Errors

An error, commonly known as absolute error is the absolute difference between exact value and approximate value.

Source of errors

(a) Rounding off

These errors that arise as a result of simply approximating the exact value of different numbers.

Example 1

Round off the following numbers to the given number of decimal places or significant figures.

(i) 3.896234 to 4 dp [3.8962]
(ii) $\frac{2}{3}$ to 3dp [0.667]
(iii) 5.002570 to 3s.f [5.00]
(iv) 0.00652673bto4 s.f [0.006527]
(v) 7.00214 to 4 s.f. [7.002]
(vi) 5415678 to 3 s.f. [5420000]

(b) Truncation

These occur when an infinite number is terminated/cutoff (without rounding off] at some point.

Example 2

Truncate the following number to the given number of decimal places (d.p) or significant figures. s.f.

(i) 4.56172 to 2dp [4.56] (ii) $\frac{2}{3}$ to 3dp [0.666] (iii) 1.345618 to 4 s.f. [1.345]

Common terms used

(a) Error or absolute error
If x represent an approximate value of X and Δx is the error approximation
$|Error| = |exact\ value - approximate\ value|$
$|\Delta x| = |X - x|$

Example 3

Round off 32.5263 to 2 dp and determine the absolute error.

Solution

X = 32.5263, x = 32.53

$|\Delta x| = |X - x|$ = $|32.5263 - 32.53|$ = 0.0037

(b) Relative error

Relative error = $\frac{absolute\ error}{exact\ value} = \frac{|\Delta x|}{X} = \frac{|X-x|}{X}$

(c) Percentage error or percentage relative error

Percentage relative error = $\frac{absolute\ error}{exact\ value} x\ 100\% = \frac{|\Delta x|}{X} x\ 100\% = \frac{|X-x|}{X}\ x\ 100\%$

Example 4

Find the percentage error in rounding off $\sqrt{3}$ 2 dp

Solution

X = $\sqrt{x}$ = 1.732050808, x = 1.73

Percentage error = $\frac{|X-x|}{X}\ x\ 100\%$ = $\frac{|1.732050808 - 1.73|}{1.732050808}\ x\ 100\%$ = 0.118%

(d) Error bound or minimum possible error in an approximated number

This depends on the number of decimal places the number is rounded to. If the number is rounded to n dp, then the maximum possible error in that number is = 0.5 x 10^{-n}.

Example 4

If a student weighs 50kg. Find the range where his weight lies

Solution

n = 0 dp, e = 0.5 x 10^{-0} = 0.5

Range = 50 $\pm$ 0.5 =(49.5, 50.5)

Example 5

If x is given to stated level of accuracy stat the lower and upper bounds of x

(a) 6.45

n = 2 dp, e = 0.5 x 10^{-2} = 0.005

Lower bound = 6.45 – 0.005 = 6.445

upper bound = 6.45 + 0.005 = 6.455

(b) 0.278

n = 3 dp, e = 0.5 x 10^{-3} = 0.0005

(c) Lower bound = 0.278 – 0.0005 = 0.2775

upper bound = 0.278 + 0.0005 = 0.2785

Example 6

A value of w = 150.58m was obtained when measuring the width of the football pitch. Given that the relative error in this value as 0.07%, find the limit within which the value w lies.

% relative error = $\frac{|\Delta w|}{w}\ x100\%$

0.07 =$\frac{|\Delta w|}{150.58}\ x\ 100$

$|\Delta w|$ = 0.105

Lower limit = 150.58 – 0.105 = 150 .475

Upper limit = 150.58 + 0.105 = 150 .685

Absolute error in an operation

When the minimum and maximum value is known then.

