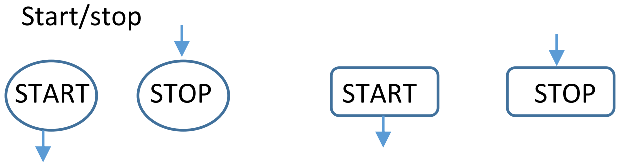
Flowcharts in mathematics

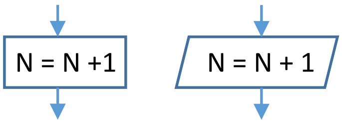
A flow chart is a diagram comprising of systematic steps followed in order to solve a problem.

Shapes used

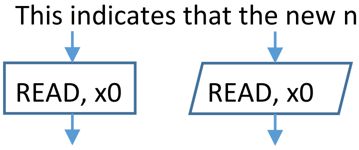
1. Start/stop



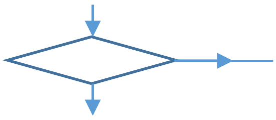
2. OPERATIONASSIGNMENT



This indicates that the new number is obtained by adding one to the previous N



3. Decision box



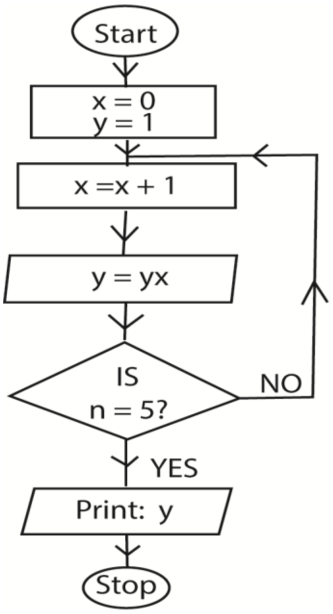
Note: all other shapes can be interchanged except for the decision box

Dry run or trace

This is the method of predicting the outcome of a given flow chart using a table

Example 1

Perform a dry run and state the purpose of the flowchart



Solution

Dry run

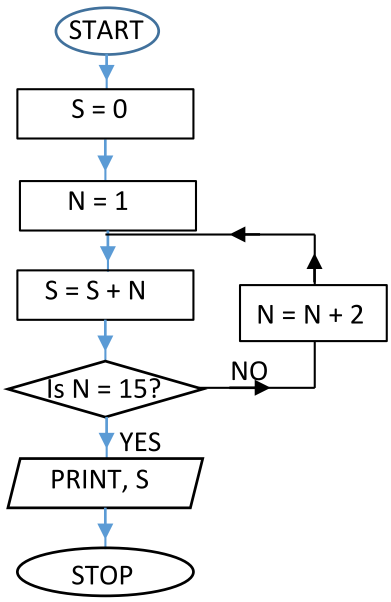
x	y
0	1
1	1
2	2
3	6
4	24
5	120
6	720

Purpose is to compute and print 6!

Relationship is y = x!

