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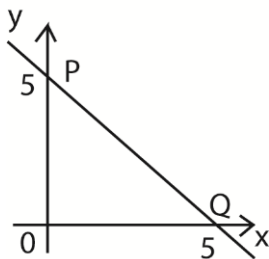
UCE MATHEMATICS PAPER 2 2018 guide

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SECTION A (40 marks)

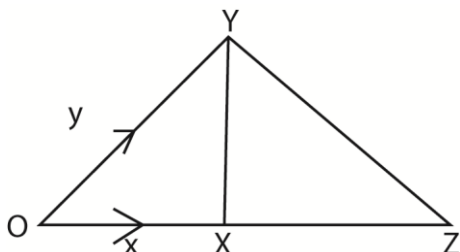
Answer all questions in this section

- Express the recurring decimal $1.633 \dots$ in the form $\frac{a}{b}$ where a and b are integers. (04marks)
- A line passes through points (3, k) and (2, 7). It is parallel to another line whose gradient is 12. Find the value of k. (04marks)
- Calculate the volume of a hemisphere whose radius is 4.9cm (04marks)
- Given that $A = \{2, 3, 5, 7, 11, 13, 17\}$ and $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$, find $n(A \cap B)'$. (04marks)
- The diagram below shows a line which cuts the y-axis at P and x-axis at Q.



Determine the equation of the line (04marks)

- If $\log_x y = 2$ and $xy = 27$, find the value of x and y. (04marks)
- Jane bought a television set at Shs. 450, 000. She sold it at Shs. 550,000. Calculate her percentage profit. (04marks)
- Mugisha, Kate, Okello and Zziwa like the following types of foods; matooke, rice, meat and matooke respectively.
 - List the elements of the domain and range of the relation "likes" (02marks)
 - Draw an arrow diagram to illustrate the relation. (02marks)
- The length of each side of a cube is $2x$ cm. the surface area of the cube is 216cm^2 . Find the length of each side. (04marks)
- In the diagram below $OX = x$, $OY = y$ and $OZ = 3OX$



Express $20Y + ZY$ in terms of x and y . (04marks)

SECTION B (60 MARKS)

Answer any **Five** questions from this section. All questions carry equal marks.

11. Two functions f and h are defined as $F(x) = x^2 - 1$ and $h(x) = x + 3$. Find
- $f'(3)$ (05marks)
 - the value of x if $hf(x) = fh(x)$ (07marks)
12. At a workshop of 150 teachers, it was found that 58 drank juice (J), 66 drank water (w) and 57 drank soda (s). 10 drank water and juice, 11 drank juice and soda and 13 drank water and soda. Some of the teachers drank all the three types of drinks. All the teachers drank at least one of the drinks.
- Show this information on a Venn diagram (07marks)
 - Find the number of teachers who drank all the three types of drinks. (02marks)
 - What is the probability that a teacher chosen at random did not drink water? (03marks)
13. (a) A car driver covered a distance of 60km at 100km/h. A lorry driver covered the same distance but took half an hour more. Calculate the
- time taken by the lorry driver
 - average speed of the lorry driver. (05marks)
- (b) A traffic police patrol car travelling at 120km/h is chasing a taxi 0.5km away and travelling at 100km/h. How far must the police car travel in order to catch up with the taxi? (07mark)
14. the table below shows the tax structure taxable income of public servants working in a certain country.

Income per annum (shs)	Tax rate %
0 – 1,200,000	12.5
1,200,001 – 2,400,000	30.0
2,400,001 – 3,600,000	36.5
3,600,001 and above	45.0

A man's gross annual income is Shs. 6, 460,000. His allowances are

Housing- 125,000 per month

Marriage - $\frac{1}{10}$ of his gross annual income

Medical = Shs. 354,000 per annum

Transport – Shs. 60,000 per annum

Family allowance per annum for only 3 children are as follows:

- Shs. 25, 000 for each child between 10 and 18 years.
- Shs. 32,000 for each child below 9 years.

