

## UACE Physics paper 1 2003 Guide

Time 2½ marks

Instructions the candidates:

Answer **five** questions, including at least **one**, but **not more than two** from each sections **A, Band C**.

Any additional question(s) answered will not be marked.

Non programmable scientific calculators may be used.

Assume where necessary

Acceleration due to gravity, $g$	$9.81\text{ms}^{-2}$
Electron charge, $e$	$1.6 \times 10^{-19}\text{C}$
Electron mass	$9.11 \times 10^{-31}\text{kg}$
Mass of the earth	$5.97 \times 10^{24}\text{kg}$
Plank's constant, $h$	$6.6 \times 10^{-34}\text{Js}$
Stefan's-Boltzmann's constant, $\sigma$	$5.67 \times 10^{-8}\text{Wm}^{-2}\text{K}^{-1}$
Radius of the earth	$6.4 \times 10^6\text{m}$
Radius of the sun	$7 \times 10^8\text{m}$
Radius of the earth's orbit about the sun	$1.5 \times 10^{11}\text{m}$
Speed of light in the vacuum, $c$	$3.0 \times 10^8\text{ms}^{-1}$
Thermal conductivity of copper	$390\text{Wm}^{-1}\text{K}^{-1}$
Thermal conductivity of aluminium	$210\text{Wm}^{-1}\text{K}^{-1}$
Specific heat capacity of water	$4.200\text{Jkg}^{-1}\text{K}^{-1}$
Universal gravitational constant	$6.67 \times 10^{-11}\text{Nm}^2\text{Kg}^{-2}$
Avogadro's number, $N_A$	$6.02 \times 10^{23}\text{mol}^{-1}$
Surface tension of water	$7.0 \times 10^{-2}\text{Nm}^{-1}$
Density of water	$1000\text{kgm}^{-3}$
Gas constant, $R$	$8.31\text{Jmol}^{-1}\text{K}^{-1}$
Charge to mass ratio, $e/m$	$1.8 \times 10^{11}\text{Ckg}^{-1}$
The constant, $\frac{1}{4\pi\epsilon_0}$	$9.0 \times 10^9\text{F}^{-1}\text{m}$
Faraday's constant, $F$	$9.65 \times 10^4\text{Cmol}^{-1}$

## SECTION A

1. (a) Distinguish between fundamental and derived physical quantities. Give two examples of each. (04marks)

**Fundamental quantities** are those physical quantities which cannot be expressed in terms of other quantities using mathematical equations. They include mass (M), length (L) and time (T).

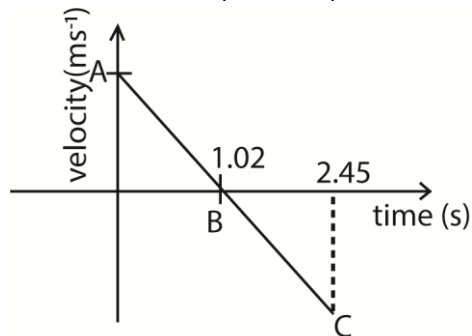
**Derived quantities** are those physical quantities which can be expressed in terms of fundamental quantities, e.g., density, velocity, pressure.

- (b) (i) What is meant by scalar and vector quantities? (02marks)

A **scalar** is a physical quantity with magnitude but no direction.

A **vector** is a physical quantity with both magnitude and direction.

- (ii) A ball is thrown vertically upwards with a velocity  $10\text{m}^{-1}$  from a point 3.0m above ground. Describe with the aid of a velocity-time sketch graph, the subsequent motion of the ball. (10marks)



The body is projected at A, rises with uniform acceleration up to B which is at a vertical height,  $h$ , from the point of projection. At B,  $v = 0$

$$\text{From } v^2 = u^2 + 2as$$

$$0 = 10^2 - 2 \times 9.81h$$

$$h = 5.1\text{m}$$

the time taken to cover the height,  $h$ , is obtained from;  $v = u + at$

$$0 = 10 - 9.81t$$

$$t = 1.02\text{s}$$

From B, the ball falls with uniform acceleration and if it takes time,  $t$  s to strike C.

$$\text{From } y = ut + \frac{1}{2}at^2$$

$$-5 = 10t - 4.9t^2$$

$$t = 2.45\text{s}$$

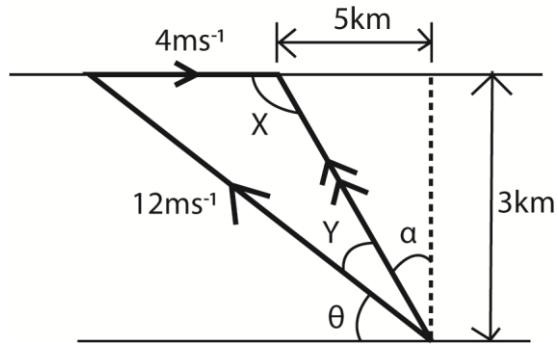
Velocity at C

$$\text{From } v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.81 \times 10.1$$

$$v = 14.1\text{ms}^{-2}$$

- (c) A boat crosses a river 3km wide flowing at  $4\text{ms}^{-1}$  to reach a point on the opposite bank 5km upstream. The boat's speed in still water is  $12\text{ms}^{-1}$ . Find the direction in which the boat must be headed. (04marks)



$$\alpha = \tan^{-1} \frac{5}{3} = 59.04^\circ$$

$$X = 90 + 59.04 = 159.04$$

Applying the sine rule to the vector triangle OQR

$$\frac{\sin Y}{4} = \frac{\sin 149.04}{12}; Y = 9.87^\circ$$

$$\text{But } \theta + Y + \alpha = 90^\circ$$

$$\theta = (90 - 9.87 - 59.04) = 21.09^\circ$$

therefore the boat should be steered in a direction making of 21.080 to the river up stream

2. (a) Define the following terms:

(i) Angular velocity (01mark)

Angular velocity is the rate of change of the angle swept out by the radius joining a body to the centre of circular path.

(ii) Centripetal acceleration (01mark)

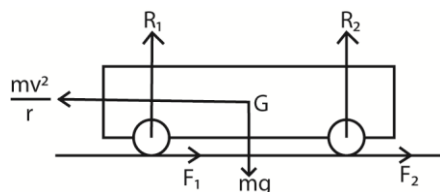
Centripetal acceleration is the rate of change of velocity for a body describing a circular path and is always directed towards the centre of the path.