Example 2

Study the flow chart below and perform dry run of the flowchart



Solution

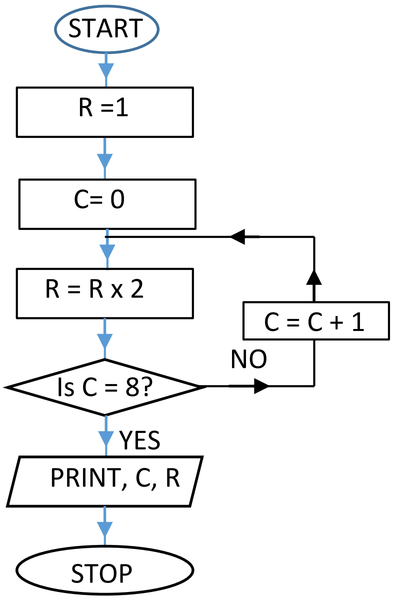
Dry run

N	S	Is N = 15?
1	1	NO
3	4	NO
5	9	NO
7	16	NO
9	25	NO
11	36	NO
13	49	NO
15	64	YES

Purpose is to compute and print the first 8 square numbers

Example 3

Perform a dry run and state the purpose of the flowchart



Solution

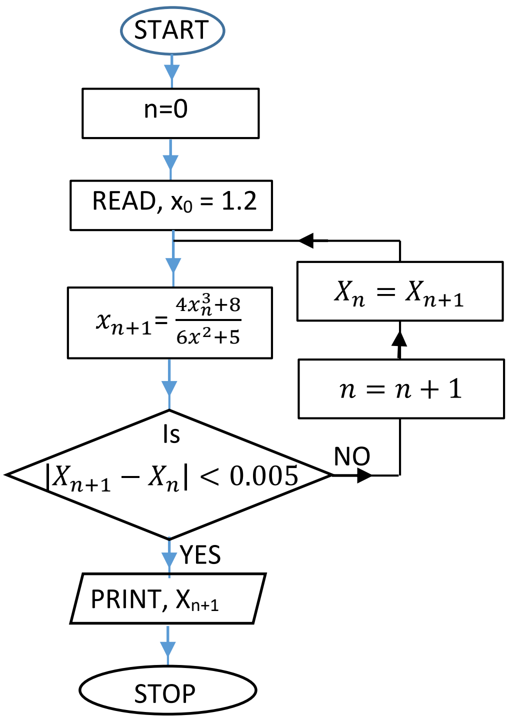
Dry run

C	R	Is C = 8?
0	1	NO
1	2	NO
2	4	NO
3	8	NO
5	32	NO
6	64	NO
7	128	NO
8	256	YES

Purpose is to compute and print 2^8

Example 4

The flowchart below is used to read the root of the equation $2x^3 + 5x - 8 = 0$



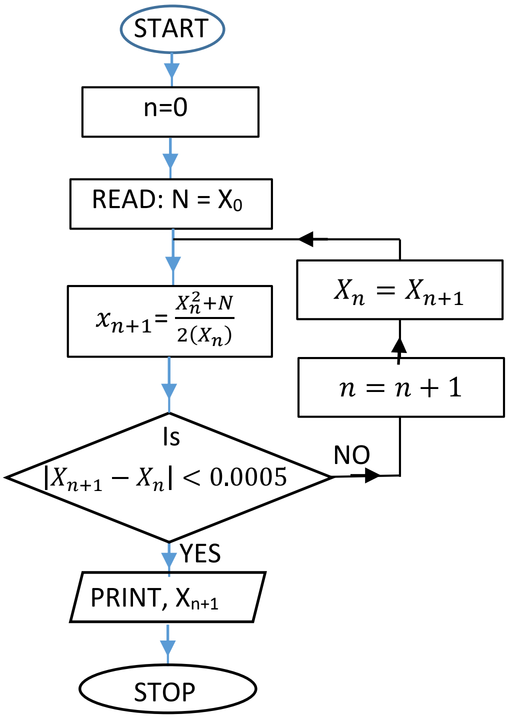
Carry out a dry run of the flow chart and obtain the root of $2x^3 + 5x - 8 = 0$ with an error less than 0.005

N	X_n	X_{n+1}	$\lvert X_{n+1} - X_n \rvert$
0	1.2	1.0933	0.1067
1	1.0933	1.0867	0.0066
2	1.0867	1.0866	0.001

Root is 1.087

Example 5

Study the flowchart below



(i) Carry out a dry run of the flowchart, taking N = 20, X0 = 4 and obtain the root of correct to 3dp.

(ii) State its purpose

Solution

N	X_n	X_{n+1}	$\lvert X_{n+1} - X_n \rvert$
0	4.0	4.5	0.5
1	4.5	4.4722	0.0278
2	4.4722	4.4721	0.0001

Root is 4.472

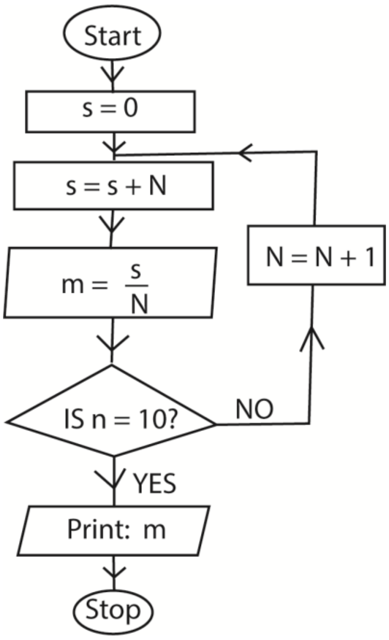
(ii) to print the square root of a number N

