Friction

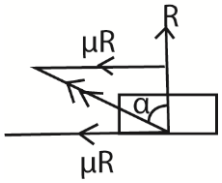
Friction is a force that opposes relative motion or attempted motion between two bodies in contact.

Friction force F = μR where R = normal reaction and μ = coefficient of friction

At limiting equilibrium, the body is at the point of moving (slip or slide) and friction force is maximum.

Angle of friction

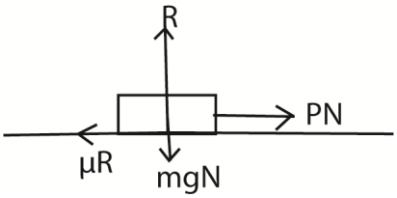
This is the angle between the resultant force and the normal reaction



$\tan\alpha = \frac{\mu R}{R} = \mu$

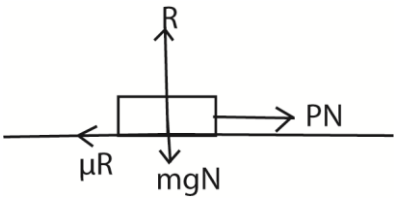
A horizontal plane

(i) at limiting equilibrium (about to slip or slid)



$P = \mu R$

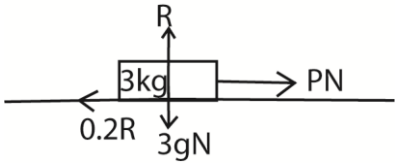
(ii) In motion



$P - \mu R = ma$

Example 1

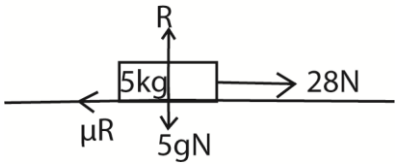
Calculate the maximum frictional force which can act when a block of mass 3kg rests on a rough horizontal surface, the coefficient of friction between the surface being 0.2



F = μR = 0.2 x 3 x 9.8 = 5.88N

Example 2

When a horizontal force of 28N is applied to a body of mass 5kg which is resting on a rough horizontal surface, the body is found to in limiting equilibrium. Find the coefficient of friction between the body and the plane



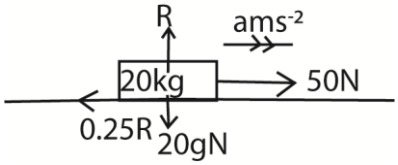
28 = μR

28 = μ x 5 x 9.8

$\mu = 0.57$

Example 3

A block of mass 20kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.25. If a horizontal force of 50N acts no the body, find the acceleration of the body.



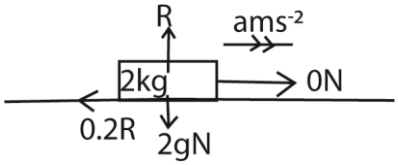
50 - μR = 20a

50 - $(0.25\ x\ 20\ x\ 9.8)$ = 20a

a = 0.05ms^{-2}

Example 4

A block of mass 2kg sliding along a smooth surface at a constant speed of 2ms-1. When the mass encounter a rough surface of coefficient of friction 0.2, it comes to rest. Find the distance the body will move across the rough surface before it comes to rest.



F = ma

0 – μR = 20a

-0.2 x 2 x 9.8 = 20a

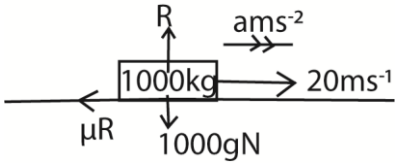
a = -1.96ms^{-2}

s = $\frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2x\ (-1.96)}$ = 1.02m

Example 5

A car of mass 1000kg moving along a straight road with speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 5m. Find the coefficient of kinetic friction between the tyres and the road.

