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Equilibrium of forces

several forces acting on a particle are said to be in equilibrium when the resultant force is equal to zero

$$\text{i.e. } F_R = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Example 1

For the following set of forces in equilibrium find the values of a and b in each case

(i) $(6i + 4j)N, (-2i - 5j)N, (ai + bj)N$

$$\begin{pmatrix} 6 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ -5 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$6 - 2 + a = 0 \Rightarrow a = -4$$

$$4 - 5 + b = 0 \Rightarrow b = 1$$

(ii) $(5i + aj + ck)N, (bi - 6j - k)N, \text{ and } (-3i + 2j + ck)$

$$\begin{pmatrix} 5 \\ a \\ c \end{pmatrix} + \begin{pmatrix} b \\ -6 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

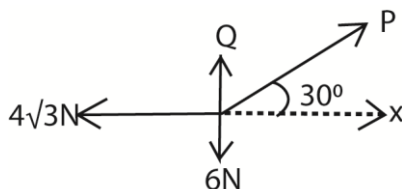
$$5 + b - 3 = 0 \Rightarrow b = -2$$

$$a - 6 + 2 = 0 \Rightarrow a = 4$$

$$2c - 1 = 0 \Rightarrow c = 0.5$$

Example 2

In the diagram below, the particle is in equilibrium, find the values of P and Q.

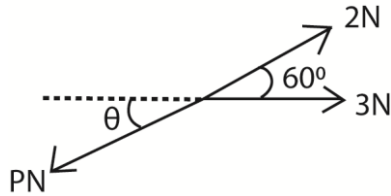


Solution

$$\begin{pmatrix} 0 \\ Q \end{pmatrix} + \begin{pmatrix} P \cos 30 \\ Q \cos 30 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \end{pmatrix} + \begin{pmatrix} -4\sqrt{3} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \left| \begin{array}{l} 0 + P \cos 30 + 0 - 4\sqrt{3} = 0; \Rightarrow P = 8N \\ Q + Q \cos 30 - 6 + 0 = 0; \Rightarrow Q = 2N \end{array} \right.$$

Example 3

Diagram below shows three coplanar forces of magnitude 2N, 3N and PN all acting at point O in the direction shown. Given that the forces are in equilibrium, Find the value of P



Solution

$$-P\cos\theta + 2\cos 60 + 3 = 0 \dots (i)$$

$$\cos\theta = \frac{2\cos 60 + 3}{P}$$

$$-P\sin\theta + 2\sin 60 = 0$$

$$\sin\theta = \frac{2\sin 60}{P} \dots\dots\dots (ii)$$

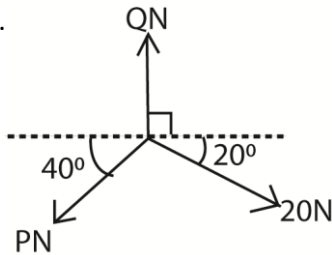
Eqn. (i) and (ii)

$$\theta = \tan^{-1}\left(\frac{2\sin 60}{2\cos 60 + 3}\right) = 23.413$$

$$\text{from (ii) } P = \frac{2\sin 60}{\sin 23.413} = 4.3589N$$

Revision Exercise

1.



- (i) The diagram above shows three coplanar forces in equilibrium. Find the value of P and Q.
- (ii) If the direction of Q is now reversed, find the magnitude and direction of the resultant
 [(i) 24.5N, 22.6N; (ii) 45.2N]

2. Forces $F_1 = (-3i + 7j)N$, $F_2 = (i - j)N$ and $F_3 = (pi + qj)$ act on a particle.

- (i) If the particle is in equilibrium, find the values of p and q. [p = 2, q = -6]
- (ii) Find the magnitude and direction of the resultant of F_1 and F_2 . [6.3246N, 71.57°]

3. Forces of 6N, 5N, 8N, 5N and 9N act on a particles in the direction N30°E, N30°W, S50°E, N60°W, N80°E and S40°W respectively. Find the additional force that will keep the system of force in equilibrium. [5.358N at 68.920 above the positive axis]

4. Forces of 7N, 2N, 4N and 5N act on a particle in directions of 0600, 1600 2000 and 3150 respectively. Find the additional force that will keep the system of forces in equilibrium. [2.3125N at 37.18° below the negative axis]

5. Forces of 2N, 1N, 3N and 4N act on a particle in the direction 0°, 90°, 270° and 330° respectively. Find the additional force that will keep the system of forces in equilibrium. [6.8N at 36° above the negative axis]

6. Forces of 6N, 5N, 7N, 4N, $3\sqrt{2} N$ and $7\sqrt{2}N$ act in direction AB, CB, CD, DA, CA and DB respectively on a square ABCD. Find the additional force that will keep the system of forces in equilibrium.
[19.2N at 81° above the negative axis]

7. Forces 8N, 7N, 6N, 4N, 7N and 6N act along the sides of a regular hexagon ABCDEF in direction AB, CB, CD, DE, EF and FA respectively. Find the additional force that will keep the system of forces in equilibrium. [12.49N at 76° above AB]

Thank you

Dr. Bbosa Science