P425/2
APPLIED MATHEMATICS
PAPER 2
3 HOURS

S6 BEGINNING OF TERM II 2007 EXAMS
APPLIED MATHEMATICS
PAPER 2
3 hours

Answer **all** questions in section A and any five from section B.

In numerical work take g = $9.8ms^{-2}$

1. A particle of mass 3 kg is acted upon by a force $\mathbf{F} = 6\mathbf{i} - 36t^2\mathbf{j} + 54\mathbf{k}$N. At time t = 0, the particle is at the point with position vector $\mathbf{i} - 5\mathbf{j} - \mathbf{k}$ m and its velocity is $3(\mathbf{i} + \mathbf{j})ms^{-1}$. Find the distance of the particle from the origin at t = 1s.

2. In a mathematics examination of last year, 30% failed and 10% scored a distinction. If the pass mark was 25% for a certain paper and the minimum mark required for a distinction was 80%, estimate the mean mark and standard deviation (Assume the marks to be normally distributed).

3. A scientist, working in an agricultural research station, believes that there is a relationship between the hardness(y) of the shells of the eggs laid by chickens and the amount (x) of a certain food supplement put into the diet as given in the table below.

x	7.0	9.8	11.6	17.5	7.6	8.2	12.4	17.5	9.5	19.5
y	1.2	2.1	3.4	6.1	1.3	1.7	3.4	6.2	2.1	7.1

(i) Find spearman's rank correlation coefficient between x and y
(ii) Do you believe the scientist's claim? Justify your answer.

4. A uniform beam AB of weight 20N can turn freely in a vertical plane about a hinge at A and the other end B is tied to a rope whose free end is fixed to a point C vertically above A such that AC = AB. Find the:
   (i) tension in the rope necessary to keep the beam at 60^o to the horizontal with B lower than A.
   (ii) components of the reaction at the hinge.

5. The table below shows values of f(x) for given values of x in the interval 0.2(0.2)0.8

x	0.2	0.4	0.6	0.8
f(x)	– 0.72	–0.28	0.32	1.08

Use linear interpolation and or extrapolation to

(i) find x such that f(x) = 0,

(ii) evaluate f(1.0).

6. The random variable x is the distance in metres, that a trainee soldier has moved along a given tight-rope before falling off. It is given that $p(x>x) = 1 - \frac{x^3}{64}$, $0 \leq x \leq 4$

(i) Show that E(x) = 3,

(ii) Find the standard deviation and establish that $p(\, |x - 3| < \delta) = \frac{69}{80}\sqrt{\frac{3}{5}}$

7. Show graphically that the equation $x^3 - 2x - 5 = 0$ has one real root. State the root of this equation correct to one decimal place.

8. Forces of 1,4,3 and 5N act along the sides AB, BC, CD and DA respectively of a square of side 2m. A force P acts at A, such that the system reduces to a couple.
Find the value of P, its direction and the moment of the couple.

SECTION B: (60 MRKS)

9. A brick of mass 2 kg falls freely down a well. After falling 98m from rest, it strikes the surface of the water. The brick then continues to descend through the water with acceleration $^1/_3$ g, where g is the acceleration due to gravity, until it reaches the bottom of the well 37.5m below the surface of the water. Given that the impulse at the surface of the water on the brick would completely destroy the momentum of a body of mass 0.5kg which had freely fallen 32m from rest, calculate:

(i) the time the brick takes to travel from rest to the bottom of the well,

(ii) the momentum of the brick when it reaches the bottom of the well.

10. (i) Show that the equation 2Sinx – xcosx –1 = 0 has a root in the interval $x = \frac{1}{4}\pi$ and $x = \frac{1}{3}\pi$

(ii) Derive the Newton-Raphson formula for finding the root of the equation 2Sinx – xcosx –1 = 0.

(iii) Use your formula in (ii) above to find the root of the equation with an error of less than 0.001.

11. A ship A is travelling on a course of 060^o at a speed of $30\sqrt{3}$ Kmh^{-1} and a ship B is travelling on a course of 030^o at $20Kmh^{-1}$. At noon B is 260km due east of A.
Find the;
(i) least distance between A and B to the nearest km,
(ii) time taken to be nearest.

12. (a) Find the position vector of the centre of gravity of three particles of weight 7N, 9N and 4N with position vectors $4\mathbf{i} + \mathbf{j}$, $3\mathbf{j}$ and $3\mathbf{i} + 4\mathbf{j}$
(b) PQRS is a square lamina of side a, from which triangle PST is removed, T being a point in RS distant c from R. Show that the centre of gravity of the remaining lamina is distance $\frac{a^2 + ac + a^2}{3(a+c)}$ from QR.
Deduce that, if the lamina be placed in a vertical plane with RT resting on a horizontal table, equilibrium will not be possible if c is less than the positive root of $2x^2 + 2ax - a^2 = 0$

13. (a) A small particle of mass 0.5kg, attached by a string fixed to a point O, describes a horizontal circle at a uniform angular speed ω $rads^{-1}$ below O. If the length of the string is 50 cm and the greatest tension that can with safety be allowed in the string is 250N, find the
(i) value of ω,
(ii) inclination of the string to the vertical at the instant the string snaps.

(b) A particle is moving in a straight line with simple harmonic motion. At t = 0, its equation of motion is $\ddot{x} = -16x$, its velocity is $4\sqrt{3}ms^{-1}$ and its displacement from the centre of motion x = 3m. Find;
(i) the amplitude of the oscillation,
(ii) the least positive value of t for which the velocity is zero,
(iii) x when $t = \frac{3}{4}\pi$

14. (a) The germination of bean seeds is not easy. From experience, Mpanga the expert bean grower, knows that on average only 40% of the seeds germinate. Six seeds are planted.
Determine:
(i) the probability that only one seed germinates,
(ii) the most likely number of germinating seeds.

(b) Given $X \sim B(12,0.5)$
Find
(i) $P(2 \,\text{¸} x \leq 5)$
(ii) $P(3 < x < 7)$
(iii) $P(x = 2)$

15. (a) Chicken eggs have mean mass 60g with standard deviation 15g, and the distribution of their masses may be taken to be normal. Eggs of less than 45g are classified as ‘small’. The rest are classified as either ‘standard’ or ‘large’. It is desired that ‘small’ and ‘large’ should

occur with approximately equal frequency. Suggest the mass at which the division between standard and large should be made.

(b) A random sample of 16 observations is to be drawn from a normal distribution having mean 11 and variance 9. If $\overline{X}$ is the sample mean, find

(i) $P(9.2 < \overline{X} < 12.2)$

(ii) The value of C for which $P(\overline{X} < C) = 0.95$.

16. (a) Mr. Lambert frequently allows his daughter to borrow his car. When he leaves the car at home after driving it, the amount of petrol in the tank is uniformly distributed between 10 litres and 50 litres. When his daughter leaves the car at home after having borrowed it, the amount of petrol in the tank is uniformly distributed between 0 and 20 litres. Given that they randomly choose who is to drive, find the probability that there is less than 15 litres of petrol in the tank on the day when Mr Lambert checks the car tank and he is not very sure of who drove the car last.

(b) The continuous random variable T uniformly distributed on the interval $0 < t < 100$
Find $P(\,|T - \mu| < \delta)$. Given that $\mu = E(T)$ and $\delta^2 = Var(T)$.