{4.7C+C_1}{C+C_1} = \frac{V}{V_1} = \frac{100}{24} = 4$

$0.7\,C = 3C_1$

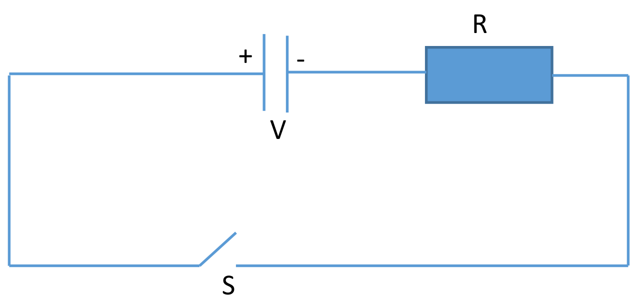
$C_1 = \frac{0.7C}{3} = \frac{7}{30}C$

But C = $\varepsilon_0 A/d$, where A in meter^2 and d in meter

C = $\frac{8.85\,x\,10^{-12}\,x\,\pi\,x\,(10\,x\,10^2)^2}{2\,x\,10^{-3}}$ = 1.4 x 10^{-10} F

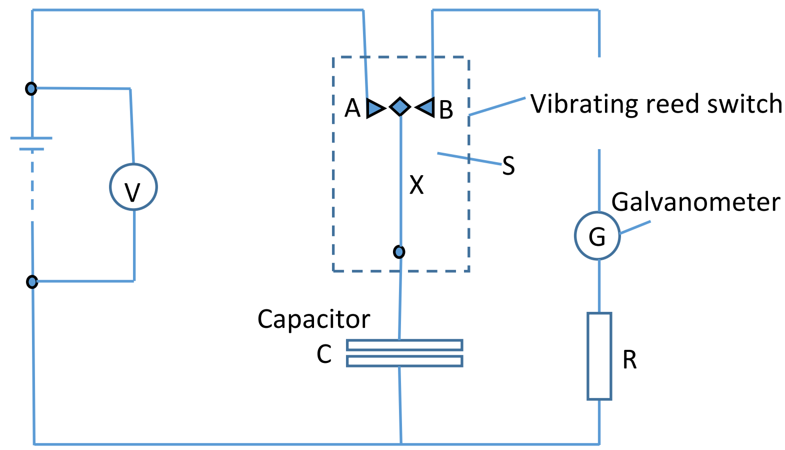
$C_1 = \frac{7}{30}\,x\,1.4\,x10^{-10}$ = 3.3 x 10^{-11} F

Discharging a capacitor through a resistor.



When switch, S is closed, the resistance R limits the current flow and discharge is not intataneous which makes the capacitor to discharge slowly. Thi

Determining capacitance of a capacitorusing a reed switch



- The circuit isconnected as shown above.
- The capacitor is discharged by connecting the contact of the vibrating reed switch X with A.
- The p.d across the capacitor plates is recorded as V from the voltmeter.
- When the contact of the reed switch is ta B, the capacitor discharges rapidly and the current I flows through galvanometer is noted
- If the frequency of the vibrating switch is f;

  I = charge per second= fQ ………………………………. (i)

  But Q = CV

  Eqn (i) becomes I = fCV

  $$C = \frac{I}{fV}$$

  Hence the capacitance of the capacitor can be determined.

Exercise

1. A 10.0μF capacitor charged to 200V is connected across an uncharged 50μF capacitor. Calculate the total energy stored in both capacitors before and after connection **(Ans. 0.2J, 0.033J)**
2. A 2.5μF capacitor is charged to a p.d of 100V and is disconneted from th supply. Its terminals are then connected to shose os an uncgarged 10μF capacitor. Calculate the
   (a) Resulting p.d across the two capacitors. **(Ans. 20V)**
   (b) Initial and final total energy stored in then **(0.0125J, 0.0025J)**
3. Three capacitorsof capaciances 4.0μF, 6.0μF and 12.0μF are connected in series across a 6V d.c supply. Calculate
   (a) The charge on 12.0μF capacitor
   (b) Total energy stored in the capacitors
   (c) P.d across the 4.0μF capacitor
   [(a) 12 x 10-6C, (b) 3.6 x 10^{-5}J, C 3.0V]
4. The capacitance of variable radio capacitor can be charged continuously from 25pF to 500pF by turning the dial from 0^0 to 180^0. With the dial set at 180^0, the capacitor is connected to a 600V

battery. After charging, the capacitor is disconnected from the battery and then the dial is turned to 0^0. Calculate the

(a) Charge on the capacitor’

(b) Energy stored in the capacitor,

(c) The work required to turn the dial from 180^0 if friction I s neglected.

[ans. (a) 3.0 x 10-7C, (b) 9.0 x 10^{-5}J, (c) 1.71 x 10^{-3}J]

Thank You

Compiled by Dr. Bbosa SCience