Constructing flowcharts

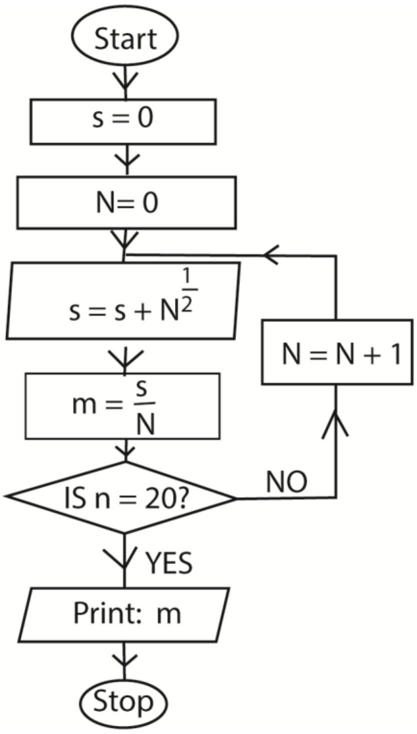
1. Draw a flow chart that reads and prints the mean of the first ten counting numbers

Solution

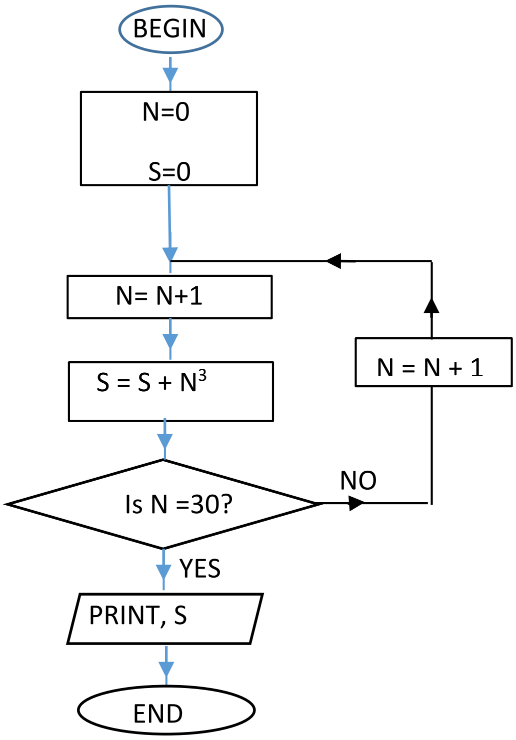
Let S be sum and m the mean



2. Draw a flowchart for computing and printing the mean of the square roots of the first 20 natural numbers

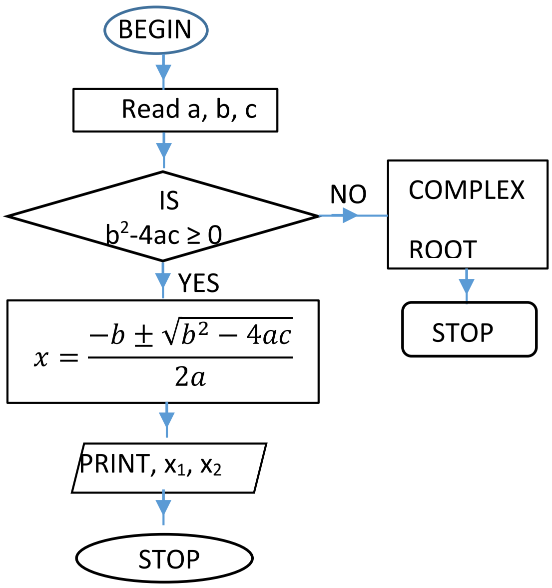


3. Draw a flowchart that computes and prints the sum of the cubes of the first 30 natural numbers



4. Draw a flowchart that computes the root of the equation ax2 + bx + c = 0

Solution



Newton Raphson's method and Flowcharts

Example 6

(a) Show that the iterative formula based on Newton Raphson's method for approximating the root of the equation 2Inx – x + 1 = 0 is given by

$x_{n+1} = x_n\left(\frac{2Inx_n - 1}{x_n - 2}\right)$, n = 0, 1,2 ………………………. (03marks)

f(x) = 2Inx – x + 1

$f'(x) = \frac{2}{x} - 1$

also

$f(x_n) = 2Inx_n - x_n + 1$

$f'(x_n) = \frac{2}{x_n} - 1$

Using $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

By substitution, we get

$$x_{n+1} = x_n - \frac{2Inx_n - x_n + 1}{\left(\frac{2}{x_n} - 1\right)}$$

$$= \frac{x_n\left(\frac{2}{x_n} - 1\right) - 2Inx_n - x_n + 1}{\left(\frac{2}{x_n} - 1\right)}$$