absolute error = $\frac{1}{2}[maximum\ value - minimum\ value]$

Absolute error in addition

Given two numbers a and b with errors Δa + Δb

$(a + b)_{max} = a_{max} + b_{max} = (a + \Delta a) + (b + \Delta b)$

$(a + b)_{min} = a_{min} + b_{min} = (a - \Delta a) + (b - \Delta b)$

Absolute error in subtraction

Given two numbers a and b with errors Δa + Δb

$(a - b)_{max} = a_{max} - b_{min} = (a + \Delta a) - (b - \Delta b)$

$(a + b)_{min} = a_{min} - b_{max} = (a - \Delta a) + (b + \Delta b)$

Example 7

Given that a = 2.453, b = 6.79, find the limits and hence absolute error of

(i) a + b

solution

a = 2.453, Δa = 0.0005 and b = 6.79, Δb = 0.005

$(a + b)_{max} = a_{max} + b_{max} = (a + \Delta a) + (b + \Delta b)$

$= (2.453 + 0.0005) + (6.79 + 0.005) = 9.2485$

$(a + b)_{min} = a_{min} + b_{min} = (a - \Delta a) + (b - \Delta b)$

$= (2.453 - 0.0005) + (6.79 - 0.005) = 9.2375$

lower limit = 9.2375; upper limit = 9.2485

absolute error $= \frac{1}{2}[9.2485 - 9.2375] = 0.0055$

(ii) a - b

solution

$(a + b)_{max} = a_{max} - b_{min} = (a + \Delta a) - (b - \Delta b)$

$= (2.453 + 0.0005) - (6.79 - 0.005) = -4.3315$

$(a + b)_{min} = a_{min} - b_{max} = (a - \Delta a) - (b + \Delta b)$

$= (2.453 - 0.0005) - (6.79 + 0.005) = -4.3425$

lower limit = -4.3425; upper limit = -4.3315

absolute error $= \frac{1}{2}[-4.3315 - -4.3425] = 0.0055$

Absolute error in multiplication

Given two numbers a and b with errors Δa + Δb
$(ab)_{max} = a_{max}b_{max}$ = (a + Δa)(b+Δb)
$(ab)_{min} = a_{min}b_{min}$ = (a - Δa)(b - Δb)

Example 8

Given that a = 4.617, and b = 3.65 find the absolute error in ab

solution

a = 4.617, Δa= 0.0005, b = 3.65, Δb = 0.005

$(ab)_{max} = a_{max}b_{max}$ = (a + Δa)(b+Δb) = (4.617 + 0.0005)(3.65b+ 0.005) =16.87696

$(ab)_{min} = a_{min}b_{min}$ = (a - Δa)(b - Δb) = (4.617 - 0.0005)(3.65b - 0.005)=16.82853

absolute error = $\frac{1}{2}[maximum\ value - minimum\ value]$ = $\frac{1}{2}(16.87696 - 16.82853)$ =0.02422

Example 9

Given that a = 4.617, and b = -3.65 find the

(i) Limits of values where ab lies
Solution

a = 4.617, Δa= 0.0005, b = -3.65, Δb = 0.005

$(ab)_{max} = a_{max}b_{max}$ = (4.617 + 0.0005)(-3.65b+ 0.005) = -16.83079
$(ab)_{min} = a_{min}b_{min}$ = (4.617 - 0.0005)(-3.65b - 0.005 = -16.87331
Lower limit = -16.87331; upper limit = -16.83079

(ii) the interval of values where ab lies
(-16.87331, -16.83079)

(iii) the absolute error
Absolute error =$\frac{1}{2}[-16.83079 - \ -16.87331]$= 0.02126

Absolute error n division

Given two numbers a and b with errors Δa + Δb

$$\left(\frac{a}{b}\right)_{max} = \frac{a_{max}}{b_{min}} = \frac{(a + \Delta a)}{(b - \Delta b)}$$

$$\left(\frac{a}{b}\right)_{min} = \frac{a_{min}}{b_{max}} = \frac{(a - \Delta a)}{(b + \Delta b)}$$

Example 10

Given a = 1.26, b = 0.435. Find the absolute error of

(i) Range of value where $\frac{a}{b}$ lies

$$\left(\frac{a}{b}\right)_{max} = \frac{a_{max}}{b_{min}} = \frac{(1.25+0.005)}{(0.435-0.0005)} = 2.91139$$

$\left(\frac{a}{b}\right)_{min} = \frac{a_{min}}{b_{max}} = \frac{(1.25-0.005)}{(0.435+0.0005)} = 2.88175$