u = $\frac{72\ x\ 1000}{2600}$ = 20ms^{-1}



a = $\frac{v^2 - u^2}{2s} = \frac{0^2 - 20^2}{2x\ 5}$ = -4ms^{-2}

ma = μR

4 x 1000 = 1000 x 9.8 x μ

μ = 0.41

Alternatively

Work done against friction = loss in kinetic energy

$\mu(mg)x\ s = \frac{1}{2}mv^2$

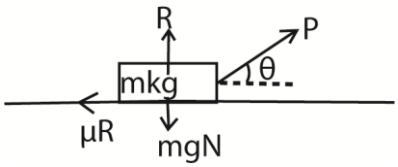
$\mu \times 9.8 \times 50 = \frac{1}{2}x\ 20^2$

$\mu = 0.408$

Revision exercise 1

1. When a horizontal force of 0.245N is applied to a body of mass 250g which is resting on a rough horizontal plane, the body is found to be in limiting equilibrium. Find the coefficient of friction between the body and the plane. [0.1]
2. A body of mass 40kg is resting on a rough horizontal plane and can just move by a force of 98N acting horizontally. Find the coefficient of friction. [0.25]
3. A block of mass 0.5kg rests on a rough horizontal plane. The coefficient of friction between the block and the table is 0.1. When a horizontal force of 1N acts on the block, find
   (i) friction force experienced by the block. [0.49N]
   (ii) acceleration with which the block will move. [1.02ms^{-2}]
4. When a horizontal force of 37N is applied to the body of mass 10kg which is resting on a rough horizontal surface, the body moves along the surface with acceleration 1.25ms^{-2}. Find the coefficient of friction between the body and the surface. [0.25]

5. A force inclined at an angle θ to the horizontal



At limiting equilibrium

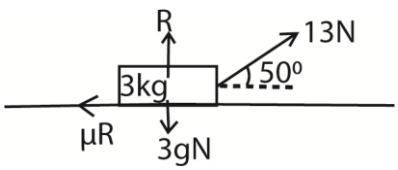
(→): Pcosθ = μR

(↑): R + Psinθ = mg

Example 6

A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 13N inclined at an angle 50^0 to the horizontal, if the particle does not move find the

(i) Normal reaction (ii) coefficient of friction

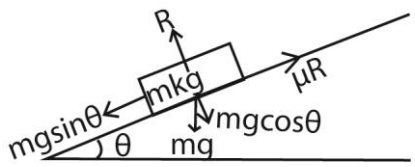


(↑):R = 3 x 9.8 – 13sin 50^0 = 19.4414N

(→): 13cos50^0 = μ x 19.4414

μ = 0.4298

Friction and inclined planes



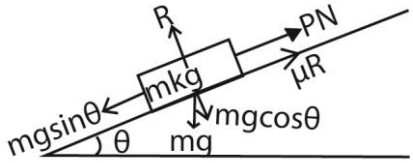
At limiting equilibrium

mgsinθ = μR

mgsinθ = μmgcosθ

μ = tanθ

(ii) A force P applied parallel to and up the plane to just move the particle upwards



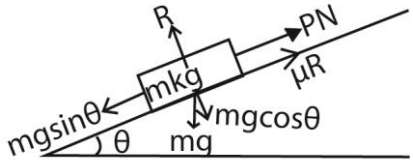
Normal to the plane: $mg\cos\theta = R$

Parallel to the plane; $mg\sin\theta + \mu R = P$

$P = mg\sin\theta + \mu mg\cos\theta$

(iii)

(iv) A force P applied parallel to and up the plane so that the particle is on the point of moving downwards (prevent moving downwards)



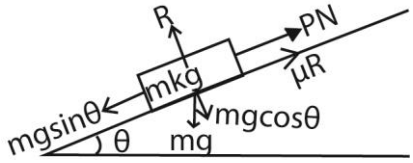
Normal to the plane: $mg\cos\theta = R$

Parallel to the plane; $mg\sin\theta = P + \mu R$

$P = mg\sin\theta - \mu mg\cos\theta$

(v)

(vi) A force P applied parallel to and up the plane to move the particle upwards



Normal to the plane: $mg\cos\theta = R$

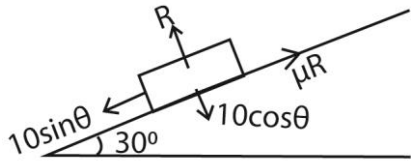
Parallel to the plane; $P - (mg\sin\theta + \mu R) = ma$

$P - (mg\sin\theta + \mu mg\cos\theta) = ma$

(vii)

Example 7

A particle of weight 10N rests on a rough plane inclined at $30^0$to the horizontal and is just about to slip. Find the value of coefficient of friction between the plane and the particle.



