Force and Newton's laws of motion

Law I: A body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force

Law II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

$F = m\left(\frac{v-u}{t}\right)$ but $\left(\frac{v-u}{t}\right)$ =a

= ma

NB: F must be the resultant force

Example 1

Find the acceleration produced when a body of mass 5kg experiences a resultant force of 10N

F = ma | 10 = 5a | $a = 2ms^{-2}$

Example 2

A car of mass 600kg travels a distance of 24m while uniformly accelerated from rest to $12ms^{-1}$

(i) Find the acceleration of the car
$v^2 = u^2 + 2as$
$12^2 = 0^2 + 2a \times 24$
$a = 3ms^{-2}$

(ii) determine the accelerating force
F = ma = 600 x 3 = 1800N

Example 3

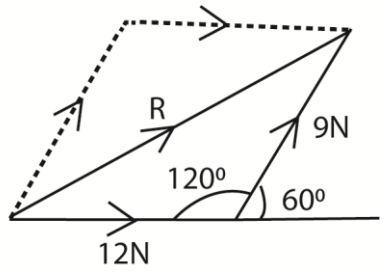
A body of mass 500g experiences a resultant force 3N. Find

(i) Acceleration produced
F = ma | 3 = 0.5 x a | $a = 6ms^{-2}$

(ii) Distance travelled by the body while increasing speed from $1ms^{-1}$ to $7ms^{-1}$
$v^2 = u^2 + 2as$
$7^2 = 1^2 + 2x\ 6 \times s$
s = 4m

Example 4

Two forces of magnitude 12N and 9N act on a particle producing an acceleration of $3.65ms^{-2}$. The two forces act at an angle of 600 to each other. Find the mass of the particle.



$R^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \cos 120^0$

R = 18.25N

F = ma

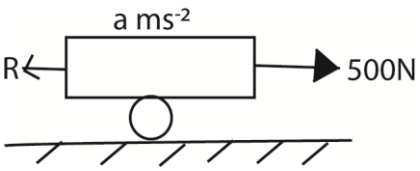
18.25N = 3.65m

m = 5kg

When resistance or friction is involved

Example 5

A car moves along a level road at constant velocity of $22ms^{-2}$. If its engine is exerting a forward force of 500N, what resistance is the car experiencing.



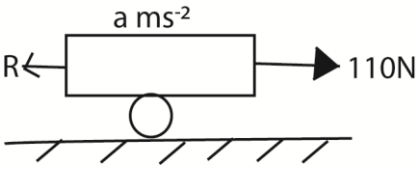
F = ma

500 – R = m x 0

R = 500N

Example 6

A car of mass 500kg moves along a level road with acceleration of $2ms^{-2}$. Its Engine is exerting a forward force of 110N. What is the resistance a car is experiencing?



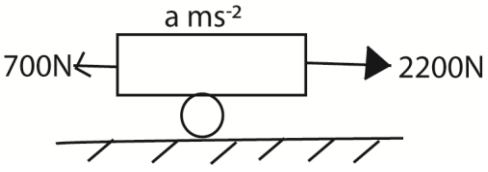
F = ma

500 – R = 500 x 2

R = 100N

Example 7

A van of mass 2tonnes moves along a level road against resistance of 700N. If its engine is exerting a forward force of 2200N. Find the acceleration of the van



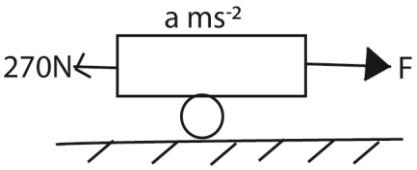
F = ma

2200 – 700 = 2000 x a

$a = 0.75ms^{-2}$

Example 8

Find the constant force necessary to accelerate a car of mass 1000kg from $15ms^{-1}$ to $20ms^{-1}$ in 10s against a resistance of 270N



v = u + at

20 = 15 + 10a

a = $0.5ms^{-2}$

F = ma

F – 270 = 1000 x 0.5

F = 770N

Calculations involving vector form

Find the resultant force required to make a body of mass 2kg at $(5i + 2j)ms^{-2}$.