(b) (i) Explain why a racing car can travel faster on a banked road than on flat track of the same curvature. (04marks)

(ii) Derive an expression for the speed with which a car can negotiate a bend on a banked track without skidding. (03marks)

- **On a flat track**

For Suppose the car is moving with velocity,  $v$ , around a horizontal circular track of radius,  $r$ . if  $m$  is the mass of the car and  $R_1$  and  $R_2$  are normal reactions at the inner and outer wheels respectively and  $F_1$  and  $F_2$  are the corresponding frictional forces, then for circular motion:



$$F_1 + F_2 = \frac{mv^2}{r} \dots\dots\dots (i)$$

For vertical equilibrium

$$R_1 + R_2 = mg \dots\dots\dots(ii)$$

Taking moments about G

Clockwise moments = anticlockwise moments

$(F_1 + F_2)h + R_1 \frac{a}{2} = R_2 \frac{a}{2}$  (a = distance between the wheels, h = the height of the centre of gravity from the ground)

$$(F_1 + F_2)h = \frac{a}{2} (R_2 - R_1) = \dots\dots\dots (iii)$$

Substituting Eqn. (i) into Eqn. (iii)

$$\frac{mv^2 h}{r} = \frac{a}{2} (R_2 - R_1) \dots\dots\dots(iv)$$

From (ii)

$$R_1 = mg - R_2$$

From equation (iv)

$$\frac{mv^2 h}{r} = \frac{a}{2} (R_2 - (mg - R_2)) = \frac{a}{2} (2R_2 - mg)$$

$$2R_2 = \frac{2mv^2 h}{ar} + mg = m \left( \frac{2v^2 h}{ar} + g \right)$$

$$R_2 = \frac{m}{2} \left( \frac{2v^2 h}{ar} + g \right)$$

Also  $R_2 = mg - R_1$

From equation (iv)

$$\frac{mv^2 h}{r} = \frac{a}{2} ((mg - R_1) - R_1) = \frac{a}{2} (mg - 2R_1)$$

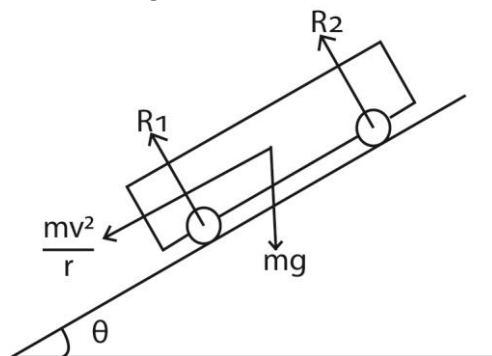
$$2R_1 = mg - \frac{2mv^2 h}{ar}$$

$$R_1 = \frac{m}{2} \left( g - \frac{2v^2 h}{ar} \right)$$

When the car is about to overturn,  $g = \frac{2v^2 h}{ar}$ ,  $R_1 = 0$ ,  $v^2 = \frac{gar}{2h}$

The maximum velocity of a car to negotiate a bend of radius r on a flat track,  $v = \sqrt{\frac{gar}{2h}}$

- On banked ground



Resolving horizontally

$$(R_1 + R_2) \sin \theta = \frac{mv^2}{r} \dots\dots\dots (i)$$

Resolving vertically;

$$(R_1 + R_2) \cos \theta = mg \dots\dots\dots (ii)$$

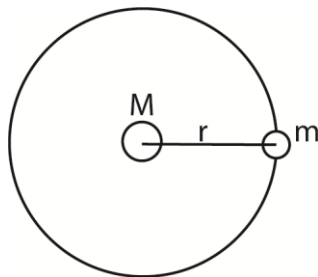
Divide equation (i) by Eqn. (ii)

$$\tan \theta = \frac{v^2}{rg}$$

$$v = \sqrt{rg \tan \theta}$$

- (c) a Show how to estimate the mass of the sun if the period and radius of one of its planets are known. (03marks)

Suppose the sun of mass M and a planet has the mass, m, in a circular orbit of radius, r



The centripetal force is provided by the gravitational force between the sun and the planet.

$$\Rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$M = \frac{v^2 r}{G}; \text{ but } v = \omega r = \left(\frac{2\pi}{T}\right) r$$

$$M = \frac{4\pi^2 r^3}{GT^2}$$

So if r and T are known, the mass of the sun M can be calculated.

- (d) The gravitational potential, U, at the surface of a planet of mass M and radius R is given by  $U = -\frac{GM}{R}$ , where G is gravitational constant.

Derive an expression for the lowest velocity, V, which an object of mass, m, must have at the surface of the planet if it is to escape from the planet. (04marks)

$$W = -\frac{GMm}{\infty} - \frac{-GMm}{R} \text{ but } \frac{GMm}{\infty} = 0$$

$$= \frac{GMm}{R}$$

$$\Rightarrow \frac{1}{2}mv^2 \geq \frac{GMm}{R}$$

$$v = \sqrt{\frac{2GM}{R}} \text{ where } r \text{ is the radius of the earth and } v \text{ is the velocity of projection}$$

- (e) Communication satellites orbit the earth in synchronous orbits. Calculate the height of communication satellite above the earth. (04marks)

If the satellite has a mass,  $m$ , and moves in an orbit of radius,  $r$ , about the earth of mass  $M$ , then

$$\frac{GMm}{R} = \frac{mv^2}{r} \text{ but } v = \omega r = \left(\frac{2\pi}{T}\right)r$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} \text{ but } GM = gr_e^2$$