$$= \frac{x_n(2 - x_n) - x_n(2Inx_n - x_n + 1)}{(2 - x_n)}$$

$$= \frac{x_n(2 - x_n - 2Inx_n + x_n - 1)}{(2 - x_n)}$$

$$= \frac{x_n(1 - 2Inx_n)}{(2 - x_n)}$$

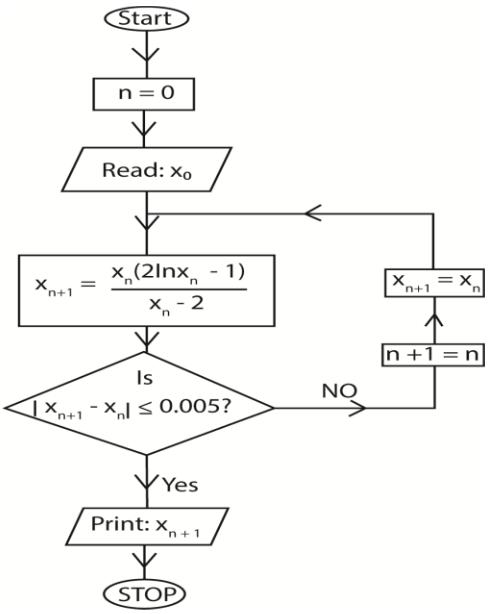
$$= \frac{-x_n(2Inx_n - 1)}{-(x_n - 2)}$$

$$= \frac{x_n(2Inx_n - 1)}{(x_n - 2)}$$

(b) Draw a flow chart that:

(i) reads the initial approximation x_0 of the root

(ii) computes and prints the root correct to two decimal places, using the formula in (a) (05marks)



(ii) Taking $x_0 = 3.4$, perform a dry run to find the root of the equation (04marks)

Dry run

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	3.4	3.51548	0.11548
1	3.51548	3.51286	0.00262
2	3.51286	3.51286	0.0000

Example 7

(a) Show that the Newton-Raphson formula for finding the root of the equation x = $N^{\frac{1}{5}}$ is given by $X_{n+1} = \frac{4X_n^5+N}{5X_n^4}$, n = 0, 1, 2, … (04marks)

$$x = N^{\frac{1}{5}}$$
$$x^5 = N$$
$$x^5 - N = 0$$

Let f(x) = $x^5 - N$

$$f(x_n) = x_n^5 - N$$
$$f'(x_n) = 5x_n^4$$

Using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^5-N}{5x_n^4} = \frac{5x_n^5 - x_n^5 - N}{5x_n^4}$$

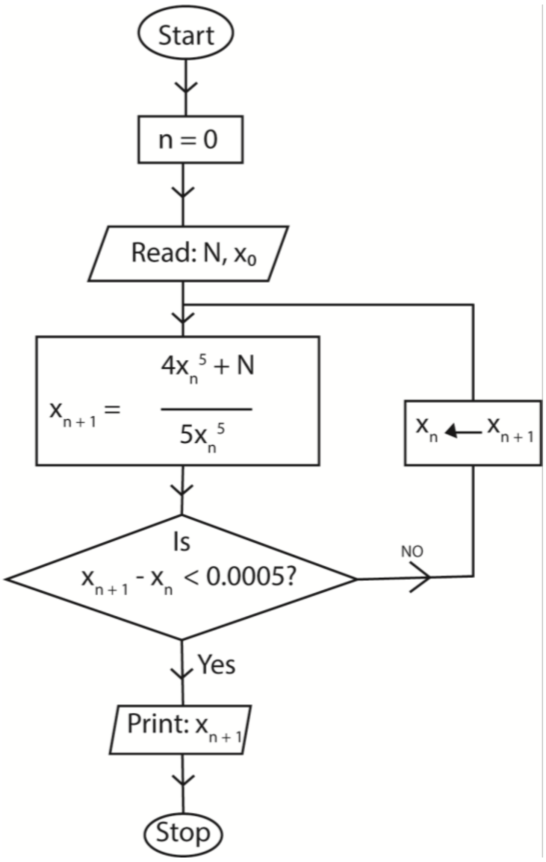
$$x_{n+1} = \frac{4x_n^5+N}{5x_n^4}$$, n = 0, 1, 2 …

(b) Construct a flow chart that

(i) reads N and the first approximation x_0.