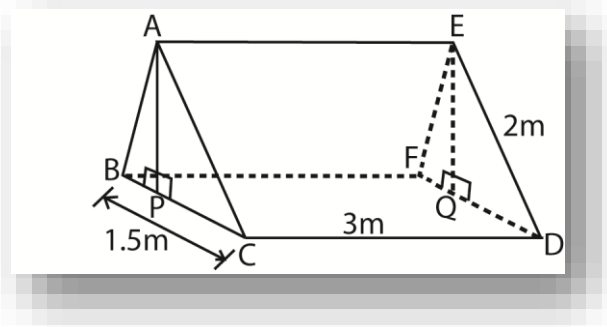
He has to pay an insurance premium of shs. 48,900 per annum.

He has four children with two of them below eight years, one is 16 years and the oldest is 20 years.

Calculate

- His taxable income. (07 marks)
- Income tax paid annually. (05marks)

15. The cost C of operating a day school for one day is partly constant and partly varies as the number of students, n . It costs Shs. 40,000 to run the school when there are 500 students and Shs. 64,000 when there are 900 students.
- Form an equation for cost C and the number of students, n . (08marks)
 - What would be the cost of running the school when there are 700 students? (02marks)
 - If the cost of running the school is Shs. 82,000 per day, how many students are in the school? (02marks)
16. The position vectors of points P , Q and R are $OP = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, $OQ = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$ and $OR = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$. M is a point such that $OM = xOQ$ and $OM = OP + yPR$. Determine the
- Vector PR (03marks)
 - Values of x and y (07 marks)
 - Position vector OM . (02marks)
17. The figure below represents a tent in the form of a triangular prism $ABCDEF$. $\overline{BC} = 1.5m$, $\overline{CD} = 3m$ and slanting edges are $2m$ long.



Calculate

- Height of the tent, AP (02marks)
- Angle between the lines BC and AC (02marks)
- Angle between the planes $ABFE$ and $ACDE$ (03marks)
- Angle between the line CE and base $BCDF$ (03marks)

Solutions

SECTION A (40 marks)

Answer all questions in this section

- Express the recurring decimal $1.633 \dots$ in the form $\frac{a}{b}$ where a and b are integers. (04marks)
 Let $x = 1.633 \dots$
 $10x = 16.33 \dots$
 $10x - x = 16.33 \dots - 1.633$
 $9x = 14.7$
 $x = \frac{14.7}{9} = \frac{147}{90} = \frac{49}{30}$
- A line passes through points $(3, k)$ and $(2, 7)$. It is parallel to another line whose gradient is 12 . Find the value of k . (04marks)
 Parallel lines have equal gradient

$$\frac{7-k}{2-3} = 12$$

$$7 - k = 12 \times 1 = -12$$

$$k = 19$$

3. Calculate the volume of a hemisphere whose radius is 4.9cm (04marks)

Volume of hemisphere = $\frac{1}{2} \times \text{volume of sphere}$

$$= \frac{1}{2} \times \frac{4}{3} \pi r^3 = \frac{1}{2} \times \frac{4}{3} \pi (4.9)^3 = 246.3 \text{cm}^3$$

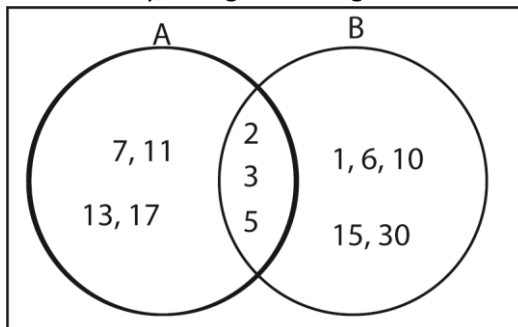
4. Given that $A = \{2, 3, 5, 7, 11, 13, 17\}$ and $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$, find $n(A \cap B)'$. (04marks)

$$(A \cap B) = \{2, 3, 5\}$$

$$(A \cap B)' = \{1, 6, 7, 10, 11, 13, 15, 17, 30\}$$

$$n(A \cap B) = 9$$

Alternatively; Using Venn diagram

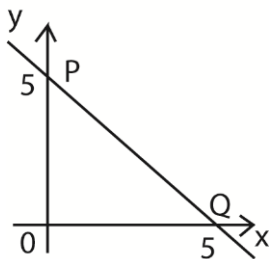


$$(A \cap B) = \{2, 3, 5\}$$

$$(A \cap B)' = \{1, 6, 7, 10, 11, 13, 15, 17, 30\}$$

$$n(A \cap B) = 9$$

5. The diagram below shows a line which cuts the y-axis at P and x-axis at Q.



Determine the equation of the line (04marks)