F = ma | F = $2\left(\frac{5}{2}\right) = \begin{pmatrix}10\\4\end{pmatrix} N$

Example 9

F = ma $\quad \begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix} = 0.5a$ | a = $\begin{pmatrix}6\\6\end{pmatrix} ms^2$

Find the acceleration produced in a body of mass 500N is subjected to forces of (4i + 2j)N and (-i + j)N

F = ma | $\begin{pmatrix}4\\2\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix} = 0.5a$ | a = $\begin{pmatrix}6\\6\end{pmatrix} ms^2$

Example 10

Find the magnitude of the acceleration produced in a body of mass 2kg subjected to forces of (2i – 3j +4k)N and (i + 5j + 2k)N

F = ma | $\begin{pmatrix}2\\-3\\4\end{pmatrix} + \begin{pmatrix}1\\5\\2\end{pmatrix} = 2a$ | a = $\begin{pmatrix}1.5\\1\\3\end{pmatrix} ms^2$ | $|a| = \sqrt{1.5^2 + 1^2 + 3^2} = 2.3ms^{-2}$

Example 10

A particle of mass 2.5kg is acted on by a resultant force of 15N acting in the direction (2i – j – 2k). find the magnitude of the acceleration

F = 15 x $\frac{2i-j-2k}{\sqrt{2^2+(-1)^2+(-2)^2}}$

= 15 x $\frac{2i-j-2k}{3}$

F = 10i – 5j -10k

F = ma

$\begin{pmatrix}10\\-3\\-10\end{pmatrix} = 2a$

a = $\begin{pmatrix}4\\-2\\-4\end{pmatrix}$

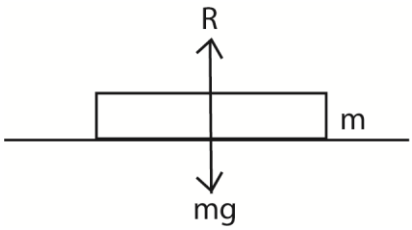
$|a| = \sqrt{10^2 + (-3)^2 + (-10)^2}$

=6ms-2

Law III: To every action there is an equal but opposite reaction

Consider

1. a body of mass m place on a smooth horizontal surface

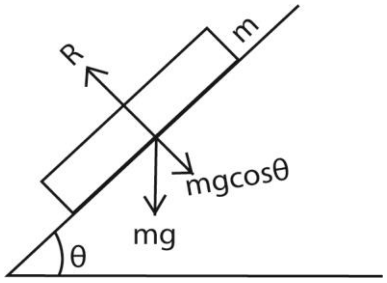


R = mg

R = normal reaction

mg gravitational pull (weight)

2. Mass m laced on a smooth inclined lane of angle of inclination θ



R = mgcosθ

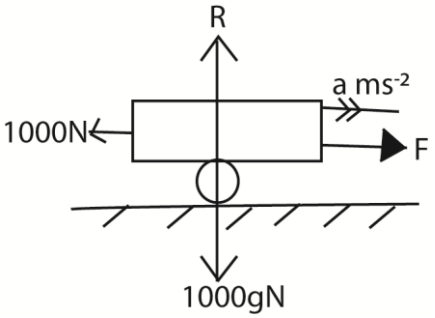
- All objects placed on or moving on an inclined plane experience a force mgsinθ **down** the plane no matter the direction of movement.
- If the plane is rough, the body experience a frictional force whose direction is opposite of the direction of motion

Motion on horizontal plane

Example 11

A car of 1000kg is accelerating at $2ms^{-2}$. If the resistance to motion is 100N

(i) Find the normal reaction of the car on the road surface



R = 1000gN

= 1000 x 9.8 = 9800N

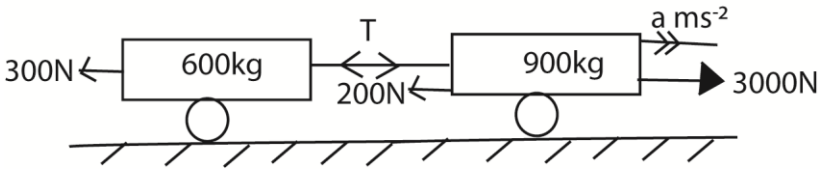
(ii) What accelerating force acts on the car?