(ii) computes the root to three decimal places

(iii) Prints the root (x_n) and the number of iteration (n) (05marks)



(c) Taking N = 50, x_0 = 2.2, perform a dry run for the flow chart. Give your root correct to three decimal places.(03marks)

N = 50, x_o = 2.2

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.2	2.18688	0.01312
1	218688	2.18672	0.00016

Root = 2.187(3D)

Example 8

(a) Show that the iterative formula based on Newton Raphson's method for solving the equation

Inx + x – 2 = 0 is given by

$$X_{n+1} = \frac{x_n(3-Inx_n)}{1+x_n}, n = 0, 1,2 \ldots.$$ (04marks)

$let\ f(x) = Inx + x - 2$

$f(x_n) = Inx_n + x_n - 2$

$f'(x) = \frac{1}{x_n} + 1 = \frac{1+x_n}{x_n}$

Using N.R.M

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{Inx_n + x_n - 2}{\frac{1+x_n}{x_n}}\right)$$

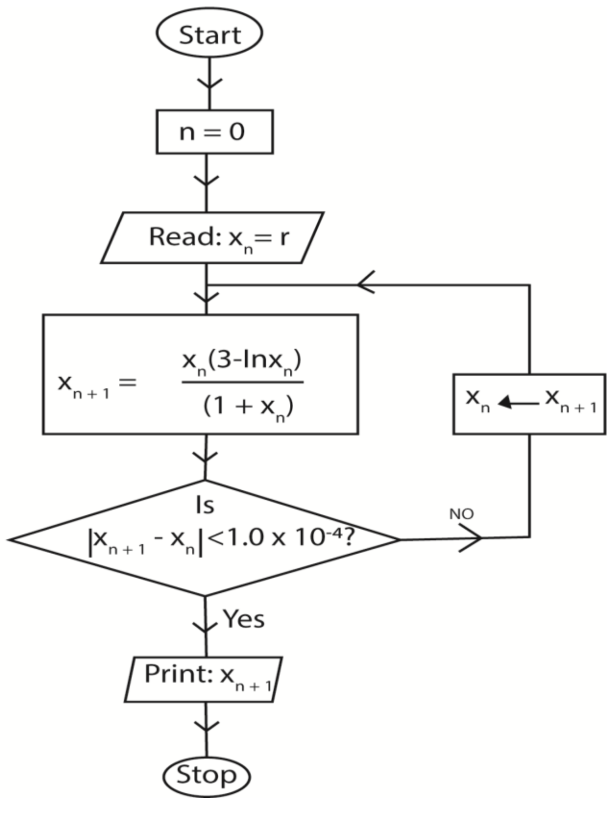
$$= \frac{x_n}{1} - \frac{x_n(Inx_n + x_n - 2)}{1+x_n}$$

$$= \frac{x_n(1+x_n - Inx_n - x_n + 2)}{1+x_n}$$

(b)(i) Construct a flow chart that ;

- reads the initial approximation as r

- computes using the interactive formula in (a), and prints the root of equation Inx + x -2 = 0, when the error is less than 1.0 x 10^{-4}.



(ii) Perform a dry run of the flow chart when r = 1.6. (08marks)

n	x_n	X_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.6	1.5569	0.0431
1	1.5569	1.5571	0.0002
2	1.5571	1.5571	0.0000

Hence the root = 1.557(3D)

Example 9

(a) Show that iterative formula based on Newton Raphson's method for approximating the sixth root of a number N is given by $x_{n+1} = \frac{1}{6}\left(5x_n + \frac{N}{{x_n}^5}\right)$

(b) Draw a flowchart that
  (i) Reads N and the initial approximation x_0 of the root
  (ii) computes and prints the root to three decimal places

(c) Taking N = 60, x0 = 1.9, perform a dry run for the flow chart, give your root correct to three decimal places.

Solution

$x = N^{\frac{1}{6}}$
$x^6 = N$
$x^6 - N = 0$
Let f(x) = $x^6 - N$
$f(x_n) = x_n^6 - N$
$f'(x_n) = 6x_n^5$