The equation of the line is the form

$$y = mx + c$$

Substituting for coordinates of P(0,5)

$$5 = c$$

Hence the equation becomes $y = mx + 5$

Substituting for coordinates of Q(5,0)

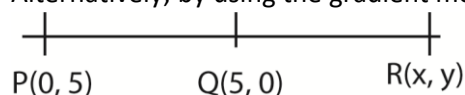
$$0 = 5m + 5$$

$$m = -1$$

Hence equation of line becomes

$$y = -x + 5$$

Alternatively; by using the gradient method



$$\text{Grad of } \overline{PQ} = \text{Grad of } \overline{PR}$$

$$\frac{0-5}{5-0} = \frac{y-5}{x-0}$$

$$-1 = \frac{y-5}{x}$$

$$y = -x + 5$$

6. If $\log_x y = 2$ and $xy = 27$, find the value of x and y . (04marks)

$$x^2 = y$$

also

$$y = \frac{27}{x}$$

$$\Rightarrow x^2 = \frac{27}{x}$$

$$x^3 = 27; x = 3$$

$$y = 3^2 = 9$$

Hence $x = 3$ and $y = 9$

7. Jane bought a television set at Shs. 450, 000. She sold it at Shs. 550,000.

Calculate her percentage profit. (04marks)

Profit = selling price – cost price

$$= 550,000 - 450,000 = 100, 000$$

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$= \frac{100,000}{450,000} \times 100\% = 22.22\%$$

8. Mugisha, Kate, Okello and Zziwa like the following types of foods;

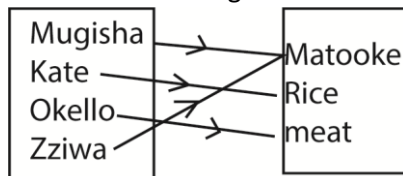
matooke, rice, meat and matooke respectively.

- (a) List the elements of the domain and range of the relation “likes” (02marks)

Domain = { Mugisha and Zziwa, Kate, Okello}

Range = {matooke, rice, meat}

- (b) Draw an arrow diagram to illustrate the relation. (02marks)



9. The length of each side of a cube is $2x$ cm. the surface area of the cube is 216cm^2 . Find the length of each side. (04marks)

$$\text{Surface area for cube} = 2(l^2 + l^2 + l^2) = 6l^2$$

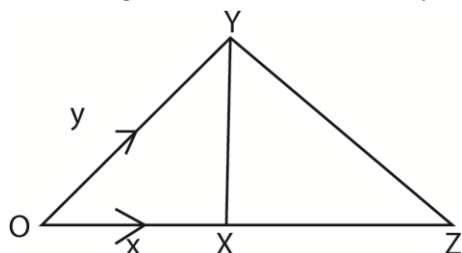
$$6(2x)^2 = 216$$

$$24x^2 = 216$$

$$x = \sqrt{\frac{216}{24}} = 3\text{cm}$$

$$l = 2x = 2(3) = 6$$

10. In the diagram below $OX = x$, $OY = y$ and $OZ = 3OX$



Express $2OY + ZY$ in terms of x and y . (04marks)

$$\begin{aligned} 2OX + ZY &= 2OX + ZO + OY \\ &= 2x - 3OX + y \\ &= 2x - 3x + y \\ &= y - x \end{aligned}$$

SECTION B (60 MARKS)

Answer any **Five** questions from this section. All questions carry equal marks.

11. Two functions f and h are defined as $F(x) = x^2 - 1$ and $h(x) = x + 3$. Find

(a) $f'(3)$ (05marks)

$$\begin{aligned} \text{Let } y &= x^2 - 1 \\ x^2 &= y + 1 \\ x &= \sqrt{y + 1} \\ \therefore f'(x) &= \sqrt{y + 1} \\ f'(3) &= \sqrt{3 + 1} = \pm 2 \end{aligned}$$

