S6 MOCK 2 EXAMINATIONS 2006
P425/1 MATHEMATICS PAPER 1

TIME: 3 HOURS

**INSTRUCTION TO CANDIDATES:**

Attempt all the **eight** questions in section **A** and not more than **five** from section **B.**

Show all the necessary working up to the last step of the solution.

Mathematical tables and squared papers are provided.

Silent Simple non-programmable calculators may be used.

State the degree of accuracy of questions attempted using calculator or tables.
Indicate **Cal** for calculator or **Tab** for Mathematical tables.

**SECTION A: (40 marks)**

1. A polynomial P(x), when divided by (x + 1) leaves a remainder 3 and when divided by (x – 2 ) leaves a remainder 1.
   Find the remainder when it is divided by (x + 1)(x – 2) (5 marks)

2. Find $\int \frac{x\sin^{-1}x \, dx}{\sqrt{(1-x^2)}}$ (5 marks)

3. A circle, centre C, cuts the circle $x^2 + y^2 - 4x + 6y - 7 = 0$ at right angles and passes through the point (1,3). Find the locus of C. (5 marks)

4. Solve $6\tan^2\theta - 4\sin^2\theta = 1$; for $0^o \leq \theta \leq 360^o$. (5 marks)

5. Air is being pumped in a spherical balloon at a constant rate. If, when the radius is 21m, it is increasing at the rate of $0.01ms^{-1}$, find the rates at which the surface area and volume are increasing at the same time. (5 marks)

6. Given the line $\mathbf{r} = \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 12 \\ 4 \end{pmatrix}$ and the plane $\mathbf{r}.\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = 66$

   Find ;
   (i) the coordinates of their point of intersection,
   (ii) the acute angle between them. (5 marks)

7. By putting $Z = x + \frac{2}{x}$, solve the equation

$$x^4 - 5x^3 + 10x^2 - 10x + 4 = 0.$$ (5 marks)

8. Using the substitution $u^2 = x^3 + 1$, show that

(iii) $\int_1^2 \frac{3dx}{x\sqrt{(x^3 + 1)}} = \log_e \frac{1}{2}(3+2\sqrt{2})$ (5 marks)

**SECTION B: (60 MARKS)**

9. (a) Expand $(1 - 3x)^{1/3}$ in ascending powers of x as far as the term in $x^4$.
By taking $x = {}^1/_8$ evaluate $5^{1/3}$ to three significant figures. (6 marks)

(b) Solve the inequality

$$\frac{x^2 - 3x + 1}{x - 2} > 1$$ (6 marks)

10. Prove that the equation of the tangent to the parabola $y^2 = 4ax$ at the point $T(at^2, 2at)$ is $ty = x + at^2$. The tangents to the above parabola at points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ intersect at R, find the coordinates of R. If the tangent at P and Q are inclined to one another at an angle of $45^o$, Show that the locus of R is the curve $y^2 = x^2 + 6ax + a^2$ (12 marks)

11. (a) The position vectors of the points A and B are $2\mathbf{i} - \mathbf{j}$ and $-\mathbf{i} + 3\mathbf{j}$ respectively with respect to the origin O. Find the position vector of point H in the plane OAB such that **OH** is perpendicular to **AB** and **AH** is perpendicular to **OB** (6 marks)

(b) Given two sets of collinear points $OAA^1$ and $OBB^1$ with $\mathbf{OA} = \mathbf{a}$, $\mathbf{OA}^1 = \mathbf{a}$, $\mathbf{OA}^1 = \lambda\mathbf{a}$, $\mathbf{OB} = \mathbf{b}$, $\mathbf{OB}^1 = \mu\mathbf{b}$ where the vectors **a** and **b** are not parallel, show that the vector **OX** where X is the point of intersection of $\mathbf{AB}^1$ and $\mathbf{A^1B}$ is given by

$$OX = \frac{\lambda(1 - \mu)}{1 - \lambda\mu}\mathbf{a} + = \frac{\mu(1 - \lambda)}{1 - \lambda\mu}$$

Provided $1 - \lambda\mu \neq 0$ (6 marks)

12. Express $\frac{2x^2 + 14x + 10}{2x^2 + 9x + 4}$ in partial fractions.

Hence determine

$$\int \frac{2x^2 + 14x + 10}{2x^2 + 9x + 4}dx$$ (12 marks)

13. (a) Given that y = 2 when x = 0, solve the differential equation.

$$\frac{dy}{dx} + \frac{2xy}{1 + x^2} = \text{Cos}x$$ (6 marks)

(b) A body of unit mass falls under gravity in a medium in which the resistance is proportional to the velocity of the body. If the body was initial at rest, show that the velocity V after time t is $^{g}/_{k}\,(1 - e^{-kt})$, where k is a constant of proportionality.
(6 marks)

14. The top of a tower is observed simultaneously from two points A and B, at the same level, A being at a distance C due north of B. From A the bearing of the tower is θ east of south at an elevation α and from B the bearing of ϕ east of north. Show that the height of the tower is
$$\frac{C \tan \alpha \, \text{Sin}\phi}{\text{Sin}\,(\Theta+\phi)}$$ and find the elevation of the top of the tower from B. (12 marks)

15. (a) Given the complex number Z, find the roots of the equation $Z^3 + 64 = 0$ and show these roots on an Argand diagram.

(b) If $Z_1 = 2 + i$ , $Z_2 = 4 + 3i$ and A and B are points representing $Z_1$ and $Z_2$ on an argand diagram, find the complex numbers $C_1$ and $C_2$ where $ABC_1$ and $ABC_2$ are equilateral triangles. (6 marks)

16. (a) If $y = e^{\tan^{-1}x}$; show that $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx}$

(b) The parametric equations of a curve are x = Cos2Θ , y =1 + Sin 2Θ. Find $\frac{dy}{dx}$ ad $\frac{d^2y}{dx^2}$ at $\Theta = {}^{\pi}/_{6}$ . Find also the equation of the curve as a relationship between x and y .

*End*