UACE MATHEMATICS PAPER 2 2017 guide

SECTION A (40 marks)

Answer all questions in this section

1. A particle is projected from a point O with speed20m/s at an angle 60^0 to the horizontal. Express in vector form its velocity v and its displacement, r, from O at any time t seconds. (05marks)
2. The probability that a patient suffering from a certain disease recovers is 0.4. If 15 people contract the disease, find the probability that:
   (a) More than 9 will recover. (02marks)
   (b) Between five and eight will recover. (03marks)
3. The table below gives values of x and corresponding values of f(x).

X	0.1	0.2	0.3	0.4	0.5	0.7
f(x)	4.21	3.83	3.25	2.85	2.25	1.43

   Use linear interpolation/extrapolation to find
   (a) f(x) when x = 0.6 (03marks)
   (b) the value of x when f(x) = 0.75 (02mars)
4. In a square ABCD, three forces of magnitudes 4N, 10N and 7N and CA respectively. Their directions are in the order of the letters. Find magnitude of the resultant force. (05marks)
5. A box A contains 1 white ball and I blue ball. Box B contains only 2 white balls. If a ball is pice at random, find the probability that it is
   (a) White (02mark)
   (b) From box A given that it is white(03marks)
6. Given that y = $\frac{1}{x} + x$ and x = 2.4 correct to one decimal place, find the limits within which y lies. (05marks)
7. The table below shows the retail prices (shs) and amount of each item bought weekly by a restaurant in 2002 and 2003.

Item	Prices(shs)		Amount bought
	2002	2003	
Milk (per litre)	400	500	200
Eggs (per tray)	2,500	3,000	18
Cooking oil (per litre)	2,400	2,100	2
Baking flour (per packet)	2,000	2,200	15

   (a) Taking 2002 as the base year, calculate the weighed aggregate price index. (03 marks)

(b) In 2003, the restaurant spent shs. 450,000 on buying these items. Using the weighed aggregate price index obtained in (a), calculate what the restaurant could have spent in 2002. (02marks)

8. The engine of a lorry of mass 5,000kg is working at a steady rate of 350 kW against a constant resistance force of 1,000N. The lorry ascends a slope of inclination θ to the horizontal. If the maximum speed of the lorry is 20ms^{-1}, find the value of θ (05marks)

SECTON B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. A discrete random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4. 5 \\ 0. & \text{Otherwise,} \end{cases}$$

Where k is a constant

Determine;

(a) the value of k (03 marks)
(b) P(2 < X < 5) (02marks)
(c) Expectation, E (X) (03marks)
(d) Variance, Var(X) (04mars)

10. A particle of mass 3g is acted on by a force F = 6i – 36t^{2}j + 54k Newton at time t. At time t = 0 the particle is at the point with position vector i – 5j –k and its velocity is 3i + 3j ms^{-1}. Determine the
(a) position vector of the particle at time t= 1second (09marks)
(b) distance of the particle from the origin at time t = 1 second

11. A student used the trapezium rule with five sub-intervals to estimate $\int_2^3 \frac{x}{(x^2-3)} dx$ correct to **three** decimal places

Determine;

(a) The value the student obtained (06marks)
(b) The actual value of the integral (03marks)
(c) (i) the error the student made in the estimate
(ii) how the student can reduce the error(03marks)

12. The times taken for 55 students to have their lunch to the nearest minute are given in the table below

Time (minutes)	3 -4	5-9	10-19	20 – 29	30 – 44
Number of students	2	7	16	21	9

(a) Calculate the mean time for the student to have lunch (04marks)
(b) (i) Draw a histogram for the given data
(ii) Use your histogram to estimate the modal time for the students to have lunch. (08marks)