F = ma

F – 1000 = 1000 x 2; F = 3000N

Example 12

A car of mass 900kg tows a trailer of mass 600kg along a level road by means of a rigid bar. The car experiences a resistance of 200N and the trailer a resistance of 300N, if the car engine exerts a force of 3kN, find the acceleration produced and the tension in the tow bar



For 900kg: 3000 – (T + 200) = 900a … (i)

For 600kg: T – 300 = 600a ……. (ii)

(i) and (ii) a= 1.6667ms^{-2}

Alternatively

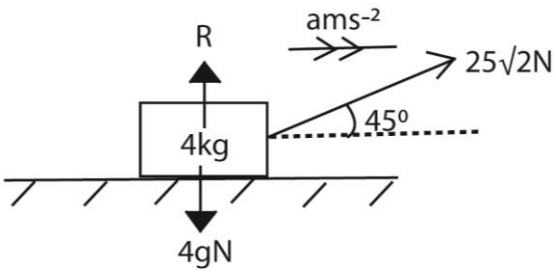
3000 – (200 + 300) = (900 + 600)a

a = 1.6667ms^{-2}

Force inclined at an angle to the horizontal

Example 13

A body of mass 4kg is acted on by force of $25\sqrt{2}$N which is inclined at 450 to a smooth horizontal surface. Find the acceleration of the body and the normal reaction between the body and the surface.



(→) $25\sqrt{2}$Ncos45 = 4a

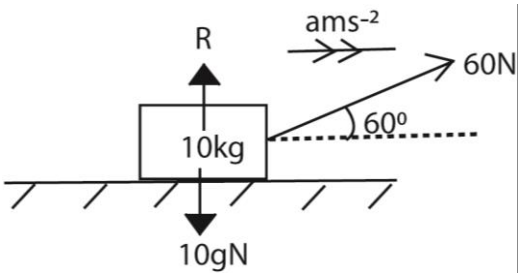
a = 6.25ms^{-2}

(↑) R + $25\sqrt{2}$Nsin 45 - = 0

R = 14.2N

Example 14

A body of mass 10kg is initially at rest on a rough horizontal surface. It is pulled along the surface by constant force of 60N inclined at 600 above the horizontal. If the resistance to motion totals 10n, find the acceleration of the body and the distance travelled in the first 3s.



(→) 60sos60 - 10 = 10a

a = 2ms^{-2}

s = ut + $\frac{1}{2}at^2$

s = 0 x 3 + $\frac{1}{2}$ x 2 x 3^2 =9m

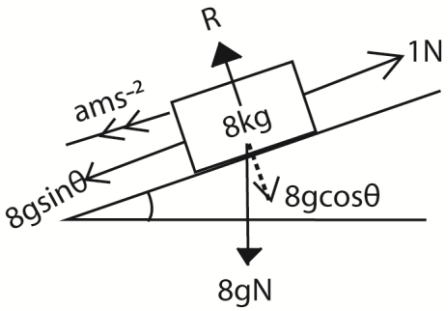
Revision exercise 1

1. A railway engine of mass 100tonnes is attached to a line of trucks of total mass 80 tonnes. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train has acceleration of 0.020ms^{-2} [25.6kN]
2. A body of mass 5kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at 450 above the horizontal. In the first 5 seconds of motion, the body moves a distance of 10m along the surface. Find the
   (i) acceleration of the body [0.8ms^{-2}]
   (ii) magnitude of P [$4\sqrt{2}$N]
   (iii) normal reaction between the body and the surface. [45N]
3. A body of mass m kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. Show that the body moves a distance in time t along the surface given by $\frac{Pt^2cos\theta}{2m}$.
4. A body of mass m kg, initially at rest on a rough horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. If the mass acquire velocity v in a distance d. Show that the resistance to motion is given by Pcosθ = $\frac{mv^2}{2d}$

Motion on an inclined plane

Example 15

A body of mass 8kg is released from on the surface of a plane at 1 in 40. If the resistance to motion is 1N, find the acceleration of the body and the speed it acquired after 6s.



sinθ= $\frac{1}{40}$

F = ma

8gsinθ – 1 = 8a

a = 0.12ms^{-2}

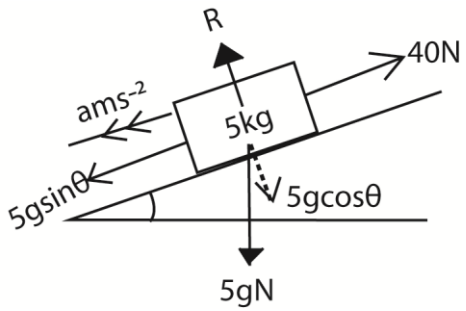
v = u + at

= 0 + 0.12 x 6 =0.72ms^{-2}

Example 16

A body of mass 5kg is pulled u a smooth plane inclined at 30^0 to the horizontal by a force of 40N acting parallel to the plane. Find