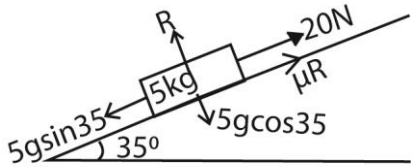
$R = 10\cos 30$ and $\mu R = 10\sin 30$

$\mu(10\cos 30) = 10\sin 30$

$\mu = 0.5774$

Example 8

A body of mass 5kg lies on a rough plane which is inclined at 35^0 to the horizontal. When a force of 20N is applied to the body parallel to and up the plane, the body is on the point of moving down the plane. Find the coefficient of friction between the body and the plane.



At limiting equilibrium

$R = 5g\cos 35$

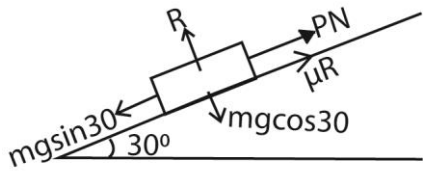
$20 + \mu R = 5g\sin 35$

$20 + \mu(5g\cos 35) = 5g\sin 35$

$\mu = 0.2$

Example 9

A block of wood of mass 150g rest on an inclined plane. If the coefficient of friction between the surface of contact is 0.3. Find the force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30^0.



At limiting equilibrium

R =0.15 x 9.8cos 30

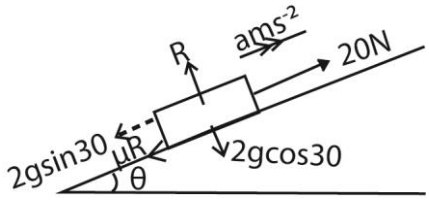
P +μR = 0.15 x 9.8sin30

P +0.3(0.15 x 9.8cos 30) =5gsin30;

P= 0.353N

Example 10

A body of mass 2kg lies on a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$ to the horizontal. A force of 20N is applied to the body, parallel to and up the plane. If the body accelerates up the plane at $1.5ms^{-2}$, find the coefficient of friction between the body and the plane.



Sin θ =$\frac{5}{13}$, cos θ = $\frac{12}{13}$

R = 2 x 9.8 cosθ = 2 x 9.8 x $\frac{12}{13}$ =18.09N

F = ma

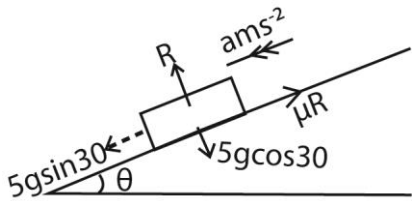
20 – (2gsinθ + μR) = 2a

20 –(2 x 9.8 x $\frac{5}{13}$ + 2 x 9.8 x $\frac{12}{13}$ x μ) = 2 x 1.5

μ =0.523

Example 11

A body of mass 5kg is released from rest on a rough surface of a plane inclined at 30^0 to the horizontal. If the body takes2.5s to acquire a speed of 4ms-2 from rest, find the frictional force and coefficient of friction.



v = u + at

4 = 0 + 2.5a

a = $1.6ms^{-2}$

F = ma

5 x 9.8sin30 – μR = 5 x 1.6;

Frictional force, μR = 16.5N

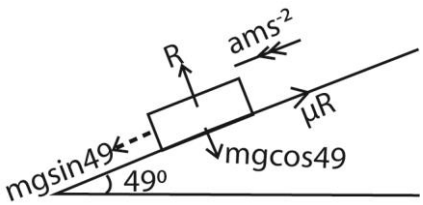
μR = 16.5N

μ = $\frac{16.6}{5\ x\ 9.8cos30}$ = 0.243

Example 12

A car of mass 500kg moves from rest with engine switched off down a road which is inclined at an angle 490 to the horizontal.

(a) calculate the normal reaction
(b) if the coefficient of friction between the tyres and the surface of the road is 0.32. Find the acceleration of the car.