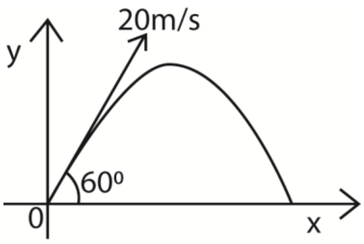
13. A non-uniform rod AB of mass 10k has its centre of gravity a distance ¼ AB. The rod is smoothly hinged at A. it is maintained in equilibrium at 60^0 above the horizontal by a light inextensible string tied at B and at a right angle to AB. Calculate the magnitude and direction of the reaction at A. (12marks)
14. By plotting graphs of y = x and y = 4siin x on the same axes. Show that the root of the equation x – 4sinx = 0 lies between 2 and 3.
    Hence use Newton Raphson's method to find the root of the equation correct to 3 decimal places. (12marks)
15. The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows, 5% of the residents have over90 cows.
    (a) Determine the values of the mean and standard deviation of the cows. (08marks)
    (b) If there are 200 residents, find how many have more than 80 cows. (04marks)
16. At 12 noon a ship A is moving with constant velocity of $20.4kmh^{-1}$ in the direction $N\theta^0E$ where tan θ = $\frac{1}{5}$. A second ship B is 15km due to north of A. Ship B is moving with constant velocity of 5kmh-1 in the direction Sα0W, where tan α= ¼. If the shortest distance between the ships is 4.2km, find the time to the nearest minute when the distance between the ships is shortest. (12mars)

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. A particle is projected from a point O with speed20m/s at an angle 60^0 to the horizontal. Express in vector form its velocity v and its displacement, r, from O at any time t seconds . (05marks)



At time t = 0

V = $20\cos60^0$i + $20\sin60^0$j = 10i + $10\sqrt{3}$j

But A = -gj

At any time t,

V = $\int adt$ = -gj$\int dt$

= -gtj + c

At t = 0

10i + $10\sqrt{3}$j = 0 + c

⇨ At time t

V= 10i + $(10\sqrt{3} - gt)$j = 10i + $(10\sqrt{3} - 9.8t)$j

Or

V = $\begin{pmatrix} 10 \\ 10\sqrt{3} - 9.8t \end{pmatrix}$

r = $\int vdt = \int \begin{pmatrix} 10 \\ 10\sqrt{3} - 9.8t \end{pmatrix} dt$ = $\begin{pmatrix} 10t \\ 10\sqrt{3}t - 4.9t^2 \end{pmatrix}$+ c

at t = 0, r = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

⇨ $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + c$, c = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Hence at time t, r = $\begin{pmatrix} 10t \\ 10\sqrt{3}t - 4.9t^2 \end{pmatrix}$

2. The probability that a patient suffering from a certain disease recovers is 0.4. If 15 people contract the disease, find the probability that:
   (a) More than 9 will recover. (02marks)
   Given n = 15, p = 0.4
   Let x = number of people who will recover
   P($X \geq 10$)= B($X \geq 10, 15, 0.4$)
   (cumulative probabilities)
   P($X \geq 10) = 0.0338$
   (b) Between five and eight will recover. (03marks)
   P(5 < X < 8) = P(X = 6,7)
   = P(X = 6) + P(X =7)

= B(6, 15, 0.4) + B(7, 15, 0.4)
= 0.2066 + 0.1771
= 0.3837
Or
P(5 < X < 8) = P(X = 6,7)
= P(X $\geq$ 6) – P(X $\geq$ 8)
= 0.5968 – 0.2131
= 0.3837

3. The table below gives values of x and corresponding values of f(x).

X	0.1	0.2	0.3	0.4	0.5	0.7
f(x)	4.21	3.83	3.25	2.85	2.25	1.43

Use linear interpolation/extrapolation to find
(a) f(x) when x = 0.6 (03marks)
Using linear interpolation
Extract

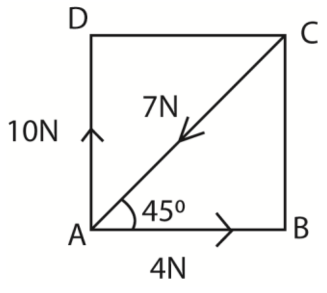
0.5	0.6	0.7
2.25	y	1.43

$\frac{1.43-2.25}{0.7-0.5} = \frac{y-2.25}{0.6-0.5}$
y = 1.84
(b) the value of x when f(x) = 0.75 (02mars)
Using linear extrapolation
Extract

0.5	0.7	X
2.25	1.43	0.75

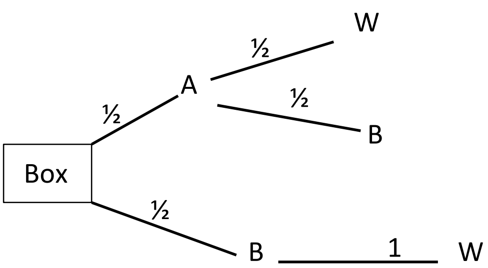
$\frac{x-0.5}{0.75-2.25} = \frac{0.7-0.5}{1.43-2.25}$; x = 0.9