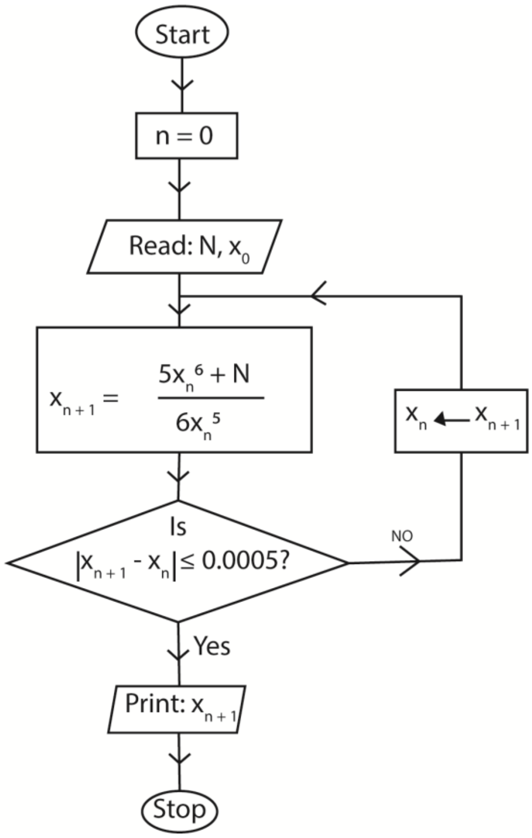
Using NRM

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^6 - N}{6x_n^5}$$

$$= \frac{x_n(6x_n^5) - (x_n^6 - N)}{6x_n^5}$$

$$x_{n+1} = \frac{5x_n^6 + N}{6x_n^5}, \text{ n = 0, 1, 2 ...}$$

(b) Flowchart



(c) Dry run

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	1.9	1.9872	0.082
1	1.9872	1.9787	0.0085
2	1.9787	1.9786	0.0001

Example 10

(a) Show that the iterative formula based on Newton Raphson's method for approximating the fourth root of a number N is given by $x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3x_n{}^3}\right)$.

(b) Draw a flowchart that

  (i) Records N and initial approximation x_0 of the root

  (ii) computes and prints the root after four iterations.

(c) Taking N = 39.0, x0 = 2.0, perform a dry run for the flowchart, give your root correct to three decimal places

Solution

$x = N^{\frac{1}{4}}$

$x^4 = N$

$x^4 - N = 0$

Let f(x) = $x^4 - N$

$f(x_n) = x_n^4 - N$

$f'(x_n) = 4x_n^3$

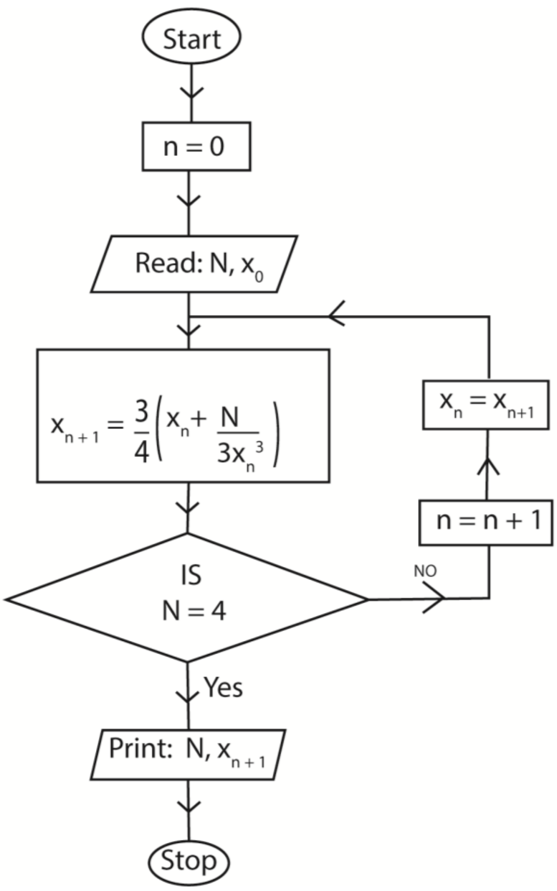
Using NRM

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - N}{4x_n^3}$$

$$= \frac{x_n(4x_n^3) - (x_n^3 - N)}{4x_n^3}$$

$$x_{n+1} = \frac{3x_n^3 + N}{4x_n^4} = \frac{3}{4}\left(x_n + \frac{N}{3x_n{}^3}\right)$$, n = 0, 1, 2 …

(b) Flowchart



(c) Dry run

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.0	2.7188	0.7188
1	2.7188	2.5242	0.1945
2	2.5242	2.4994	0.0249
3	2.4994	4.4990	0.0004

Example 11

(a) Show that the iterative formula based on Newton's Raphson's method for finding the natural logarithm of a number N is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+N}{e^{x_n}}$, n = 0, 1, 2, …

(b) Draw a flowchart that
   (i) Records N and initial approximation x_0 of the root
   (ii) computes and prints the natural logarithm after four iteration and gives natural logarithm to 3 decimal places.