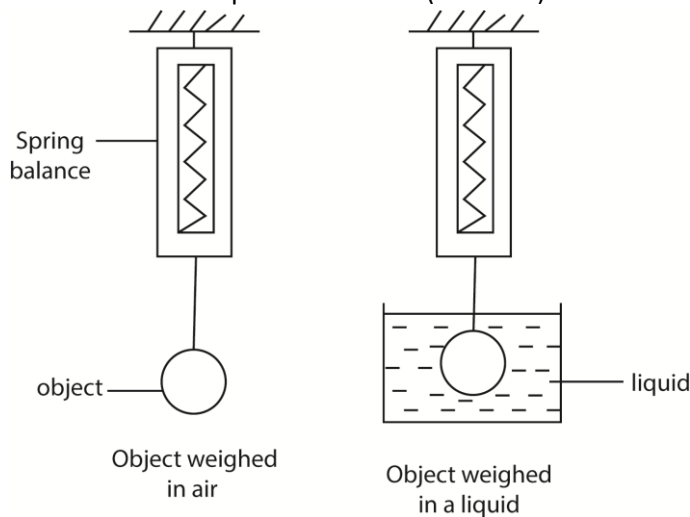
$$r = \sqrt[3]{\frac{gr_e^2 T^2}{4\pi^2}} = \sqrt[3]{\frac{9.81 \times (6.4 \times 10^6)^2 \times (24 \times 60 \times 60)^2}{4\pi^2}} = 4.24 \times 10^7 \text{ m}$$

Hence height above the earth =  $(4.24 \times 10^7 - 6.4 \times 10^6) = 3.6 \times 10^7 \text{ m}$

3. (a) State the law of floatation. (01mark)

The law of floatation state that a floating body displaces its own weight of the fluid in which it floats

(b) With the aid of a diagram, describe how to measure the relative density of a liquid using Archimedes' Principle of moments (06marks)



- A solid is weighed in air =  $M \text{ g}$
- Then a solid  $s$  weighed when totally immersed in a liquid whose relative density is required =  $m_1 \text{ g}$
- Then a solid  $s$  weighed when totally immersed in water =  $m_2 \text{ g}$

Calculations

$$\text{Upthrust in the liquid} = M - m_1$$

$$\text{Upthrust in water} = M - m_2$$

$$\text{Relative density of the liquid} = \frac{M - m_1}{M - m_2}$$

$$\text{Or density of the liquid} = \left(\frac{M - m_1}{M - m_2}\right) \times \text{density of water}$$

(c) A cross sectional area of a ferry at its water-line is  $720 \text{ m}^2$ . if sixteen cars of average mass  $1100 \text{ kg}$  are placed on board, to what extra depth will the boat sink in the water?

(04marks)

Let the extra depth be  $x$ ;

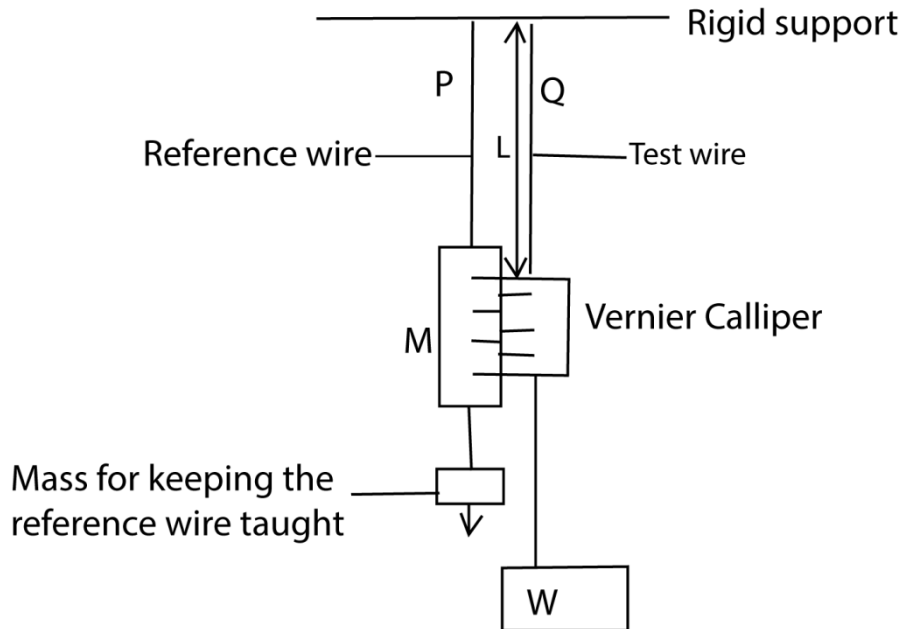
$$\text{From } \rho = \frac{m}{V}; \Rightarrow m = \rho V; \text{ but } V = 720x$$

$$\therefore 1000 \times 729x = 16 \times 110$$

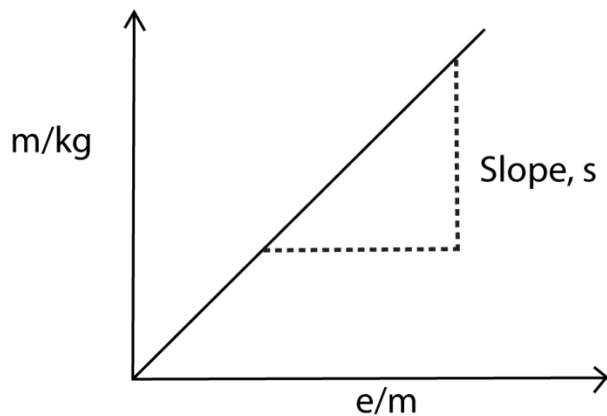
$$x = 0.034m$$

- (d) (i) Define longitudinal stress and Young's Modulus of elasticity. (02marks)  
 Longitudinal stress is the force acting on a cross-section area of  $1m^2$ .  
 (ii) Describe how to determine Young's Modulus for steel wire. (07marks)

**Experiment to determine Young's Modulus for a metal wire**



- (i) Two thin, long wires of the same material and length P and Q are suspended from a rigid support.
- (ii) P carries a scale M in mm and it's straightened by attaching a weight at its end.
- (iii) Q carries a Vernier scale which is alongside scale M
- (iv) Various loads are added to the test wire and corresponding extensions caused are read off from a vernier scale.
- (v) The diameter ( $2r$ ) of the wire is obtained by a micrometre screw gauge, and the cross section area of the wire  $A = 4\pi r^2$
- (vi) A graph of mass ( $m$ ) of the load against extension  $e$  is plotted



Young's modulus,  $Y = \frac{gsL}{A}$

4. (a) A mass of 0.1kg is suspended from a light spring of force constant  $24.5\text{Nm}^{-1}$ . Calculate the potential energy of the mass. (04marks)

Elastic potential energy, P.E =  $\frac{1}{2}kx^2$  ..... (i)

From Hooke's law,  $mg = kx$

$\Rightarrow x = \frac{mg}{k}$  .....(ii)

Substituting eqn. (ii) in eqn. (i)

P.E =  $\frac{(mg)^2}{2k} = \frac{(0.1 \times 9.81)^2}{2 \times 24.5} = 1.96 \times 10^{-2}\text{J}$

- (b) (i) State four characteristics of simple harmonic motion. (04marks)

- The acceleration is always directed towards a fixed point in the motion line
- The acceleration is directly proportional to the displacement from a fixed point
- It is periodic
- Total mechanical energy is conserved.