(a) R =mgcos49 = 500 x 9.8cos49 = 3217.97N

(b) F = ma

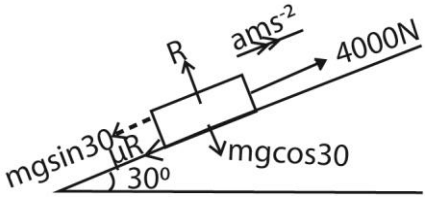
mgsin49 – μR = 500a

500 x 9.8sin49 – 0.32 x 3217.97 = 500a

a = 5.34ms^{-2}

Example 13

A car of mass 1000kg climbs a plane which is inclined at 30^0 to the horizontal. The speed of the car at the bottom of the incline is 36kmh^{-1}. If the coefficient of friction between the plane and the car tyres is 0.3 and the engine exerts a force of 4000N, how far up the incline does the car move in 5s



u = 36kh^{-1} = $\frac{36\ x\ 1000}{3600}$ = 10ms^{-1}

F = ma

4000 – (mgsin30 + μR) = ma

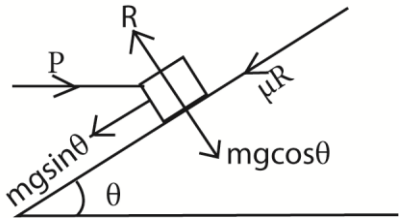
4000 – (1000 x 9.8sin30 + 0.3 x 1000 x 9.8cos30) = 1000a

a = -3.45ms^{-2}

s = ut + $\frac{1}{2}at^2$ = 10 x 5 + $\frac{1}{2}\ x\ (-3.45)x\ 5^2$ = 6.9m

Horizontal force on inclined planes

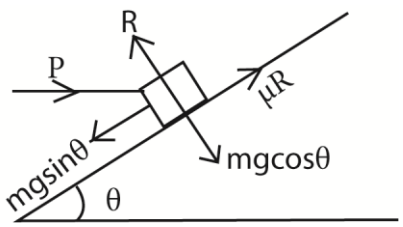
(i) A horizontal force P required to just move the particle upwards



Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ + μR = Pcosθ

(ii) a horizontal force O required to prevent the particle from moving downwards

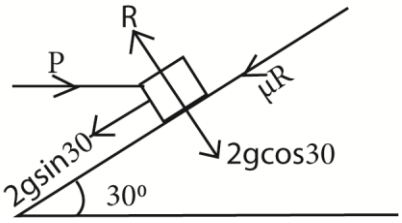


Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ - μR = Pcosθ

Example 14

A body of mass 2kg lies on a rough plane inclined at 300 to the horizontal. When a horizontal force of 20N is applied to the body in an attempt to push it up the plane, the body is found to be on the point of moving up the plane. Find the coefficient of friction between the body and the plane.



At limiting

Normal to the plane: 2gcos30 + 20sin30 = R……(i)

Parallel to the plane: 2gsin30 + μR = 20cos30 …… (ii)

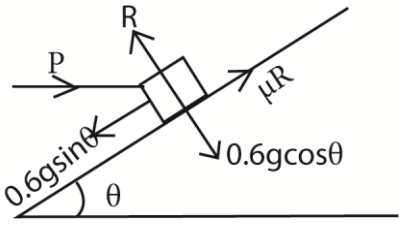
(i) and (ii)

2gsin30 +μ(2gcos30 + 20sin30) = 20cos30

μ =0.279

Example 15

A horizontal force of 1N is just sufficient to prevent a brick of mass 600g sliding down a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$to horizontal. Find the coefficient of friction between the brick and the plane.



sinθ = $\frac{5}{13}$, cosθ =$\frac{12}{13}$

At limiting

Normal to the plane: 0.6gcosθ + 1xsinθ = R……(i)

Parallel to the plane: 0.6gsinθ - μR = 1 x cosθ …… (ii)

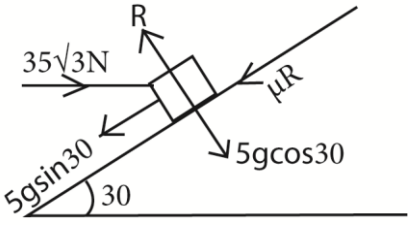
(i) and (ii)