(c) Taking N = 10, x_0 = 2, perform a dry run for the flowchart, give your root correct to three decimal places

Solution

(a) x = lnN; $e^x = N \Rightarrow e^x - N = 0$

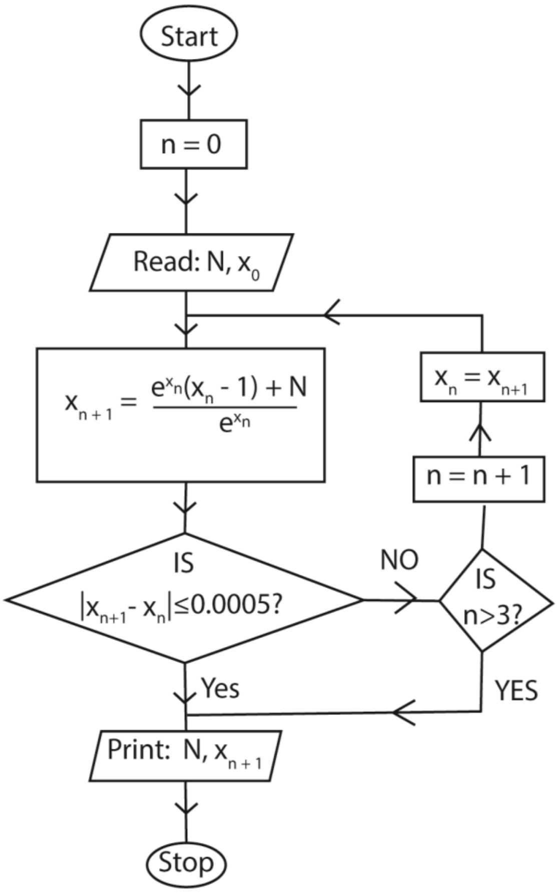
$f(x) = e^x - N$; $f^1(x) = e^x$

$x_{n+1} = x_n - \left(\frac{e^{x_n}-N}{e^x}\right)$

$= \frac{x_n e^x - (e^{x_n} - N)}{e^x}$

$= \frac{e^x(x_n-1)+N}{e^x}$

(b) Flowchart



(c) Dry run

n	x_n	x_{n+1}	$\lvert x_{n+1} - x_n \rvert$
0	2.0	2.3533	0.3533
1	2.3533	2.3039	0.0494
2	2.3039	2.3026	0.0013
3	2.3026	2.3026	0.0000

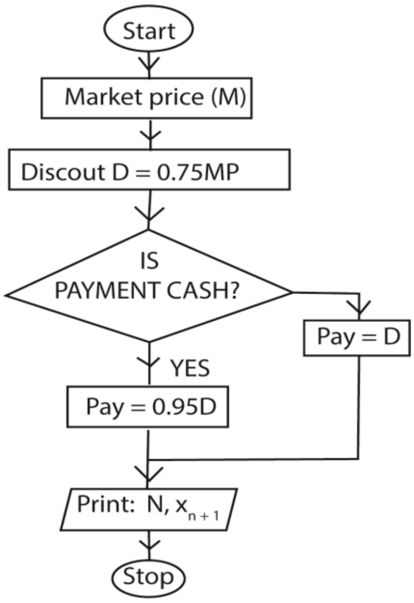
Example 12

A shop offers a 25% discount on all items in their store and a second discount of 5% for paying cash.

(a) Construct a flowchart for the above information

(b) perform a dry run for (i) a shoe of 75,000/= cash and (ii) a shirt of 45,000/= credit

(a) Flowchart



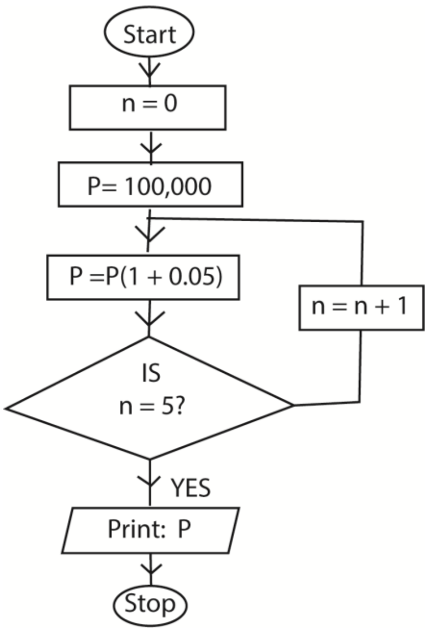
(c) dry run

MP	D= 0.75MP	Pay	Cash = 0.95D	Credit = D
75,000	56,250	cash	53437.50	--------
45,000	33,750	credit	-----------	33750

Example 13

Given that a man deposited 100,000/= to a bank which gives a compound interest of 5%. Draw a flowchart to compute the amount of money accumulated after 5 years and perform a dry run for the flowchart.