4. In a square ABCD, three forces of magnitudes 4N, 10N and 7N and CA respectively. Their directions are in the order of the letters. Find magnitude of the resultant force. (05marks)



$$\binom{X}{Y} = \binom{4}{0} + \binom{-7\cos 45^0}{-7\sin 45^0} + \binom{0}{10} = \binom{-0.9497}{5.0503}$$
$|R| = \sqrt{(-0.9497)^2 + (5.0503)^2}$ = 5.1388N

5. A box A contains 1 white ball and I blue ball. Box B contains only 2 white balls. If a ball is pice at random, find the probability that it is
(a) White (02mark)



P(W) = $\frac{1}{2}\ x\ \frac{1}{2} + \frac{1}{2}\ x\ 1$ = $\frac{3}{4}$

(b) From box A given that it is white(03marks)

P(A/W) = $\frac{P(A \cap W)}{P(W)}$ = $\frac{P(W/_A).P(A)}{P(W)} = \frac{\frac{1}{2} x \frac{1}{2}}{\frac{3}{4}} = \frac{1}{3}$

6. Given that y = $\frac{1}{x} + x$ and x = 2.4 correct to one decimal place, find the limits within which y lies. (05marks)

Error in 2.4 = $\frac{1}{2}\ x\ \frac{1}{10} = 0.05$

y_{max} = 2.45 + $\frac{1}{2.35} = 2.8755$

y_{min} = 2.35 + $\frac{1}{2.45} = 2.7582$

∴ the limits are [2.7582, 2.8755]

7. The table below shows the retail prices (shs) and amount of each item bought weekly by a restaurant in 2002 and 2003.

Item	Prices(shs)		Amount bought
	2002	2003	
Milk (per litre)	400	500	200
Eggs (per tray)	2,500	3,000	18
Cooking oil (per litre)	2,400	2,100	2
Baking flour (per packet)	2,000	2,200	15

(a) Taking 2002 as the base year, calculate the weighed aggregate price index. (03 marks)

P_0	W	P_0W	P_1	P_1W
400	200	80,000	500	100,000
2500	18	45,000	3,000	54,000
2400	2	4800	2100	4200
200	15	30,000	2200	33,000
		$\sum P_0W = 159800$		$\sum P_1W = 191200$

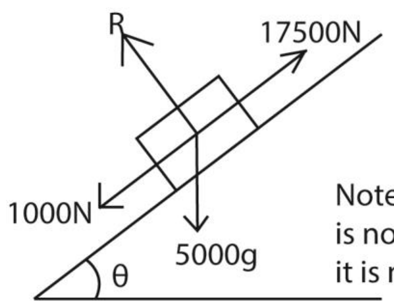
W.A.P.I = $\frac{191200}{159800}\ x\ 100$ = 119.65

(b) In 2003, the restaurant spent shs. 450,000 on buying these items. Using the weighed aggregate price index obtained in (a), calculate what the restaurant could have spent in 2002. (02marks)

$\frac{450,000}{P_0}$ x 100 = 119.65

$P_0 = 376,097$

8. The engine of a lorry of mass 5,000kg is working at a steady rate of 350 kW against a constant resistance force of 1,000N. The lorry ascends a slope of inclination θ to the horizontal. If the maximum speed of the lorry is 20ms^{-1}, find the value of θ (05marks)



Note resistance of 1000N is not on the surface because it is not friction alone

Resultant force = 17500 – (1000 + 5000sinθ)

5000a = 17500 – (1000 + 5000sinθ)

At maximum speed a = 0

⇨ 0 = 17500 – (1000 + 5000sinθ)
16500 = 5000sinθ
$\theta = 19.7^0$

SECTON B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. A discrete random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4. 5 \\ 1. \quad \text{Otherwise,} \end{cases}$$

Where k is a constant

Determine;