Range of values is (2.88175, 2.91139)

(ii) Absolute error

$=\frac{1}{2}(2.91139 - 2.88175)$ = 0.01482

Example 11

(a) Given that y = e^x and x = 0.62correct to two decimal places, find the interval within which the exact value of y lies. (05marks)

$e_x = 0.005$

$y_{max} = e^{0.625} = 1.8682$

$y_{min} = e^{0.615} = 1.8497$

The interval = (1.8497, 1.8682)

(b) Show that the maximum possible relative error in ysin^2x is

$\left|\frac{\Delta y}{y}\right| + 2\cot x\,|\Delta x|$, where$\Delta x$ and Δy are errors in x and y respectively

Hence find the percentage error in calculating ysin^2x if y = 5.2 $\pm$ 0.05 and x = $\frac{\pi}{6} \pm \frac{\pi}{360}$

(07 marks)

z = y$\sin^2$x

$e_z = \Delta y sin^2x$ + 2y$\Delta x cos\ x sinx$

$\frac{e_z}{z} = \frac{\Delta y sin^2x}{\text{y}\sin^2\text{x}} + \frac{2\text{y}\Delta x cos\ x sinx}{\text{y}\sin^2\text{x}}$

$\left|\frac{e_z}{z}\right| = \left|\frac{\Delta y}{y} + 2cotx.\Delta x\right|$

$\leq \left|\frac{\Delta y}{y}\right| + 2cotx.|\Delta x|$

$\therefore$ Maximum possible error is $\left|\frac{\Delta y}{y}\right| + 2cotx.|\Delta x|$

$percentage\ error = \left[\frac{0.05}{5.2} + 2\cot\frac{\pi}{6}.\ \left|\frac{\pi}{360}\right|\right] x\ 100\%$ = 3.9845%

Example 12

Two numbers A and B have maximum possible error e_a and e_b respectively.

(a) Write an expression for the maximum possible error in their sum

Maximum possible error = $|e_a||e_b|$

(b) If A = 2.03 and B = 1.547, find the maximum possible error in A + B (05marks)

$e_a = 0.005, e_b = 0.0005$

$|e_{(A+B)}| = |0.005| + |0.0005|$

= 0.0055

Example 13

Given that y = $\frac{1}{x} + x$ and x = 2.4 correct to one decimal place, find the limits within which y lies. (05marks)

Error in 2.4 = $\frac{1}{2}\ x\ \frac{1}{10} = 0.05$

y_{max} = 2.45 + $\frac{1}{2.35} = 2.8755$

y_{min} = 2.35 + $\frac{1}{2.45} = 2.7582$

∴ the limits are [2.7582, 2.8755]

Example 14

The numbers X = 1.2, Y = 1.33 and Z = 2.245 have been rounded off to the given decimal places. find the maximum possible value of $\frac{Y}{Z-X}$ correct to 3 decimal places

Maximum value = $\frac{(Y+\Delta Y)}{(Z-\Delta Z)-(X+\Delta X)} = \frac{(1.33+0.005)}{(2.245-0.0005)-(\ 1.2+0.05)}$ = 1.342

Revision exercise 1

1. Given the numbers x = 2.678 and y = 0.8765 measured the nearest possible decimal places indicated.
   (i) state the maximum possible error in x ans y [Δx = 0.0005, Δy = 0.00005]
   (ii) find the limits within which the product xy lie [2.3467, 2.3478]
   (iii) determine the maximum possible error in xy [0.000572]

2. The length, width and height of water all rounded off to 3.65m, 2.14m and 2.5m respectively. Determine the least and greatest amount of water the tank can contain [19.066, 19.992]
3. Given that the values x = 4, y = 6 and z = 8 each has been approximate to the nearest integer. find the maximum and minimum values of
   (i) $\frac{y}{x}$ [1.85714, 1.22222]
   (ii) $\frac{z-x}{y}$ [0.90909, 0.46154]
   (iii) $(x+y)z$ [93.5, 67.5]

Error propagation

Triangular inequality state that $|a \pm b| \leq\ |\Delta a| +\ |\Delta b|$

Addition

Consider two numbers X and Y are approximated by x and y with errors Δx and Δy.

