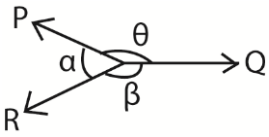
Equilibrium of three forces L ami's theorem

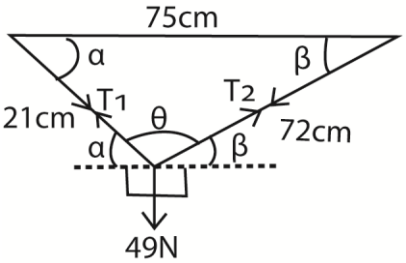
For any three forces acting on a particle in equilibrium where none of them is parallel to each other, Lami's theorem is applicable



$$\frac{P}{sin\beta} = \frac{Q}{sin\alpha} = \frac{R}{sin\theta}$$

Example 1

A weight of 49N is suspended by two strings of length 21 cm and 72cm attached to 2 points in a horizontal line a distance of 75cm apart. Find the tension in the strings so that the particle remain in equilibrium



By cosine rule:

$75^2 = 21^2 + 72^2 - 2 \times 21 \times 72\cos\theta$

$\theta = 90^0$

Similarly, $\beta = 16.26^0$ and $\alpha = 73.74^0$

$$\frac{T_1}{\sin(16.26+90)} = \frac{49}{\sin 90};$$

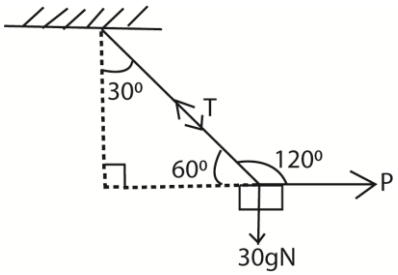
$$\therefore T_1 = 47N$$

$$\frac{T_2}{\sin(73.74+90)} = \frac{49}{\sin 90};$$

$$\therefore T_2 = 13.72N$$

Example 2

Mass of 30kg hangs vertically at the end of a light string. If the mass is pulled by a horizontal force P so that the string makes 30^0 with the vertical. Find the magnitude of the force and the tension in the string so that the particle remain in equilibrium.

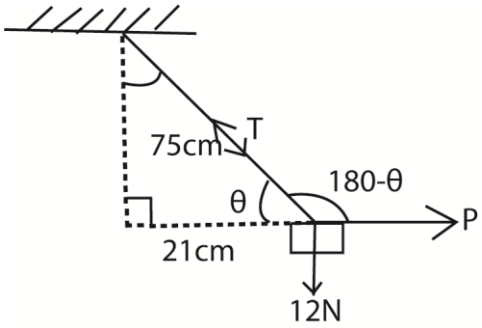


$$\frac{T}{\sin 90} = \frac{30\ x\ 9.8}{\sin 120};\ T = 339.48N$$

$$\frac{P}{\sin(60+90)} = \frac{30\ x\ 9.8}{\sin 120};\ P = 16974N$$

Example 3

One end of a light inextensible string of length 75cm is fixed to a point on a rigid pole. The particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force, P. Find the magnitude of the force, P and the tension of the string so that the particle remain in equilibrium



$\theta = \cos^{-1}\left(\frac{21}{75}\right) = 73.74^0$

$\frac{T}{\sin 90} = \frac{12}{\sin(180-73.74)}$

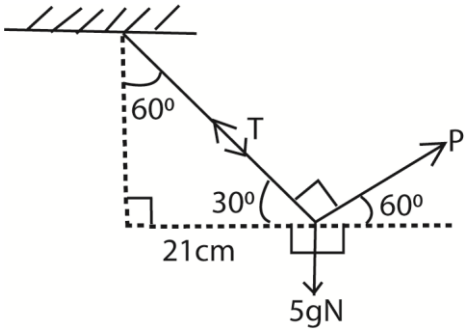
T = 12.5N

$\frac{P}{\sin(90+73.4)} = \frac{12}{\sin(180-73.74)}$

P = 3.5N

Example 4

A light inextensible string AB whose end A is fixed has end B attached to a particle of mass 5kg. A force P acting perpendicular to the string is applied on the particle keeping it in equilibrium with the string inclined at 60^0 to the vertical. Find the value of P and the tension in the string



$\frac{T}{\sin(90+60)} = \frac{5\ x\ 98}{\sin 90}$

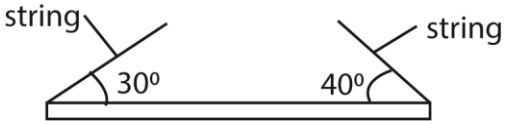
T = 24.5N

$\frac{5\ x\ 98}{\sin 90} = \frac{P}{\sin\ (90+30)}$

P = 42.44N

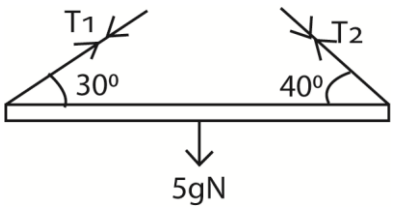
Example 5

A non-uniform beam of mass 5kg rests horizontally in equilibrium supported by two strings attached to the ends of the beam.