- (ii) Show that the speed of a body moving with simple harmonic motion of angular velocity,  $\omega$ , is given by  $V = \omega(A^2 - x^2)^{\frac{1}{2}}$ , where A is the amplitude and x is the displacement from equilibrium position. (04marks)

If x is the displacement, from  $a = -\omega^2x$

$a = \frac{dv}{dt} = \frac{dv}{dx} x \frac{dx}{dt} = V \frac{dv}{dx}$

$\Rightarrow V \frac{dv}{dx} = -\omega^2x$

$\int v dv = -\omega^2 \int x dx$

$\frac{v^2}{2} = -\frac{\omega^2x^2}{2} + c$  ..... (i)

But  $x = A, v = 0$

$\Rightarrow c = \frac{\omega^2A^2}{2}$

Substituting c in eqn.(i)

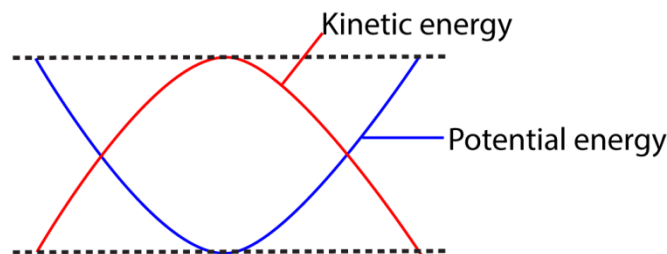
$V^2 = \omega^2(A^2 - x^2)$

Or

$V = \omega(A^2 - x^2)^{\frac{1}{2}}$ ,

- (iii) Sketch graphs to show the variation with displacement, of kinetic and potential energies of a body moving with simple harmonic motion (02marks)

Variation of kinetic and potential energy in S.H.M



- (c) A mass of 0.1kg suspended from a spring of force constant  $24.5\text{Nm}^{-1}$  is pulled vertically downwards through a distance 5.0cm and released. Find the

- (i) period of acceleration (02marks)

From  $T = 2\pi \sqrt{\frac{m}{k}}$



$$T = 2\pi\sqrt{\frac{0.1}{24.5}} = 0.4\text{s}$$

(ii) position of the mass 0.3s after release. (04marks)

From  $x = A\sin(\omega t + \epsilon)$ ; where  $\epsilon$  is the phase angle which is dependent on the position of the particle when the timing starts.

Given that  $A = -5\text{cm} = -5 \times 10^{-2}\text{m}$  (displacement is negative because it is below the equilibrium position).

$$\therefore -5 \times 10^{-2} = 5 \times 10^{-2} \times \sin \epsilon$$

$$\epsilon = \frac{\pi}{2} \text{ radians}$$

At  $t = 0.3\text{s}$

$$x = 5 \times 10^{-2} \sin(0.3\omega - \frac{\pi}{2})$$

$$\text{But } \omega = \frac{2\pi}{T} = \frac{2\pi}{0.4} = 15.7 \text{ rads}^{-1}$$

$$x = 5 \times 10^{-2} \sin(0.3 \times 15.7 - \frac{\pi}{2}) = 1.19 \times 10^{-4} \text{m}$$

## SECTION B

5. (a) (i) Define molar heat capacity of a gas at constant volume. (01mark)

Molar heat capacity of a gas at constant volume is the amount of heat required to raise 1 mole of the gas through 1K at constant volume.

(ii) The specific heat capacity of oxygen at constant volume is  $719\text{Jkg}^{-1}\text{K}^{-1}$ . If the density of oxygen at s.t.p is  $1.429\text{kgm}^{-3}$ , calculate the specific heat capacity of oxygen at constant pressure. (04marks)

$$PV = nRT$$

$$\text{At s.t.p } 1 \times 10^5 V = 273R$$

$$V = 273R \times 10^{-5} \text{m}^3$$

$$\text{Since } \rho = \frac{m}{V}; m = \rho V \dots\dots\dots(i)$$

$$\text{But } mC_p - mC_v = R \dots\dots\dots(ii)$$

Where  $C_p$  and  $C_v$  are specific heat capacities at constant pressure and volume respectively

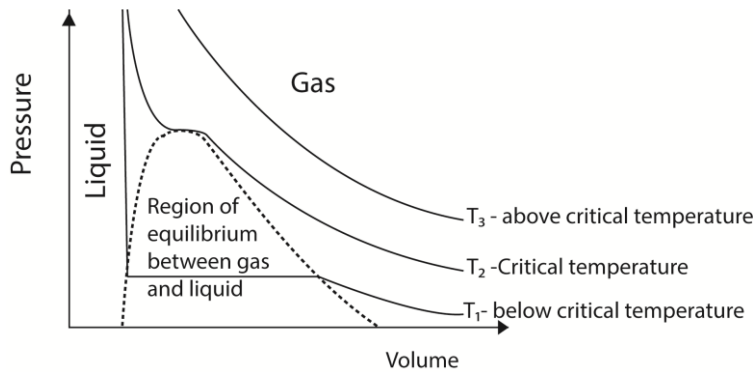
From equation (ii)

$$C_p = \frac{R + mC_v}{m} \dots\dots\dots(iii)$$

Substituting for  $m$  from equation (i) into equation (iii)

$$C_p = \frac{R + \rho(273R \times 10^{-5})C_v}{273\rho R \times 10^{-5}} = \frac{1 + 273\rho C_v \times 10^{-5}}{273\rho \times 10^{-5}} = \frac{1 + (273 \times 1.429 \times 719 \times 10^{-5})}{273 \times 1.429 \times 10^{-5}} = 975.5\text{Jkg}^{-1}\text{K}^{-1}$$

(b) Indicate the different states of a real gas at different temperatures and pressure versus volume sketch graph.