$|e_{x+y}|$ = $|(x+\Delta x)+(y+\ \Delta y)-(x+y)|$

$=|\Delta x +\ \Delta y| = |\Delta x| + |\Delta y|$

R.E$_{max}$ = $\left[\left|\frac{\Delta x}{X+Y}\right| + \left|\frac{\Delta y}{X+Y}\right|\right]$

Alternatively

absolute error = $\frac{1}{2}[\max - min]$

$= \frac{1}{2}[(x+\Delta x)+(y+\ \Delta y)] - [(x-\Delta x)+(y-\ \Delta y)]$

$|e_{x+y}|$=$|\Delta x + \Delta y|$= $|\Delta x| + |\Delta y|$

R.E$_{max}$ = $\left[\left|\frac{\Delta x}{X+Y}\right| + \left|\frac{\Delta y}{X+Y}\right|\right]$

Example 15

Given numbers x = 7.824 and y = 3.36 rounded to the given number of decimal places. Find the limits within which (x + y) lies

Solution

Δx = 0.0005, Δy = 0.005

$|e_{x+y}|$=$|\Delta x| + |\Delta y|$= 0.0005 + 0.005 = 0.0055

working value (x + y) = 7.824 + 3.36 = 10.184

Upper limit = 10.184 + 0.0055 = 10.1895

Lower limit = 10.184 - 0.0055 = 10.1785

Alternatively

$(x + y)_{max}$ = 7.8245 + 3.365 =10.1895

$(x + y)_{min}$ = 7.8235 + 3.355 = 10.1785

Example 16

If x = 4.95 and y = 2.2 are each rounded off to the given number of decimal places. Calculate

(i) The percentage error in x + y

Solution

Δx = 0.005, Δy = 0.05

%error = $\left[\left|\frac{\Delta x}{X+Y}\right| + \left|\frac{\Delta y}{X+Y}\right|\right] x\ 100\%$ = $\left[\left|\frac{0.005}{4.95+2.2}\right| + \left|\frac{0.05}{4.95+2.2}\right|\right] x\ 100\%$ = 0.769

Alternatively

Working value x + y = 4.95 + 2.2 = 7.15

$|e_{x+y}|$=$|\Delta x| + |\Delta y|$= 0.005 + 0.05 = 0.055

% error = $\frac{0.055}{7.15} x\ 100\%$ = 0.769

(ii) Find the limit within which (x + y) is expected to lie. Give your answer to two decimal places.
Upper limit = 7.15 + 0.055 = 7.21; lower limit = 7.15 – 0.055 = 7.10
Alternatively
Upper limit = 4.955 + 2.25 = 7.21; lower limit = 4.945 + 2.15 = 7.10

Subtraction

Consider two numbers X and Y are approximated by x and y with errors Δx and Δy.

$|e_{x-y}|$ = $|(x + \Delta x) - (y + \Delta y) - (x - y)|$

=$|\Delta x - \Delta y|$= $|\Delta x| + |\Delta y|$

R.E$_{max}$ = $\left[\left|\frac{\Delta x}{X-Y}\right| + \left|\frac{\Delta y}{X-Y}\right|\right]$

Alternatively

absolute error = $\frac{1}{2}[\max - min]$

$=\frac{1}{2}\{[(x+\Delta x)-(y-\Delta y)]-[(x+\Delta x)-(y+\Delta y)]\}$

$|e_{x-y}|$ = $|\Delta x + \Delta y|$= $|\Delta x| + |\Delta y|$

R.E$_{max}$ = $\left[\left|\frac{\Delta x}{X-Y}\right| + \left|\frac{\Delta y}{X-Y}\right|\right]$

Example 17

Given number x = 6.375 and y 4.46 rounded off to the given number of decimal places. Find the limit within which (x – y) lies