The strings makes 300 and 400 with the horizontal beam as shown above. Find the tension in the strings.

Solution



(→) $T_1\cos30 = T_2\cos40$; $T_1 = 0.8846T_2$

(↑)$T_1\sin30 + T_2\sin40 = 5g$

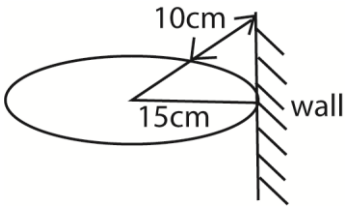
$0.8846T_2\sin30 + T_2\sin40 = 5 \times 9.8$

$T_2 = 45.159N$

$T_1 = 0.8846 \times 45.159 = 39.94N$

Example 6

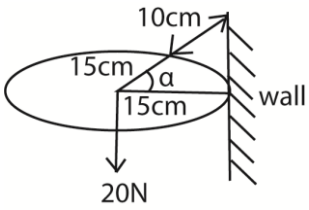
A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.



(i) calculate the reaction on the sphere due to the wall

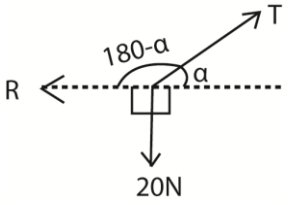
(ii) Find the tension in the string

Solution



$\alpha = \cos^{-1}\left(\frac{15}{25}\right) = 53.13^0$

Using Lami's theory



$\frac{T}{sin90} = \frac{20}{\sin(180-53.13)}$; T =25N

$\frac{R}{\sin(90+53.13)} = \frac{20}{\sin(180-53.13)}$; R = 15N

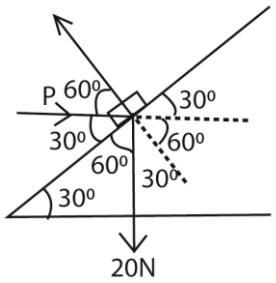
Example 7

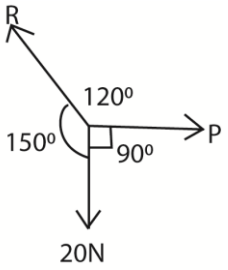
A particle of weight 20N is held at equilibrium on a smooth plane inclined at 30^0 to the horizontal by a horizontal force P.

(i) Find the value of P and the reaction between the particle and the plane.

(ii) If the force P is removed and a string parallel to the plane is used to hold the particle, find the tension in the string and the new value of the reaction.

Solution

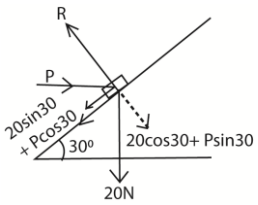




$\frac{P}{sin150} = \frac{R}{sin90} = \frac{20}{sin120}$

R = 23.09N and P = 11.55N

Alternatively: by resolving forces



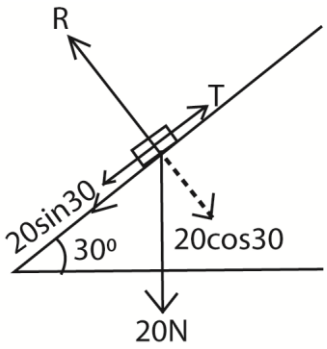
At equilibrium parallel to plane forces = 0

Pcos30 + 20sin30 = 0; P = 11.55N

R = 20cos30 + Psin30

R = 20cos30 + 11.55sin30 = 23.09N

(ii)



Parallel to the plane T = 20sin30 = 10N

Perpendicular to the plane R = 20cos30 = 13N

Alternatively by Lami's theory

$$\frac{T}{sin150} = \frac{R}{sin120} = \frac{20}{sin90}$$

T = 10N

R = 1.3N

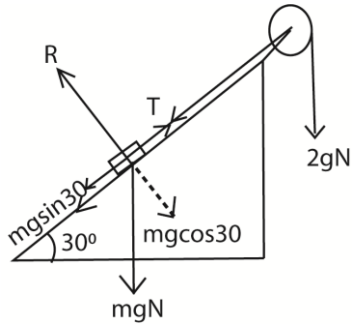
Example 8

A light inextensible string passes over a smooth fixed pulley at the top of a smooth plane inclined at 300 to the horizontal. A particle of mass 2kg is attached to one end of the string and rests vertically in equilibrium when the particle of ma resting on the surface of the plane is attached to the other end of the string. Find

(i) the normal reaction between m and the plane
(ii) tension in the string and the value of m.