(c) (i) In deriving the expression  $P = \frac{1}{3} \rho c^2$  for the pressure of an ideal gas, two of the assumptions made are not valid for a real gas. State these assumptions. (02marks)

- The intermolecular forces are negligible
- The volume of the gas is negligible compared the volume of the container
- Collision are perfectly elastic
- The duration of collision is negligible

(ii) The equation of state of one mole of a real gas is  $\left(P + \frac{a}{v^2}\right)(v - b) = RT$

Account for the terms  $\frac{a}{v^2}$  and b (02marks)

The molecules that strike the walls of the container are retarded by unbalances forces due to the molecules behind them. So the observed pressure is less than what it would be without these forces. Hence  $\frac{a}{v^2}$  corrects deficit in pressure due to intermolecular attractions of gas molecules

The volume of the molecules compared to the volume of the container occupied by the gas. Therefore, b, called the co-volume accounts for the finite volume of molecules themselves

(d) Use the expression  $P = \frac{1}{3} \rho c^2$  ; for the pressure of an ideal gas to derive Dalton's law of partial pressures (04marks)

$$P = \frac{1}{3} N \frac{m}{V} c^2 = \frac{2}{3} N \left( \frac{1}{2} m c^2 \right)$$

$$\text{For gas 1, } P_1 V_1 = \frac{2}{3} N_1 \left( \frac{1}{2} m_1 c_1^2 \right)$$

$$\Rightarrow N_1 = \frac{3}{2} P_1 V_1 \cdot \frac{1}{K_1}$$

Similarly for gas 2

$$N_2 = \frac{3}{2} P_2 V_2 \cdot \frac{1}{K_2}$$

For a mixture of gases,  $N = \frac{3}{2} PV \cdot \frac{1}{K}$ ; but  $N = N_1 + N_2$

$$\frac{3}{2} PV \cdot \frac{1}{K} = \frac{3}{2} P_1 V_1 \cdot \frac{1}{K_1} + \frac{3}{2} P_2 V_2 \cdot \frac{1}{K_2}$$

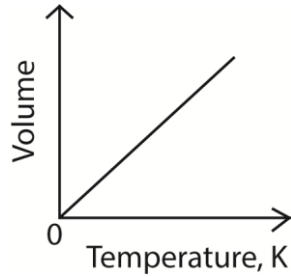
Since temperature is constant,  $K_1 = K_2 = K$

$$- PV = P_1 V_1 + P_2 V_2$$

$$- \text{But } V = V_1 = V_2$$

$$- \therefore P = P_1 + P_2$$

(e) Explain, with the aid of a volume versus temperature sketch graph, what happens to a gas cooled at constant pressure from room temperature to zero Kelvin. (04marks)

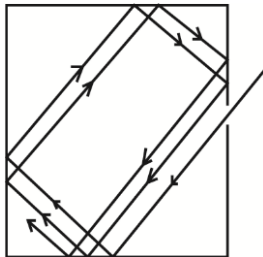


As the gas cools, the kinetic energy of its molecules reduce, implying that the internal energy of the molecules reduce until zero K where the volume is assumed to be zero and Charles laws ceases to apply.

6. (a) What is meant by black body? (01mark)

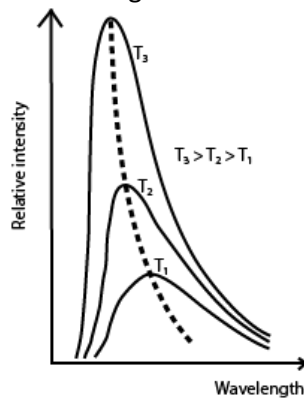
A black body is one that absorbs all incident radiation, but reflects and transmits none.

(b) Describe how an approximate black body can be realized in practice. (02marks)



When radiation enters a black container through a hole, it undergoes multiple reflections. At each reflection, part of the radiation is absorbed. After several reflections, all the radiation is retained inside the container. Hence it approximates to a black body.

(c)(i) Draw sketch graphs to show how variation of relative intensity of black body radiation with wavelength for three different temperatures. (02marks)



(ii) Describe the features of the sketch in (c)(i) above. (03marks)

As temperature increases, the intensity increases. The intensity of shorter wavelengths increase more rapidly. The wavelength of the most intense radiation decreases as temperature increases.

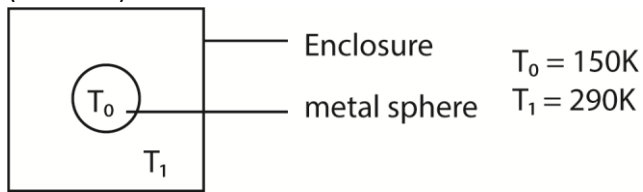
(d)(i) State Stefan's law (01mark)

Stefan's law states that the total power radiated by a black body is directly proportional to the fourth power of its absolute temperature.

(ii) A solid copper sphere of diameter 10 mm and temperature of 150K is placed in an enclosure maintained at a temperature of 290K. Calculate, stating assumptions made, the initial rate of rise of temperature of the sphere.

[Density of copper =  $8.93 \times 10^3 \text{kgm}^{-3}$ , specific heat capacity of copper =  $3.7 \times 10^2 \text{JkgK}^{-1}$ )

(07marks)



Power radiated by the sphere =  $4\pi r^2 \sigma T_0^4$

Power absorbed by the sphere =  $4\pi r^2 \sigma T_1^4$

Net power absorbed,  $P = 4\pi r^2 \sigma (T_1^4 - T_0^4)$

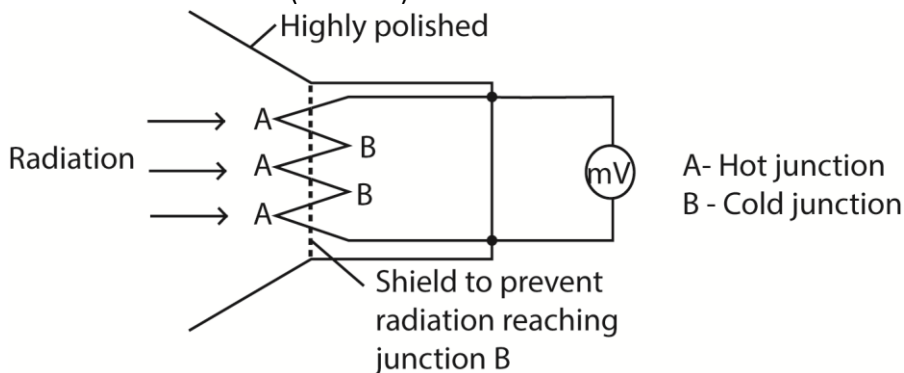
But also,  $P = mc \frac{d\theta}{dt} = \frac{4}{3} \pi r^3 \rho c \frac{d\theta}{dt}$