(a) the value of k (03 marks)

x	1	2	3	4	5	sum
P(X =x)	k	2k	3k	4k	5k	15k

15k = 1

$k = \frac{1}{15}$

x	1	2	3	4	5	sum
P(X =x)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	1
xP(X= x)	$\frac{1}{15}$	$\frac{4}{15}$	$\frac{9}{15}$	$\frac{16}{15}$	$\frac{25}{15}$	$\frac{55}{15}$
X^2P(X =x)	$\frac{1}{15}$	$\frac{8}{15}$	$\frac{27}{15}$	$\frac{64}{15}$	$\frac{125}{15}$	$\frac{225}{15}$

(b) P(2 < X < 5) (02marks)

P(2<X<5) = P(X = 3, 4)

$= \frac{3}{15} + \frac{4}{15}$

$= \frac{7}{15}$

(c) Expectation, E (X) (03marks)

$E(X) = \sum xP(X = x) = \frac{55}{15} = \frac{11}{3}$

(d) Variance, Var(X) (04mars)

$Var(X) = E(X^2) – E(X)^2$

$= \frac{225}{15} - \left(\frac{55}{15}\right)^2$

$= \frac{14}{9}$

10. A particle of mass 3kg is acted on by a force $F = 6i – 36t^2j + 54k$ Newton at time t. At time t = 0 the particle is at the point with position vector i – 5j –k and its velocity is 3i + 3j ms^{-1}.

Determine the

(a) position vector of the particle at time t= 1second (09marks)

F = ma

$6i - 36t^2j + 54k = 3a$

a = 2i + 12t2j + 18k

V = $\int adt = \int 2i - 12t^2j + 18k\,dt$

$= 2ti - 4t^3j + 9t^2k + c$

At t = 0, V = 3i + 3j

⇨ 3i + 3j = 0 + c

c = 3i + 3j

By substitution

V = $(2ti - 4t^3j + 9t^2k) + (3i + 3j) = (2t + 3)I + (4t^3 + 3)j + 9t^2k$

r = $\int vdt = \int (2t + 3)i + (-4t^3 + 3)j + 9t^2k\,dt$

$= (t^2 + 3t)i + (-t^4 + 3t)j + 3t^3k + c$

At t = 0, r = i -5j – k

⇨ i -5j – k = 0 +c

c = i – 5j –k

substituting for c

r = $[(t^2 + 3t)i + (-t^4 + 3t)j + 3t^3k] + [i - 5j - k]$

$= [(t^2 + 3t + 1)i + (-t^4 + 3t - 5)j + (3t^3 - 1) k] + [i - 5j - k]$

At t = 1

r = (1 + 3+ 1)i + (-1 + 3 -5)j + (3 – 1)k

= 5i -3j + 2k

(b) distance of the particle from the origin at time t = 1 second

$|r| = \sqrt{5^2 + (-3)^2 + 2^2} = \sqrt{38} = 6.164$

11. A student used the trapezium rule with five sub-intervals to estimate $\int_2^3 \frac{x}{(x^2-3)}dx$ correct to **three** decimal places

Determine;

(a) The value the student obtained (06marks)

h = $\frac{3-2}{5}$ =0.2

X	y_1, y_6	$y_2 ......y_5$
2.0	2.0	
2.2		1.1956
2.4		0.8696
2.6		0.6915
2.8		0.5785
3	0.5	
Sum	2.5	3.3352

$\int_2^3 \frac{x}{(x^2-3)}dx = \frac{1}{2}\ x\ 0.2[2.5 + 2(3.3352)]$

= 0.91704 = 0.917 (3D)

(b) The actual value of the integral (03marks)

$\int_2^3 \frac{x}{(x^2-3)}dx = \left[\frac{1}{2}Inx^2 - 3\right]_2^3$

$=\frac{1}{2}(In\ 6 - In\ 1)$
= 0.896

(c) (i) the error the student made in the estimate
Error = $|0.896 - 0.917|$ = 0.021
(ii) how the student can reduce the error(03marks)
Increasing on the number of sub-intervals or ordinates or reducing the width of h