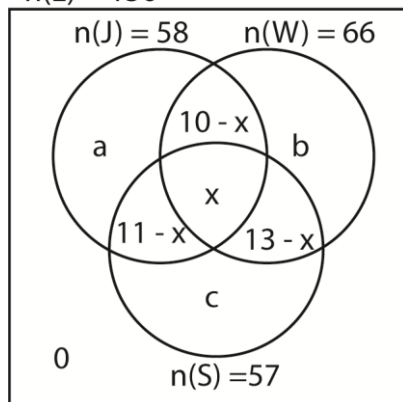
(b) the value of x if $hf(x) = fh(x)$ (07marks)

$$\begin{aligned} hf(x) &= fh(x) \\ h(x^2 - 1) &= f(x + 3) \\ x^2 - 1 + 3 &= (x + 3)^2 - 1 \\ x^2 + 2 &= x^2 + 6x + 9 - 1 \\ 2 &= 6x + 8 \\ 6x &= -6 \\ x &= -1 \end{aligned}$$

12. At a workshop of 150 teachers, it was found that 58 drank juice (J), 66 drank water (w) and 57 drank soda (s). 10 drank water and juice, 11 drank juice and soda and 13 drank water and soda. Some of the teachers drank all the three types of drinks. All the teachers drank at least one of the drinks.

(a) Show this information on a Venn diagram (07marks)

$$\begin{aligned} n(E) &= 150, n(J) = 58, n(W) = 66, n(S) = 57, n(W \cap J) = 10, n(J \cap S) = 11, n(W \cap S) = 13 \\ n(W \cup J \cup S)' &= 0, n(W \cap J \cap S) = x \\ n(E) &= 150 \end{aligned}$$



(b) Find the number of teachers who drank all the three types of drinks. (02marks)

$$\begin{aligned} n(J) \text{ only} &= a = 58 - (10 - x + x + 11 - x) = 58 - (21 - x) = 37 + x \\ n(W) \text{ only} &= b = 66 - (10 - x + x + 13 - x) = 66 - (23 - x) = 43 + x \\ n(S) \text{ only} &= c = 57 - (11 - x + x + 13 - x) = 57 - (24 - x) = 33 + x \end{aligned}$$

$$58 + 43 + x + 13 - x + 33 + x = 150$$

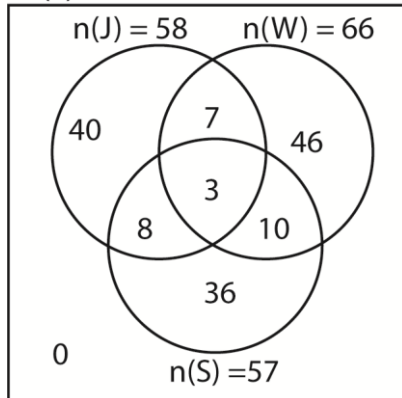
$$147 + x = 150$$

$$x = 3$$

hence the number of teachers who drank all the three types of drinks = 3

- (c) What is the probability that a teacher chosen at random did not drink water? (03marks)

$$n(E) = 150$$



$$\text{Number of those who do not drink water} = 40 + 8 + 36 = 84$$

$$\text{Probability that a teachers chosen did not drink water} = \frac{84}{150} = \frac{14}{25}$$

13. (a) A car driver covered a distance of 60km at 100km/h. A lorry driver covered the same distance but took half an hour more.

Calculate the

- (i) time taken by the lorry driver

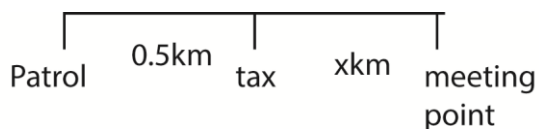
$$\begin{aligned} \text{time taken by the car} &= \frac{60}{100} = \frac{3}{5} = 0.6\text{h} \\ &= 0.6 \times 60 = 36\text{min} \end{aligned}$$

$$\text{Time taken by the lorry} = 36 + 30 = 1\text{h } 6\text{min}$$

- (ii) average speed of the lorry driver. (05marks)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time taken}} = \frac{60}{1\frac{6}{60}} = \frac{60 \times 60}{66} = 54.55\text{km/h}$$

- (b) A traffic police patrol car travelling at 120km/h is chasing a taxi 0.5km away and travelling at 100km/h. How far must the police car travel in order to catch up with the taxi? (07mark)



Let x = distance travelled by the taxi before being caught up.