Solution

By resolving forces



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

Parallel to the plane

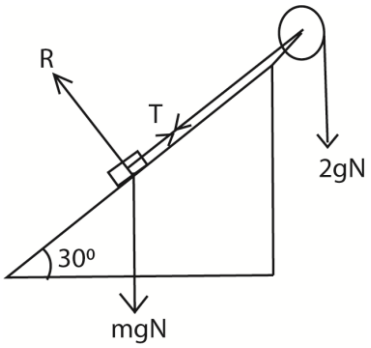
T – mgsin30 = 0; m = 4kg

Perpendicular to the plane

R = mgco30

R = 4 x 98cos30 = 33.98

Alternatively by using Lami's theorem



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

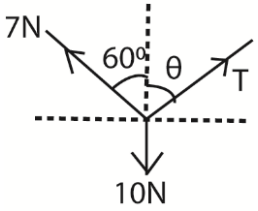
$$\frac{T}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$$

$$\frac{19.62}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$$

m = 4kg and R = 33.98N

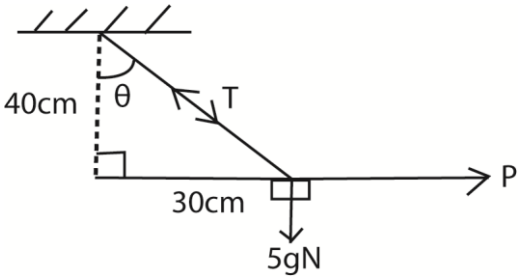
Revision exercise

1. A particle P of mass 2kg is suspended from a fixed point O by means of a light inextensible string. The string is taut and makes an angle of 300 with the downward vertical through O and a particle is held in equilibrium by means of a horizontal force of magnitude F acting on the particle. Find the value of F and the tension in the string [F = 11.3161, T = 22.6321N]
2. A particle of mass 3kg lies on a smooth plane inclined at angle θ to the horizontal, where tanθ = $\frac{3}{4}$. The particle is held in equilibrium by horizontal force of magnitude FN. The line of action of this force is the same vertical plane as a line of greatest slope of inclined plane. Find the value of F. [22.05N]
3. The diagram below shows a body of weight 10N supported in equilibrium by two light inextensible strings. The tension in the strings are 7N and T and the angle the string makes with the upward vertical are 60^0 and θ respectively.



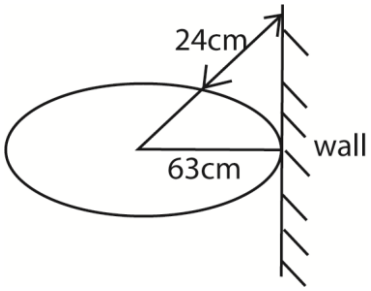
Find T and θ. [T = 8.9N, θ = 43^0]

4. A particle of weight 8N is attached to a point B by a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^0 to the downward vertical. A force F at B acting at right angles to AB, keeps the particle in equilibrium. Find the magnitude of force F and the tension in the string. [4N, $4\sqrt{3}N$]
5. The diagram shows a light inextensible string with one end fixed at A and a mass of 5kg suspended at the other end.



The mass is held in equilibrium at an angle θ to the downward vertical by a horizontal force P. Find the value of θ, P and the tension in the string [θ = 36.9^0, P = 36.75N, T = 61.25N)

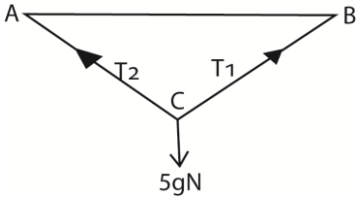
6. A sphere of mass 5kg and radius 63cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 24cm attached to a point on the sphere an to a point on the wall as shown.



Find the tension in the string. [71.05N]

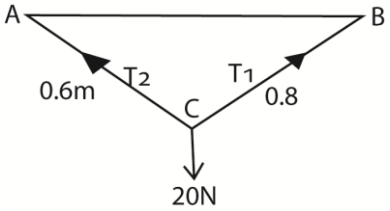
7. A particle whose weight is 50N is suspended by a light string which is 35^0 to the vertical under the action of a horizontal force F. Find the force F and the tension in the string. [35.0N, 61.0N]