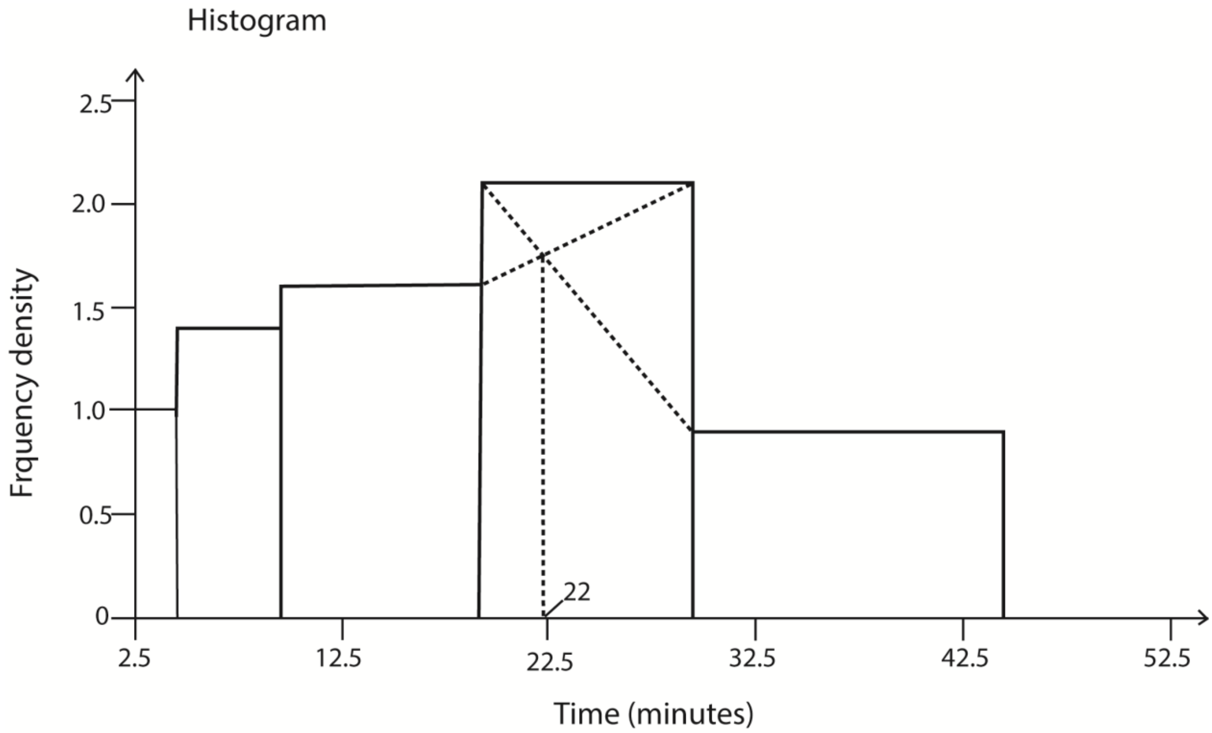
12. The times taken for 55 students to have their lunch to the nearest minute are given in the table below

Time (minutes)	3 -4	5-9	10-19	20 – 29	30 – 44
Number of students	2	7	16	21	9

Class boundaries	f	x	fx	c	fd
2.5 – 4.5	2	3.5	7	2	1.0
4.5 – 9.4	7	7	49	5	1.4
9.5 – 19.5	16	14.5	232	10	1.6
19.5 – 29.5	21	24.5	514.5	10	2.1
29.5 – 44.5	9	37	333	15	0.6
	$\sum f = 55$		$\sum fx = 1135.5$		

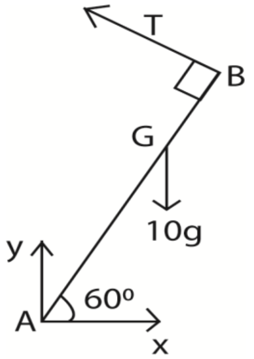
(a) Calculate the mean time for the student to have lunch (04marks)
Mean = $\frac{\sum fx}{\sum f} = \frac{1135.5}{55} = 20.65$

(b) (i) Draw a histogram for the given data



(ii) Use your histogram to estimate the modal time for the students to have lunch. (08marks)
Mode = 22 minutes

13. A non-uniform rod AB of mass 10k has its centre of gravity a distance ¼ AB. The rod is smoothly hinged at A. it is maintained in equilibrium at 60^0 above the horizontal by a light inextensible string tied at B and at a right angle to AB. Calculate the magnitude and direction of the reaction at A. (12marks)



Taking moments about point A

T x (AB) = $10g\left(\frac{3}{4}ABcos\ 60^0\right)$

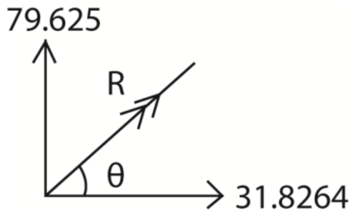
T = $\frac{15g}{4} = \frac{15\ x\ 9.8}{4}$ = 36.75N