At equilibrium

$\frac{4}{3} \pi r^3 \rho c \frac{d\theta}{dt} = 4\pi r^2 \sigma (T_1^4 - T_0^4)$

$\frac{d\theta}{dt} = \frac{3\sigma}{\rho c r} (T_1^4 - T_0^4) = \frac{3 \times 5.67 \times 10^{-8}}{8.93 \times 10^3 \times 3.7 \times 10^2 \times 5 \times 10^{-3}} = 0.068 \text{Ks}^{-1}$

(e) With the aid of a labelled diagram, describe how a thermopile can be used to determine infrared radiation. (04marks)



Radiation falling on junction A is absorbed and temperature rises above that of junction B. An e.m.f is generated and is measured by millivolt meter which deflects as a result.

7. (a) (i) What is meant by kinetic theory of gases? (03marks)

Gases are composed of molecules which are in continuous random motion. The molecules collide elastically with one another and also with the walls of the container. When heat energy is supplied, their kinetic energy increases.

(ii) Define an ideal gas (01mark)

An ideal gas is one which exactly obeys Boyle's law at all conditions

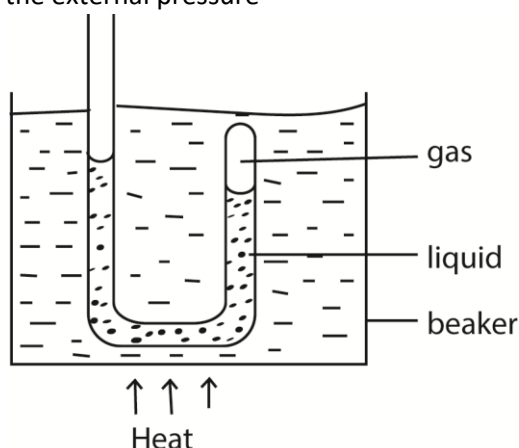
(iii) State and explain conditions under which real gases behave like ideal gases. (04mark)

At very high temperature, the intermolecular forces of attraction become negligible.

At low pressure, the volume of gas molecules becomes negligible compared to the volume of the container.

(b) (i) Describe an experiment to show that a liquid boils only when its saturated vapor pressure is equal to external pressure (05marks)

Experiment to show that a liquid boils off when its saturated vapour pressure equals the external pressure



- Air is trapped in the closed limb of the tube by water column.
- The tube is heated in water bath.
- When the water bath begins to boil, the water in the tube comes to the same level in each limb.
- This shows that the vapor pressure in closed limb is equal to external pressure.

(ii) Explain how cooking at a pressure of 76cm of mercury and temperature of 100°C may be achieved on top of high mountains. (03marks)

Cooking pans are fitted with lids that possess safety valves that open when the pressure exceeds 76cmHg. During cooking the vapour pressure inside the cooking pan increase and the temperature increases to 100°C. the safety valves prevents pressure to exceed 76cmHg and therefore boiling occurs at 100°C.

(c) (i) Define root-mean-square speed of molecules of a gas. (01mark)

Root mean square speed of the gas is the average of the square speeds of individual molecules of a gas.

(ii) The mass of hydrogen and oxygen atoms are  $1.66 \times 10^{-27}$ kg and  $2.66 \times 10^{-26}$ kg respectively. What is the ratio of the root mean square speed of hydrogen to that of oxygen molecules at the same temperature? (03marks)

$$\text{From } \frac{1}{3}Nmc^2 = RT$$

$$\frac{1}{3}N \times 1.66 \times 10^{-27} \times c_H^2 = \frac{1}{3}N \times 2.66 \times 10^{-26} \times c_O^2$$

$$\frac{c_H^2}{c_O^2} = \frac{2.66 \times 10^{-26}}{1.66 \times 10^{-27}} = 16$$

## SECTION C

8. (a) (i) State Rutherford's model of the atom. (02marks)

- The atom consists of positive charges confined to the center where most mass is concentrated
- Electrons orbit around the nucleus just like planets around the sun
- It is the electron cloud that accounts for the volume of an atom

(ii) Explain two main failures of Rutherford's model of the atom. (03marks)

- Much as electrons revolve around the nucleus, they do so only in certain allowed orbit. While in these orbits, they do not emit energy. Rutherford failed to explain this.

- Electrons can jump from one orbit to another of lower energy and energy emitted equal to energy difference between the energy levels equal  $hf$ , where  $f$  is the frequency of radiation emitted and  $h$  = Plank's constant. Rutherford failed to explain this.

(b) (i) Explain how Millikan's experiment for measuring the charge of an electron proves that charge is quantized. (04marks)

Millikan found out that the charge on every other droplet was always a multiple of basic unit  $1.6 \times 10^{-19}C$  and a whole number. Thus the charge carried by an electron is  $-1.6 \times 10^{-19}C$ . A drop carrying charge of  $N \times 10^{-19}C$  has either  $N$  electrons too many or too few. Thus Millikan concluded that charge is quantized.

(ii) Oil droplets are introduced into the space between two flat horizontal plates, set 5.0mm apart. The plate voltage is then adjusted to exactly 780V so that one of the droplets is held stationary. Then the plate voltage is switched off and the selected droplet is observed to fall a measured distance of 1.5mm in 11.2s. Given the density of oil used is  $900kgm^{-3}$  and the viscosity of air is  $1.8 \times 10^{-5}Nsm^{-2}$ , calculate the charge on the droplet. (06marks)

When the uncharged drop is falling steadily under gravity with terminal velocity.  $v_0$ .