Flowchart



Dry run

n	P	A
0	100,000	100,000
1	100,000	105,000
2	105,000	110,250
3	110,250	115,762.0
4	115,762,5	121,550.625
5	121,550.625	127,628.1563

Revision Exercise

1. (a) show that the iterative formula based on Newton Raphson's method for approximating the cube root of a number N is given by $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{N}{x_n^2}\right), n = 0, 1, 2, \dots$
   (b) Draw a flowchart that
      (i) reads N and the initial approximation x_0 of the root
      (ii) computes and prints the root to three decimal places.
   (c) Taking N = 54, x_0 = 3.7, perform a dry run for the flowchart, give your root to three decimal places [3.780]
2. (a) show that the iterative formula based on Newton Raphson's method for approximating the fourth root of a number N is given by $x_{n+1} = \frac{3}{4}\left(x_n + \frac{N}{3x_n^3}\right), n = 0, 1, 2, \dots$
   (b) Draw a flowchart that
      (i) reads N and the initial approximation x_0 of the root
      (ii) computes and prints the root to two decimal places.
   (c) Taking N = 35, x_0 = 2.3, perform a dry run for the flowchart, give your root to two decimal places. [2.43]
3. (a) show that the iterative formula based on Newton Raphson's method for finding the root of a number $N^{\frac{1}{5}}$ is given by $x_{n+1} = \left(\frac{4x_n^5+N}{5N_n^4}\right), n = 0, 1, 2, \dots$
   (c) Draw a flowchart that
      (i) reads N and the initial approximation x_0 of the root
      (ii) computes and prints the root to three decimal places.
   (d) Taking N = 50, x_0 = 2.2, perform a dry run for the flowchart, give your root to three decimal places [2.187]
4. (a) show that the iterative formula based on Newton Raphson's method for approximating the cube root of 2lnx – x + 1 = 0 is given by $x_{n+1} = x_n\left(\frac{2Inx_n-1}{x_n-2}\right), n = 0, 1, 2, \dots$
   (b) Draw a flowchart that
      (i) reads N and the initial approximation x_0 of the root
      (ii) computes and prints the root
   (c) Taking x_0 = 3.4, perform a dry run for the flowchart, give your root to three decimal places
5. (a) show that the iterative formula based on Newton Raphson's method for approximating the cube root of lnx + x – 2 = is given by $x_{n+1} = x_n\left(\frac{3-Inx_n}{1+x_n}\right), n = 0, 1, 2, \dots$
   (b) Draw a flowchart that
      (iii) reads N and the initial approximation r of the root
      (iv) computes and prints the root of the equation, when the error is less than 10. x 10^{-4}.
   (c) Taking r = 1.6, perform a dry run for the flowchart, give your root to three decimal places
6. (a) show that the iterative formula based on Newton Raphson's method for approximating the cube root of x = ln(x +2) is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+2}{e^{x_n-1}} n = 0, 1, 2, \dots$
   (b) Draw a flowchart that
      (i) reads the initial approximation x_0 of the root
      (ii) computes and prints the root to three decimal places
   (c) Taking x_0 = 1.2, perform a dry run for the flowchart, give your root to three decimal places
7. (a) show that the iterative formula based on Newton Raphson's method for finding the natural logarithm of number N is given by $x_{n+1} = \frac{e^{x_n}(x_n-1)+N}{e^{x_n}}, n = 0, 1, 2, \dots$
   (b) Draw a flowchart that

(i) reads N and the initial approximation x_0 of the root
(ii) computes and prints the root to two decimal places.

(c) Taking N = 45, x_0 = 3.5, perform a dry run for the flowchart, give your root to two decimal places [3.81]

8. (a) show that the iterative formula based on Newton Raphson's method for finding the root of the $2x^3 + 5x - 8$ is given by $x_{n+1} = \left(\frac{4x_n^3 + 8}{6x_n^2+5}\right), n = 0, 1, 2, \ldots$

(b) Draw a flowchart that
(i) reads N and the initial approximation α of the root
(ii) computes and prints the root when the error is less than 0.001.

(c) Taking α= 1.1, perform a dry run for the flowchart, give your root to three decimal places [1.087]

9. A shop offers a 25% discount on all items in their store and a second discount of 5% for paying cash.

(a) Construct a flowchart for the above information
(b) perform a dry run for (i) a radio of 125,000/= cash and (ii) a T.V of 340,000/= credit [89.062.50, 255,0000]

10. Given that a man deposited 120,000/= to a bank which gives a compound interest of 15%. Draw a flowchart to compute the amount of money accumulated after 4 years and perform a dry run for the flowchart. [209,880.75/=]

Thank you
Dr. Bbosa Science