Resolving forces horizontally

X = $Tcos30^0 = 36.75cos30^0$ = 31.8264N

Y = 10g – 36.75sin 30^0 = 79.625N

$|R| = \sqrt{(31.8264)^2 + (79.625)^2}$ = 85.75N



θ = $tan^{-1}\left(\frac{79.625}{31.8264}\right) = 68.2^0$

The direction of resultant force is 68.2^0 or $E68.2^0N$ or $N21.8^0E$

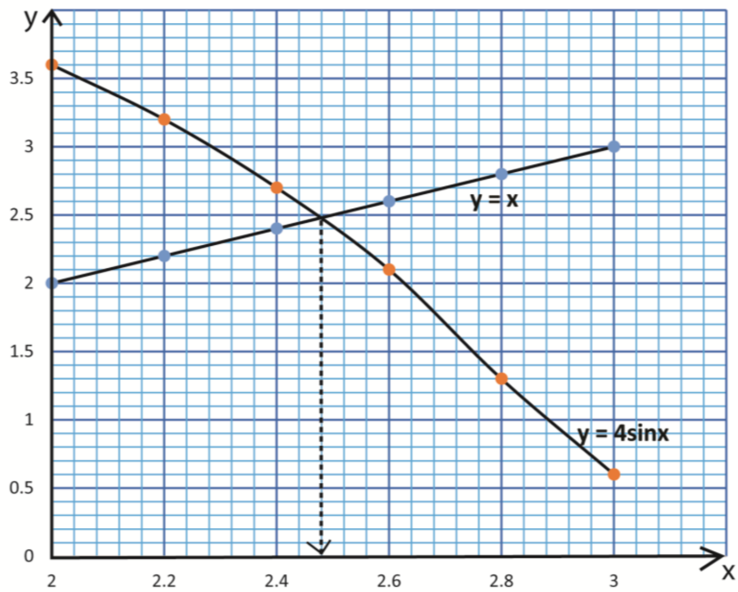
14. By plotting graphs of y = x and y = 4siin x on the same axes. Show that the root of the equation x – 4sinx = 0 lies between 2 and 3.

Hence use Newton Raphson's method to find the root of the equation correct to 3 decimal places. (12marks)

Table of results

x	2.0	2.2	2.4	2.6	2.8	3.0
y = x	2.0	2.2	2.4	2.6	2.8	3.0
Y = 4sinx	3.6	3.2	2.7	2.1	1.3	0.6

Graph



From the graph the root lies between 2.4 and 2.6

Using N.R.M

f(x) = x – 4sinx

f'(x) = 1 – 4cos x

$x_{n+1} = x_n - \frac{x_n - 4\sin x_n}{1 - 4\cos x_n}$

$Taking\ x_0 = 2.47$

$x_1 = 2.47 - \frac{2.47 - 4\sin 2.45}{1 - 4\cos 2.47}$ = 2.4746

$Error = |2.4746 - 2.47| = 0.0046 > 0.0005$

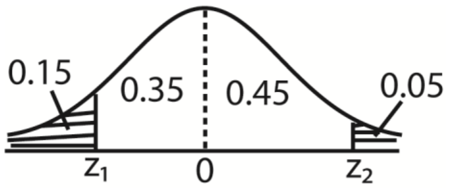
$x_2 = 2.4746 - \frac{2.4746 - 4\sin 2.4546}{1 - 4\cos 2.4746}$ = 2.4746

$Error = |2.4746 - 2.4746| = 0.000 < 0.0005$

∴ 2.475 (3D)

15. The number of cows owned by residents in a village is assumed to be normally distributed. 15% of the residents have less than 60 cows, 5% of the residents have over90 cows.