8. A particle of weight w rests on a smooth plane which inclined at 40^0 to horizontal. The particle is prevented from slipping by a force of 50.0N acting parallel to the plane and up a line of greatest slope. Calculate w and reaction due to the plane. [77.8N, 59.6N]
9. A mass of 2kg is suspended by two light inextensible strings. One making an angle of 600 with the upward vertical and the other 300with the upward vertical. Find the tension in each string. [9.8N, 17.0N]
10. A heavy uniform rod of weight W is hung from a point by two equal strings, one attached to each end of the rod. A body of weight w is hang half-way between A and the center of the rod. Prove that the ratio of tension in the string is $\frac{2W+3w}{2W+w}$.
11. A non-uniform beam AB of length 8m and its weight 10N acts from a point G between A and B such that AG = 6m. The beam is supported horizontally by strings attached to A and B. The string attached to A makes an angle of 30^0 with AB. Find the angle that the string attached to B makes with AB and find the tension in the strings. [60^0, 5N, 8.66N]
12. A light inextensible string of length 40cm has its upper end fixed to a point A and carries a mass of 2kg at its lower end. A horizontal force applied to the mass keeps it in equilibrium, 20cm from the vertical through A. Find the magnitude of this horizontal force and the tension in the string. [11.3N, 22.6N]
13. The diagram shows a body of mass 5kg supported by two light inextensible strings, the other ends of which are attached to two points A and B on same level as each other end 7m apart.



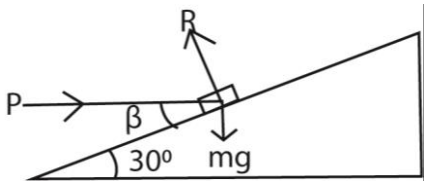
The body rests in equilibrium at 3m vertically below AB. If angle CBA = 45^0, find T_1 and T_2 the tensions in the strings. [35N, $28\sqrt{2}N$]

14. The diagram shows a body of weight 20N supported by two light inextensible strings of length 0.6m and 0.8m from two points 1m apart on a horizontal beam.



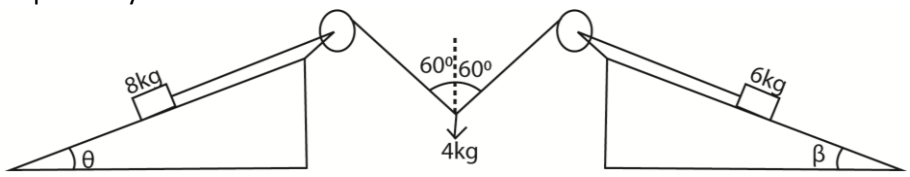
The body rests in equilibrium, find T_1 and T_2 the tensions in the strings. [16N, 12N]

15. A light inextensible string of length 50cm has its upper end fixed at point A and carries a particle of 8kg at its lower end. A horizontal force P applied to the particle in equilibrium 30cm from the vertical through A, find the magnitude of P and the tension in the string. [58.8N, 98N}
16. A article is in equilibrium under the action of forces 4N due north, 8N due west, $5\sqrt{2}$N south east and P, find the magnitude and direction of P. [3.16N, N71.6^{0}E]
17. A force P holds a particle of mass mkg in equilibrium on a smooth plane which is inclined at 300 to the horizontal.



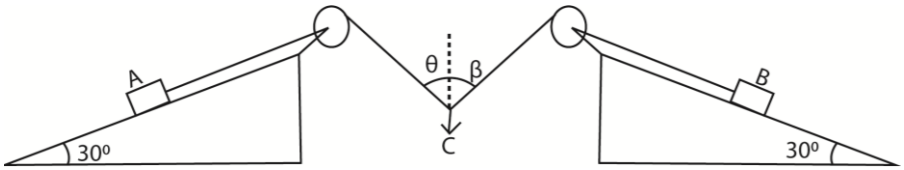
If P makes an angle β with the plane, find β when R the normal reaction between the particle and the plane is 15mg [51.7^0]

18. The diagram below shows masses of 8kg and 6kg lying on smooth planes of inclination θ and β respectively



Light inextensible strings attached to these masses pass along the line of greatest slopes over smooth pulleys and are connected to 4kg mass hanging freely. The strings both make an angle of 600 with the upward vertical as shown above. If the system rest in equilibrium find θ and β. [$\theta = 30^0$ and β 41.8^0]

19. The diagram below shows masses A and B each lying on smooth planes of inclination30^0.



Light inextensible strings attached to A and B pass along the lines of greatest slopes, over smooth pulleys and are connected to a third mass C hanging freely. The strings make angles of θ and β with the upward vertical as shown above. If A, B and C have masses 2m, m, and m respectively and the system rests in equilibrium show that $\sin\theta = 2\sin\beta$ and $\cos\beta + 2\cos\theta = 2$. Hence find θ and β. [$29.0^0$, 75.5^0]