For Patrol car

$$\text{Distance} = 0.5 + x$$

$$\text{Time taken} = \frac{\text{distance}}{\text{speed}} = \frac{0.5+x}{120}$$

For taxi

$$\text{Time taken} = \frac{x}{100}$$

But time taken is the same

$$\Rightarrow \frac{0.5+x}{120} = \frac{x}{100}; x = 2.5\text{km}$$

Distance travelled by the Patrol car = $0.5 + 2.5 = 3\text{km}$

14. The table below shows the tax structure taxable income of public servants working in a certain country.

Income per annum (shs)	Tax rate %
0 – 1,200,000	12.5
1,200,001 – 2,400,000	30.0
2,400,001 – 3,600,000	36.5
3,600,001 and above	45.0

A man's gross annual income is Shs. 6, 460,000. His allowances are

Housing- 125,000 per month

Marriage - $\frac{1}{10}$ of his gross annual income

Medical = Shs. 354,000 per annum

Transport – Shs. 60,000 per annum

Family allowance per annum for only 3 children are as follows:

- Shs. 25, 000 for each child between 10 and 18 years.
- Shs. 32,000 for each child below 9 years.

He has to pay an insurance premium of shs. 48,900 per annum.

He has four children with two of them below eight years, one is 16 years and the oldest is 20 years.

Calculate

- (a) His taxable income. (07 marks)

Taxable income = gross income – allowances

Allowances

Allowances	Rate	Total per annum
Housing	125,000 per month	1,500,000
Marriage	$\frac{1}{10} \times 6,460,000 \text{ pa}$	646,000
Medical	354,000 pa	354,000
Transport	60,000 per month	720,000
Children as follows		
<8 years = 2	2 x 32,000 pa	64, 000
16 years =	1 x 25,000pa	25,000
Insurance	48,900pa	48,900
Sum		3, 357,900

Taxable income = $6, 460,000 - 3,357,900 = 3,02,100$

- (b) Income tax paid annually. (05marks)

Taxable income = 3,102,100	Tax rate%	Income tax paid
0 – 1,200,000	12.5	$\frac{12.5}{100} \times 1,200,000 = 150,000$
1,200,001 – 2,400,000	30.0	$\frac{30}{100} \times 2,400,000 = 360,000$
(2,400,001 – 3,102,100) =702,099	36.5	$\frac{36.5}{100} \times 702,099 = 256,266.5$
Total		766,266.5

Hence total income tax = shs. 766,266.5

15. The cost C of operating a day school for one day is partly constant and partly varies as the number of students, n. it costs Shs. 40,000 to run the school when there are 500 students and Shs. 64,000 when there are 900 students.

(a) Form an equation for cost C and the number of students, n. (08marks)

Part of C = a (where a is constant)

And another part of C = kn (where k is constant)

$$\Rightarrow C = a + kn$$

Substituting for C = 40,000 and n = 500

$$40,000 = a + 500k \dots\dots\dots (i)$$

Substituting for C = 64,000 and n = 900

$$64,000 = a + 900k$$

Equation (ii) – eqn. (i)

$$400k = 24,000$$

$$k = 60$$

From (i)

$$40,000 = a + 500 \times 60$$

$$a = 10,000$$

substituting for a and k

$$C = 10,000 + 60n$$

(b) What would be the cost of running the school when there are 700 student?(02marks)

$$C = 10,000 + 60 \times 700 = \text{shs. } 52,000$$

(c) If the cost of running the school is shs. 82,000 per day, how many students are in the school? (02marks)

$$82,000 = 10,000 + 60n$$

$$n = \frac{72,000}{60} = 1,200$$

16. The position vectors of points P, Q and R are $OP = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$, $OQ = \begin{pmatrix} -7 \\ -1 \end{pmatrix}$ and $OR = \begin{pmatrix} -1 \\ 9 \end{pmatrix}$. M is a point such that $OM = xOQ$ and $OM = OP + yPR$

Determine the

(a) Vector PR (03marks)

$$\begin{aligned} \underline{PR} &= \underline{OR} - \underline{OP} \\ &= \begin{pmatrix} -1 \\ 9 \end{pmatrix} - \begin{pmatrix} -3 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 14 \end{pmatrix} \end{aligned}$$