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_1$$

Where  $\sigma$  is density of air,  $\rho$  is density of oil and  $\eta$  is viscosity of air

$$\Rightarrow r = \left[ \frac{9\eta v_1}{2g(\rho - \sigma)} \right]^{\frac{1}{2}}$$

$$\text{But } v_1 = \frac{\text{distance moved by oil drop}}{\text{time}} = \frac{1.5 \times 10^{-3}}{11.2} = 1.339 \times 10^{-4}ms^{-1}$$

Neglecting density of air, i.e.  $\sigma = 0$

$$r = \frac{9 \times 1.8 \times 10^{-5} \times 1.339 \times 10^{-4}}{2 \times 9.81 \times 900} = 1.09 \times 10^{-16}m$$

When an electric field is applied across the plates,

$$\frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi r^3 \sigma g + 6\pi r \eta v_2 - Eq \text{ where } v_2 \text{ is the new speed}$$

$$Eq = \frac{4}{3}\pi r^3 (\rho - \sigma) - 6\pi r \eta v_2$$

$$= 6\pi r \eta v_1 - 6\pi r \eta v_2$$

$$\text{Also } E = \frac{V}{d} = \frac{780}{50 \times 10^{-3}} = 1.56 \times 10^5 Vm^{-1}$$

Since the charged drop is held stationary by the field  $v_2 = 0$

$$\text{Hence } q = \frac{6\pi r \eta v_1}{E}$$

$$= \frac{6\pi \times 1.8 \times 10^5 \times 1.1 \times 10^{-6} \times 1.339 \times 10^{-4}}{1.56 \times 10^5}$$

$$= 3.204 \times 10^{-19}C$$

$$= 2e$$

(c) A beam of positive ions is accelerated through a potential difference of  $1 \times 10^3V$  into a region of uniform magnetic field of flux density 0.2T. While in magnetic field it moves in a circle of radius 2.3cm. Derive an expression for the charge to mass ratio of the ions, and calculate the value. (05marks)

When positive ions are accelerated through a p.d they acquire kinetic energy equal to  $\frac{1}{2}mv^2$ .

The force,  $F$ , on an electron moving normal to magnetic field is given by  $F = Bqv$ . This force provides the required centripetal force,  $F_c$ .

Thus  $F_c = F_B$

$$\text{Now } \frac{mu^2}{r} = Bqu$$

$$u = \frac{Bqr}{m}$$

The kinetic energy should balance the energy due to the electric field

$$\text{K.E} = \frac{1}{2} mu^2 = qV$$

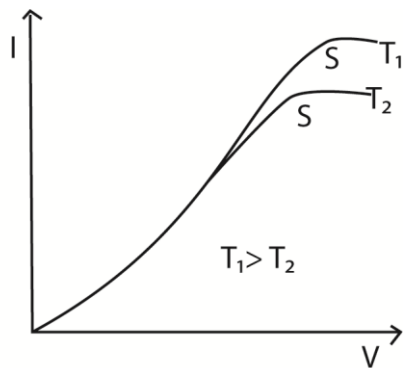
$$\frac{B^2 q^2 r^2}{2m} = qV$$

$$\frac{q}{m} = \frac{2V}{r^2 B^2} = \frac{2 \times 1 \times 10^3}{(2.3 \times 10^{-2})^2 \times 0.2^2} = 9.5 \times 10^7 \text{Ckg}^{-1}$$

9. (a) (i) What is meant by thermionic emission? (01mark)

It emission of electrons from a metal surface when heated.

(ii) Sketch the current-potential difference characteristics of a thermionic diode for two different operating temperature and explain their main features. (05marks)



The current increases with the positive anode potential as far as the point S. Beyond this point the current does not increase, because the anode is collecting all the electrons emitted by the filament ; the current is said to be saturated. More electrons are emitted at a higher temperature  $T_1$  than  $T_2$ .

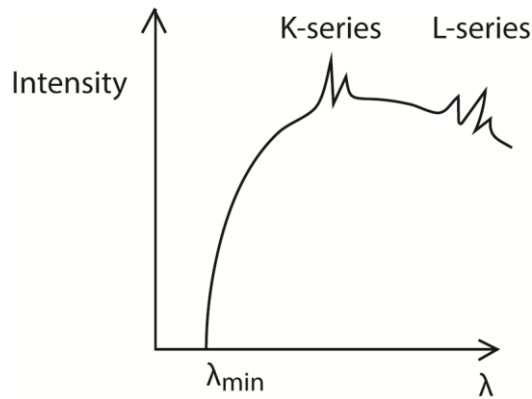
(i) Describe one application of a diode. (02marks)

Diodes are used in rectification of a.c into d.c since they conduct in one direction only.

(b) (i) What features of an x-ray tube make it suitable for continuous production of X-rays. (03marks)

- low voltage source for heating the cathode an electron source by thermionic emission
- accelerating p.d
- high melting point target
- highly conducting copper anode and cooling fins to dissipate heat by radiation and convection to prevent damage to apparatus due to overheating.

(ii) Sketch a graph of intensity versus frequency of a radiation produced in an X-ray tube and explain its main feature. (05marks)



The spectrum consists of two major components, i.e. the continuous (background) spectrum and the very sharp line spectrum superimposed onto the background spectrum.

The continuous spectrum is produced when electrons make multiple collisions with the target atoms in which they are decelerated. At each deceleration, X-rays of differing wavelength are produced.

The shortest Wavelength X-rays are produced when electrons lose all their energy as X-ray photon in a single encounter with the target atoms. The wavelength of the X-rays at this point is known as the cut off wavelength. At cut off wavelength, energy in an X-ray photon equals kinetic energy of the electron; i.e.  $hf = eV$  or  $\frac{hc}{\lambda_{max}} = eV$  where  $V = p.d$

### The line spectrum

At high tube voltages, the bombarding electrons penetrate deep into the target atoms and knock out electrons from inner shell. The knocked out electrons occupy vacant spaces in higher unfilled shells putting the atom in excited state and making them unstable.

Transition of an electron from higher to lower energy levels results in an emission of X-ray photon of energy equal to energy difference between the energy levels.

If the transition ends in the K-shell, it produces K-series and if the transition ends in L-shell. It produces L-series.