(a) Determine the values of the mean and standard deviation of the cows. (08marks)



P(0 < Z < Z_1) = 0.35; Z_1 = -1.036

$\frac{60 - \mu}{\sigma}$ = -1.036

$60 - \mu$ = -1.036σ …………… (i)

P(0 < Z < Z_2) = 0.35; Z_2 = 1.645

$\frac{90 - \mu}{\sigma}$ = 1.645

$90 - \mu$ = 1.645σ ………………(ii)

Eqn. (ii) – Eqn. (i)

30 = 2.681σ; σ= 11.1899

From eqn. (i)

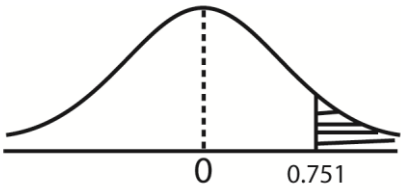
$60 - \mu$ = -1.036 x 11.1899

$\mu = 71.5926$

If there are 200 residents, find how many have more than 80 cows. (04marks)

P(X > 80) = P$\left(Z > \frac{80-71.5926}{11.1899}\right)$

= P(Z > 0.751)



P(Z > 0.751) = 0.5 – (0 < Z < 0.751)

= 0.5 – 0.2737

=0.2263

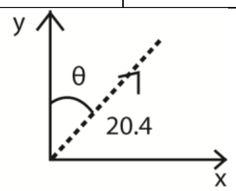
Number of residents =200 x 0.2263 = 45

16. At 12 noon a ship A is moving with constant velocity of 20.4kmh^{-1} in the direction Nθ^0E where tan θ = $\frac{1}{5}$. A second ship B is 15km due to north of A. Ship B is moving with constant velocity of 5kmh-1 in the direction Sα0W, where tan α= ¼. If the shortest distance between the ships is 4.2km, find the time to the nearest minute when the distance between the ships is shortest. (12mars)

Table of results

Vector	Magnitude	Direction
V_A	20.4kmh^{-1}	NθE
V_B	5kmh^{-1}	SαW

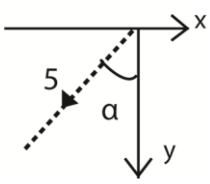
V_A Ξ



tan θ = $\frac{1}{5}$; θ = 11.3^0

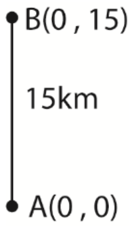
$$V_A = \begin{pmatrix} 20.4\sin 11.3^0 \\ 20.4\cos\ 11.3^0 \end{pmatrix} = \begin{pmatrix} 4 \\ 20 \end{pmatrix}$$

V_B Ξ



tan α = $\frac{1}{5}$; α = 36.87^0

$$V_{-B} = \begin{pmatrix} -5\sin 36.87^0 \\ -5\cos\ 36.87^0 \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$



$r_A = \int V_A dt = \int \begin{pmatrix} 4 \\ 20 \end{pmatrix} dt = \begin{pmatrix} 4t \\ 20t \end{pmatrix} + c$

At t = 0, $r_A = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ => c = $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Hence $r_A = \begin{pmatrix} 4t \\ 20t \end{pmatrix}$

$R_B = \int V_B dt = \int \begin{pmatrix} -3 \\ -4 \end{pmatrix} dt = \begin{pmatrix} -3t \\ -4t \end{pmatrix} + c$

At t = 0, $r_B = \begin{pmatrix} 0 \\ 15 \end{pmatrix}$ => c = $\begin{pmatrix} 0 \\ 15 \end{pmatrix}$

Hence $r_B = \begin{pmatrix} -3t \\ 15 - 4t \end{pmatrix}$

${}_Ar_B = r_A - r_B = \begin{pmatrix} 4t \\ 20t \end{pmatrix} - \begin{pmatrix} -3t \\ 15 - 4t \end{pmatrix} = \begin{pmatrix} 7t \\ 24t - 15 \end{pmatrix}$

$d_s = |{}_Ar_B| = \sqrt{(7t)^2 + (24t - 15)^5}$

but ds = 4.2

⇨ $\sqrt{(7t)^2 + (24t - 15)^5} = 4.2$

$\left(\sqrt{(7t)^2 + (24t - 15)^5}\right)^2 = 4.2^2$

$(7t)^2 + (24t - 15)^5 = 4.2^2$

$47t^2 + 576t^2 - 720t + 225 = 17.64$

$625t^2 - 720t + 207.36 = 0$

t = $\frac{720 \pm \sqrt{(-720)^2 - 4(625)(207.36)}}{2(625)}$ = 0.576hours

= 0.576 x 60 = 35minutes

Hence the time at which the distance is shortest is 12:35pm

Thank you

Yours Dr. Bbosa Science