Solution

Δx = 0.0005, Δy = 0.005

$|e_{x-y}|$ = $|\Delta x| + |\Delta y| =$ |0.0005| + |0.005| = 0.0055

working value = 6.375 – 4.46 = 1.915

Upper limit = 1.915 + 0.0055 = 1.9205; Lower limit = 1.915 – 0.0055 = 1.9095

Alternatively

$(x-y)_{max}$ = 6.3755 – 4.455 = 1.9205

$(x-y)_{min}$ = 6.3745 – 4.465 = 1.9095

Example 18

If x = 1.563 and y = 9.87 are each rounded off to the given number of decimal places. Calculate

(i) the percentage error in (x – y)
% error = $\left[\left|\frac{\Delta x}{X-Y}\right| + \left|\frac{\Delta y}{X-Y}\right|\right] x\ 100\% = \left[\left|\frac{0.0005}{1.563-9.87}\right| + \left|\frac{0.005}{1.563-9.87}\right|\right] x 100\%$ = 0.0662
Alternatively
Working value = x – y = 1.563 9.87 = -8.307
$|e_{x-y}|$ = $|\Delta x| + |\Delta y| = |0.0005| + |0.005|$ =0.0055
$\% \ error = \frac{0.0055}{-8.307}\ x\ 100\% = 0.0662$

(ii) the limit within which (x – y) is expected to lie. Give your answer to three decimal places
Upper limit = -8.307 + 0.0055 = -8.302
Lower limit = -8.307 - 0.0055 = -8.313
Alternatively
$(x-y)_{max}$ = 1.5635 – 9.865 =-8.302
$(x-y)_{min}$ =1.5625 – 9.875 =-8.313

Multiplication

Consider two numbers X and Y are approximated by x and y with errors Δx and Δy.

$|e_{xy}| = |(x + \Delta x)(y + \Delta y) - (xy)|$

$= |xy + y\Delta x + x\Delta y + \Delta x\Delta y - xy|$

Since Δx and Δy are very small, $\Delta x\Delta y \approx 0$

$|e_{xy}| = |y\Delta x + x\Delta y| = |y\Delta x| + |x\Delta y|$

R.E$_{max}$ = $\left|\frac{y\Delta x}{xy}\right| + \left|\frac{x\Delta y}{xy}\right|$

$= \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$

Alternatively

absolute error = $\frac{1}{2}|\max - min|$

$= \frac{1}{2}\left[(x + \Delta x)(y + \Delta y) - [(x - \Delta x)(y - \Delta y)]\right]$

$|e_{xy}| = |y\Delta x + x\Delta y| = |y\Delta x| + |x\Delta y|$

R.E$_{max}$ = $\left|\frac{y\Delta x}{xy}\right| + \left|\frac{x\Delta y}{xy}\right|$

Example 19

Given numbers x = 6.375 and y = 4.46 rounded off to eh given number of decimal places. Find the limit within which (xy) lies

Solution

Δx =0.0005 Δy = 0.005

$|e_{xy}| = |y\Delta x| + |x\Delta y| = |6.375\ x\ 0.005| + |4.46\ x\ 0.0005|$ = 0.0341

working value = xy = 6.375 x 4.46 = 28.4325

Upper limit = 28.4325 + 0.0341 =28.4666

Lower limit = 28.4325 - 0.0341 = 28.3984

Alternatively

$(xy)_{max}$ = 6.3755 x 4.465 =28.4666

$(xy)_{min}$ = 6.3745 x 4.455 = 28.3984

Example 20

If x = 1.563 and y = 9.87 are each rounded off to the given number of decimal places. Calculate

(i) Percentage error in (xy)

Δx = 0.0005, Δy = 0.005

% error $=\left\{\left|\frac{\Delta x}{x}\right|+\left|\frac{\Delta y}{y}\right|\right\} x\ 100\% = \left\{\left|\frac{0.0006}{1.563}\right|+\left|\frac{0.005}{9.87}\right|\right\} x\ 100\% = 0.0826$

Alternatively

Working value = 1.563 x 9.87 = 15.4268

$|e_{xy}| = |y\Delta x| + |x\Delta y|$ = 9.87 x 0.0005 + 1.563 x 0.005 =0.0128

$$\%\ error = \frac{0.0128}{15.4268}\ x\ 100\% = 0.0826$$

(ii) the limit within which (xy) is expected to lie. Give your answer to three decimal places.