- (iii) A mono chromatic X-ray beam of wavelength  $1.0 \times 10^{-10} \text{cm}$  is incident on a set of planes in a crystal of spacing  $2.8 \times 10^{-10} \text{m}$ . What is the maximum order possible with these X-rays? (04marks)

$$\text{From } 2d\sin\theta = n\lambda$$

$$\text{For } n \text{ maximum } \sin\theta = 1$$

$$n = \frac{2 \times 2.8 \times 10^{-10}}{1 \times 10^{-10}} = 5$$

10. (a) What is meant by the following terms:

- (i) nuclear number (01mark)

Nuclear number is the number of nucleons in the nucleus of an atom



(ii) binding energy (01marks)

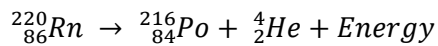
This is the energy required to split the nucleus into its constituent nucleons

(b) Calculate the energy released during the decay of  ${}^{220}_{86}\text{Rn}$  nucleus into  ${}^{216}_{84}\text{Po}$  and  $\alpha$ -particle

$$\left. \begin{array}{l} \text{Mass of } {}^{220}_{86}\text{Rn} = 219.964176\text{u} \\ \text{Mass of } {}^{216}_{84}\text{Po} = 215.955794\text{u} \\ \text{Mass of } {}^4_2\text{He} = 4.001566\text{u} \end{array} \right\}$$

(1u = 931eV)

(04marks)



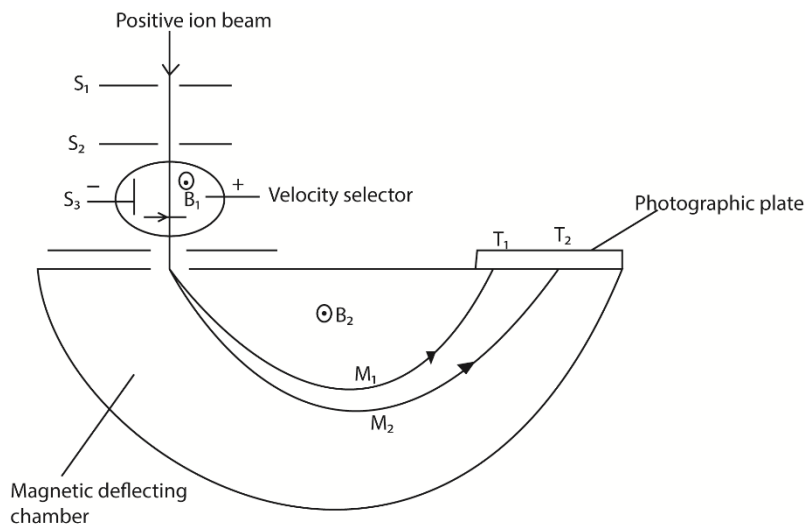
$$219.964\text{U} = 215.955\text{U} + 4.0015\text{U} + E$$

$$E = 6.816 \times 10^{-3}\text{U}$$

$$= 6.816 \times 10^{-3} \times 931\text{MeV}$$

$$= 6.35\text{MeV}$$

(c) Describe the Bainbridge mass spectrometer and explain how it can be used to distinguish between isotopes (07marks)



$T_1$  and  $T_2$  are tracers on photographic plate,  $S_1$ ,  $S_2$  and  $S_3$  are slits

### Mode of Action

- Positive ions are produced in a discharge tube and admitted as a beam through slits  $S_1$  and  $S_2$ .
- The beam then passes between insulated plates P, Q, connected to a battery, which create an electric field of intensity E.
- A uniform magnetic field  $B_1$ , perpendicular to E is applied over the region of the plates and all ions, charge e with the same velocity, v given by  $B_1ev = Ee$  will then pass undeflected through the plates and through a slit  $S_3$ .

- The selected ions are deflected in a circular path of radius  $r$  by a uniform perpendicular magnetic field  $B_2$  and an image is produced on a photographic plate as shown.

In this case

$$\frac{mv^2}{r} = B_2 ev$$

But for the ions selected  $v = \frac{E}{B_1}$  from above

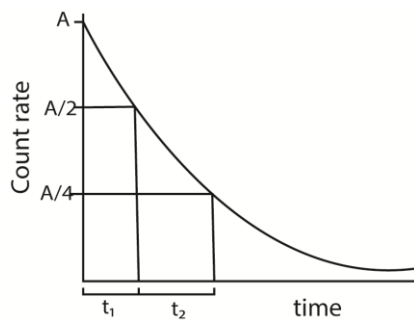
$$\therefore \frac{m}{e} = \frac{rB_2B_1}{E}$$

$$r = \left( \frac{E}{B_1B_2Q} \right) m$$

thus different isotopes strike the photographic plate at different points.

- (d) (i) Explain how you would use a decay curve for a radioactive material to determine its half-life. (02marks)

The curve is defined by  $N = N_0e^{-\lambda t}$



Find time  $t_1$  taken for the activity to reduce to  $A/2$  and  $t_2$  taken for activity to reduce to  $A/4$  from  $A/2$ , where  $A$  is the initial count rate.

$$\text{Half-life} = \frac{1}{2}(t_1 + t_2),$$

- (ii) A radioactive source contain  $1.0\mu\text{g}$  of plutonium of mass number 239. If the source emits 2300  $\alpha$ -particles per second, calculate the half-life of plutonium.

[Assume the decay law  $N = N_0e^{-\lambda t}$ ] (05mark)

$$\text{Activity } A = \lambda N$$

$$\Rightarrow 2300 = \lambda N \dots\dots\dots(i)$$

$$239\text{g of Plutonium} \equiv 6.02 \times 10^{23} \text{ atoms}$$

$$1\mu\text{g Of Plutonium} = \frac{6.02 \times 10^{23}}{239} \times 10^{-6} = 2.52 \times 10^{15} \text{ atoms}$$

$$\therefore \text{number of atoms } N = 2.52 \times 10^{15}$$

From eqn. (i)

$$\lambda = \frac{2300}{2.52 \times 10^{15}} = 9.126 \times 10^{-13}\text{s}^{-1}$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{9.126 \times 10^{-13}} = 7.50 \times 10^{11}\text{seconds}$$