(b) Values of x and y (07 marks)

$$\underline{OM} = x\underline{OQ} = x \begin{pmatrix} -7 \\ -1 \end{pmatrix} = \begin{pmatrix} -7x \\ -x \end{pmatrix}$$

$$\text{Also } \underline{OM} = \underline{OP} + y\underline{PR} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} + y \begin{pmatrix} 2 \\ 14 \end{pmatrix} = \begin{pmatrix} -3 + 2y \\ -5 + 14y \end{pmatrix}$$

Equating the two equation

$$\begin{pmatrix} -7x \\ -x \end{pmatrix} = \begin{pmatrix} -3 + 2y \\ -5 + 14y \end{pmatrix}$$

$$-7x = -3 + 2y \dots\dots\dots (i)$$

$$-x = -5 + 14y \dots\dots\dots (ii)$$

Eqn.(ii) – 7 x eqn. (i)

$$-97y = -32$$

$$y = \frac{-32}{-97} = \frac{1}{3}$$

From eqn. (ii)

$$5 = x + 14y = x + 14 \times \frac{1}{3} = x + \frac{14}{3}$$

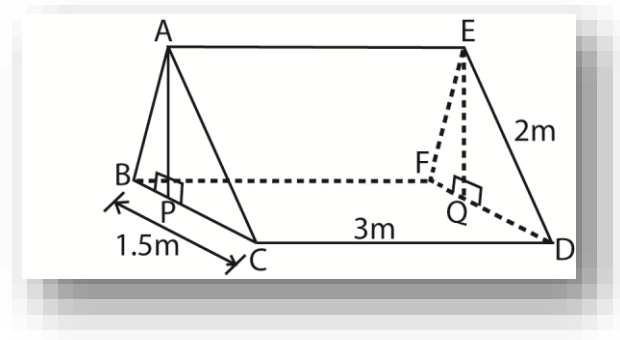
$$x = 5 - \frac{14}{3} = \frac{15-14}{3} = \frac{1}{3}$$

Hence $x = \frac{1}{3}$ and $y = \frac{1}{3}$

(c) Position vector OM. (02marks)

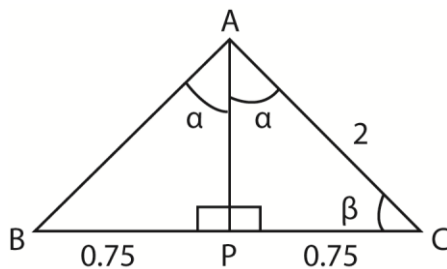
(d) $\underline{OM} = \begin{pmatrix} -7x \\ -x \end{pmatrix} = \begin{pmatrix} -\frac{7}{3} \\ -\frac{1}{3} \end{pmatrix}$

17. The figure below represents a tent in a form of a triangular prism ABCDEF. $\overline{BC} = 1.5m$, $\overline{CD} = 3m$ and slanting edges are 2m long.



Calculate

(a) Height of the tent, AP (02marks)



$$AP^2 = 2^2 - (0.75)^2 = 3.4375$$

$$AP = \sqrt{3.4375} = 1.854m$$

(b) Angle between the lines BC and AC (02marks)

$$\cos \beta = \frac{0.75}{2}$$

$$\beta = \cos^{-1} \left(\frac{0.75}{2} \right) = 67.98^\circ$$

(c) Angle between the planes ABFE and ACDE (03marks)

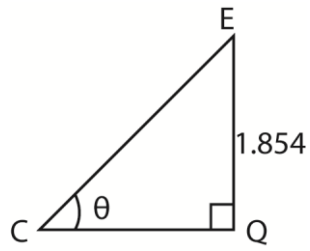
$$\theta + 90 + 67.98 = 180$$

$$\theta = 22.02$$

$$2\theta = 44.04$$

Hence the angle between planes ABFE and ACDE = 44.04°

(d) Angle between the line CE and base BCDF (03marks)



$$CE^2 = 3^2 + 2^2 = 9 + 4 = 13$$

$$CE = \sqrt{13}$$

$$\sin \theta = \frac{1.854}{\sqrt{13}}$$

$$\theta = \sin^{-1} \left(\frac{1.854}{\sqrt{13}} \right) = 30.9^\circ$$

Thank you

Dr. Bbosa Science