Upper limit = 15.4268 + 0.0128 = 15.440

Lower limit =15.4268 - 0.0128 =15.414

Alternatively

Upper limit = 1.5635 x 9.875 = 15.440

Lower limit = 1.5625 x 9.865 =15.414

Division

Consider two numbers X and Y are approximated by x and y with errors Δx and Δy.

$\left|e x/_y\right| = \left|\frac{x+\Delta x}{y+\Delta y} - \frac{x}{y}\right| = \left|\frac{xy+y\Delta x-x\Delta y-xy}{y^2+y\Delta y}\right|$

$= \left|\frac{y\Delta x-x\,\Delta y}{y^2\left(1+\frac{\Delta y}{y}\right)}\right|$

Since Δx and Δy are very small, then $\frac{\Delta y}{y} \approx 0$

$\left|e x/_y\right| = \left|\frac{y\Delta x-x\,\Delta y}{y^2}\right|$

$\left|e x/_y\right| \leq \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|}$

$e_{max} = \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|}$

$R.E_{max} = \frac{|y\Delta x|-|x\,\Delta y|}{|y^2|} \div \frac{x}{y}$

$R.E_{max} = \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$

Alternatively

absolute error = $\frac{1}{2}|\max - min|$

$= \frac{1}{2}\left|\frac{(x+\Delta x)}{(y-\Delta y)} - \frac{(x-\Delta x)}{(y+\Delta y)}\right|$

$e x/_y = \left|\frac{x\Delta y+y\Delta x}{y^2-\Delta y^2}\right|$

Since Δx and Δy are very small, then $\Delta y^2 \approx 0$

$\left|e x/_y\right| = \left|\frac{y\Delta x-x\,\Delta y}{y^2}\right|$

$\left|e x/_y\right| \leq \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|}$

$e_{max} = \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|}$

$R.E_{max} = \frac{|y\Delta x|-|x\,\Delta y|}{|y^2|} \div \frac{x}{y}$

$R.E_{max} = \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$

Example 21

Given numbers x = 5.794 and y = 0.28 rounded off to the given number of decimal places. Find limit within which $\frac{x}{y}$ lies

Solution

Δx = 0.0005, Δy = 0.005

$\left|e_{x/y}\right| = \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|}$

$= \frac{|0.28\ x\ 0.0005|+\ |5.794\ x\ 0.005|}{|0.28^2|}$

= 0.3713

Working value $=\frac{x}{y} = \frac{5.794}{0.28}$ = 20.6929

Upper limit = 20.6929 + 0.3713 =21.0642

Lower limit =20.6929 - 0.3713 = 20 .3198

Alternatively

Upper limit = $\frac{5.7945}{0.275}$ =21.079

Lower limit = $\frac{5.7935}{0.285}$ = 20.3281

Example 22

If x = 7.37 and y = 2.00 are each rounded off to the given number of decimal places. Calculate

(i) Percentage error

% error = $=\left\{\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|\right\} x100\% = \left\{\left|\frac{0.005}{7.37}\right| + \left|\frac{0.005}{2.00}\right|\right\} x100\% = 0.318$

Alternatively

$\left|e_{x/y}\right| = \frac{|y\Delta x|+|x\,\Delta y|}{|y^2|} = \frac{|2.00\ x\ 0.005|+\ |7.37\ x\ 0.005|}{|2.00^2|}$ = 0.0117

Working value = $\frac{x}{y} = \frac{7.37}{2.00} = 3.685$

% error = $\frac{0.0117}{3.685}\ x\ 100 = \ 0.318$

(ii) the limit within which $\left(\frac{x}{y}\right)$ is expected to lie. Give your answer to three decimal places.

Upper limit = $3.685 + \ 0.318$ = 3.697

Lower limit = $3.685 - \ 0.318$ = 3.673

Alternatively

Upper limit = $\frac{7.375}{1.995} = 3.697$

Lower limit = $\frac{7.365}{2.005} = 3.673$

Error in functions

Given a function f(x) with a maximum possible error Δx.