(i) acceleration of the body
(ii) force exerted on the body by the plane R



F = ma

40 -5gsin30^0 = 5a

a = 3.095ms^{-2}

(ii) R = 5gcos30

= 5 x 9.8cos30 = 42.4N

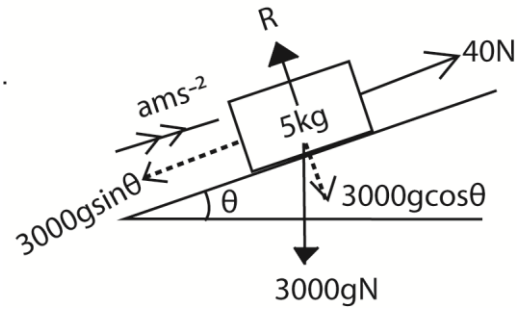
Example 17

A lorry of mass 3tonnes travelling at 90k/h starts to climb an incline of 1 in 5. Assuming the attractive pull between its tyres and the road remains constant and that its velocity reduces to 54kmh in a distance of 500m. Find the attractive pull.

u = 90kmh = $\frac{90\ x\ 1000}{3600}$ = 25ms^{-1}

V = 54kmh = $\frac{54\ x\ 1000}{3600}$ = 15ms^{-1}

a = $\frac{v^2 - u^2}{2s} = \frac{15^2 - 25^2}{2\ x\ 500}$ = -0.4ms^{-2}



sinθ = $\frac{1}{40}$

F = ma

F - 3000gsinθ = 3000a

F – 3000 x 9.8 x $\frac{1}{5}$ = 3000 x 0.4

F = 4686N

Revision exercise 2

1. A particle of mass 5kg resting on smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$to the horizontal. Find the magnitude of the horizontal force required to keep the particle in equilibrium and the normal reaction. [28.29N, 56.58N]
2. The engine of a train exerts a force of 35,000N on a train of mass 240 tonnes and draws up a slope of 1 in 120 against resistance totalling to 60N per tonne. Find the acceleration of the train. [0.004167ms^{-2}]
3. A car of mass 2.5 metric tonnes is drawn up a slope of 1 in 10 from rest with acceleration of 1.2ms^{-2} against a constant frictional force of $\frac{1}{100}$ of the weight of the vehicle using a cable. Find the tension in the cable. [5695N]
4. A mass 5kg is initially at the bottom of a smooth slope which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. The mass is pushed up the slope by horizontal force 50N, find
   (i) the normal reaction between the mass and the plane [69.2N]
   (ii) calculate the acceleration up the slope [2.12ms^{-2}]
   (iii) how far up the slope the mass travels in the first 4s [16.96m]
5. A body of mass 100kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. Find
   (i) velocity of the body when it has travelled 20m down the slope. [14ms^{-1}]
   (ii) velocity, if the mass of the body was 50kg. [14ms^{-1}]
6. A body of mass 20kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. If the body accelerates down the slope at 3ms^{-2}, find the constant resistance to motion experienced by the body. [38N]
7. A body of mass 20kg is released from rest at the top of a rough slope which is inclined at 30^0 to the horizontal. 6s later the body has a velocity of 21ms^{-1} down the slope, find the constant resistance to motion experienced by the body. [28N]

8. A car of 1 tonne accelerated from 36kmh^{-1} to 72kmh^{-1} while moving 0.5km up a road inclined at an angle of α to the horizontal where sinα = $\frac{1}{20}$. If the total resistive force to its motion is 0,3kN, find the driving force of the car engine. [1090N]
9. A railway truck of mass 6.0 tonnes moves with an acceleration of 0.050ms^{-2} down a track which is inclined to the horizontal at an angle α where sinα = $\frac{1}{120}$. find the resistance to motion. [190N]
10. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of 40N acting at 60^0 above the horizontal. find
    (i) Accelerating force [4ms^{-2}]
    (ii) Force the body exerts on the ground [14.4N]
11. A body of mass 3.0kgslides down a plane which is inclined at 300 to the horizontal. Find the acceleration of the body if
    (i) the plane is smooth [4.9ms^{-2}]
    (ii) there is frictional resistance of 9.0N [1.9ms^{-2}]
12. A car of mass 1000kg tows a caravan of mass 600kg u a road which rises 1m vertically for every 20m of its length. There is a constant frictional resistance of 200N and 100N to the motion of the car and caravan respectively. The combination has an acceleration of 1.2ms^{-2} with the engine exerting a constant driving force. (Take g = 10ms^{-2}). Find
    (a) driving force[3020N]
    (b) Tension in the tow-bar [1120N]

Thank You

Dr. Bbosa Science