Absolute error, $|e|$ = $|\Delta x| f^1(x)$

Maximum possible relative error, R.E = $\frac{|\Delta x|f^1(x)}{f(x)}$

Example 23

Find the absolute error and maximum relative error in each of the following functions

(i) $y = x^4$

$|e|=|\Delta x|f^1(x) = 4x^3|\Delta x|$

R.E = $\frac{|\Delta x|f^1(x)}{f(x)} = \frac{4x^3|\Delta x|}{x^4} = \frac{4|\Delta x|}{x}$

(ii) $y = x^{\frac{3}{2}}$

$|e|=|\Delta x|f^1(x) = \frac{3}{2}x^{\frac{1}{2}}|\Delta x|$

R.E = $\frac{|\Delta x|f^1(x)}{f(x)} = \frac{\frac{3}{2}x^{\frac{1}{2}}|\Delta x|}{x^{\frac{3}{2}}} = \frac{3}{2}\frac{|\Delta x|}{x}$

(iii) $y = \sin x$

$|e|=|\Delta x|f^1(x) = \cos x|\Delta x|$

R.E = $\frac{|\Delta x|f^1(x)}{f(x)} = \frac{\cos x|\Delta x|}{\sin x} = |\Delta x||\cot x|$

Example 24

Given that the error in measuring an angle is 0.4^0. find the maximum possible error and relative error in tanx if $x = 60^0$.

Solution

$|e|=|\Delta x|f^1(x) = (1 + \tan^2 x)|\Delta x|$

$|e|=(1 + \tan^2 60)\left|\frac{0.4}{180}\pi\right| = 0.0280$

R.E = $\frac{0.0280}{\tan 60} = 0.0162$

Error in a function that has more variables

Given a function f(x, y) with a maximum possible error Δx and Δy respectively

Absolute error, $|e| = |\Delta x|f^1(x) + |\Delta y|f^1(y)$

Maximum possible relative error = $\frac{|\Delta x|f^1(x)+|\Delta y|f^1(y)}{f(x,y)}$

Example 25

Given that X and Y are rounded off to give x and y with error Δx and Δy respectively. Show that the maximum relative error recorded in x^4y is given by $4\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$

Solution

$|e| = |\Delta x|f^1(x) + |\Delta y|f^1(y) = |\Delta x 4x^3y| + \Delta y x^4$

$|e| \leq 4|x^3y||\Delta x| + |x^4||\Delta y|$

$|e_{max}| = 4|x^3y||\Delta x| + |x^4||\Delta y|$

R.E = $\frac{4|x^3y||\Delta x| + |x^4||\Delta y|}{x^4y}$

$= 4\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$

Example 26

Show that the maximum possible relative error in ysin^2x is

$\left|\frac{\Delta y}{y}\right| + 2\cot x\,|\Delta x|$, where$\Delta x$ and Δy are errors in x and y respectively

Hence find the percentage error in calculating ysin^2x if y = 5.2 $\pm$ 0.05 and x = $\frac{\pi}{6} \pm \frac{\pi}{360}$ (07 marks)

z = y$\sin^2$x

$e_z = \Delta y sin^2x$ + 2y$\Delta x cos\ x sinx$

$$\frac{e_z}{z} = \frac{\Delta y sin^2x}{\text{ysin}^2\text{x}} + \frac{2\text{y}\Delta x cos\ x sinx}{\text{ysin}^2\text{x}}$$

$$\left|\frac{e_z}{z}\right| = \left|\frac{\Delta y}{y} + 2cotx.\Delta x\right|$$

$$\leq \left|\frac{\Delta y}{y}\right| + 2cotx.|\Delta x|$$

$\therefore$ Maximum possible error is $\left|\frac{\Delta y}{y}\right| + 2cotx.|\Delta x|$

$percentage\ error = \left[\frac{0.05}{5.2} + 2\cot\frac{\pi}{6}.\ \left|\frac{\pi}{360}\right|\right]\ x\ 100\%$ = 3.9845%

Thank you

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