0.6x 9.8 x $\frac{5}{13}$ - μ(0.6 x 9.8 x $\frac{12}{13}$ + 1 x $\frac{5}{13}$) = 1 x $\frac{12}{13}$

μ =0.23

Example 16

A body of mass 5kg is initially at the bottom of a rough inclined plane of length 6.3m. The plane is inclined to the horizontal and the coefficient of friction between the body and the plane is $\frac{1}{2}\sqrt{3}$. A constant horizontal force of $35\sqrt{3}N$is applied to the body causing it to accelerate up the plane. Find the time taken for the body to reach the top and its speed on arrival.



R = 5gcos 30 + $35\sqrt{3}$sin30

$35\sqrt{3}$cos 30 – (μR + 5gsin30) = 5a

a = 1.39ms^{-2}

s = ut + $\frac{1}{2}at^2$

6.3 = 0 x t + $\frac{1}{2}x1.39\ x\ t^2$; t = 3s

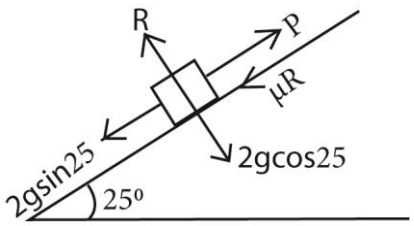
v = u + at

v = 0 + 1.39 x 3 =4.17ms^{-1}

Example 17

A box of mass 2kg at rest on a plane inclined at 25^0 to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane?

Let the minimum force required be P



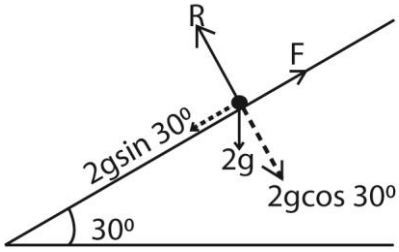
Perpendicular to the plane: R = 2gcos25 …. (i)
Along the plane: P = 2gsin25 + μR ……(ii)

(i) and (i)

P = 2gsin25 + 0.4(2gcos25) = 8.3 + 7.1 = 15.388N

Example 18

A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.

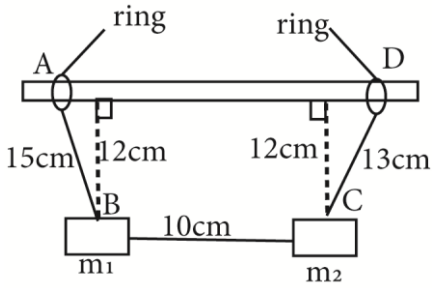


R = 2gcos 30^0
F = 2gsin 3^0
μR = 2gsin30^0
$\mu[2gcos\ 30^0]$ = 2gsin30^0
$\mu = \frac{2gsin\ 30^0}{2gcos\ 30^0}$ = tan 30^0 = 0.57735

Revision exercise

1. The resistance to motion of a lorry of mass mkg is 1/200 of its weight. When travelling at 108kmh^{-1} on a level road and ascends a hill inclined at 1 in 100. Its engine fails to work. Find how far up the hill (in km) the lorry moves before it comes to rest. [36.12m]
2. A vehicle of mass 2.5metric tonnes is drawn up on a slope of 1 in 10 from rest with an acceleration of 1.2 ms-2 against a constant frictional resistance of $\frac{1}{100}$ of the weight of the vehicle, using a cable. Find the tension in the cable. [T = 5695N]
3. (a) A particle of mass, m kg is projected with a velocity of 10ms-1 up a rough plane of inclination 300 to the horizontal. If the coefficient of friction between the particle and the plane is $\frac{1}{4}$. Calculate how far up the plane the particle travels.[s = 7.121m]
   (b) A car is working at 5kW and is travelling at a constant speed of 72knh^{-1}. Find the resistance to motion. [250N]

4. A body of mass 8kg rests on a rough plane inclined at θ to horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium. [$\frac{8g(sin\theta-\mu cos\theta)}{(cos\theta+\mu sin\theta)}$]
5. A carton of 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal. The coefficient of friction between the carton and the plane is $\frac{1}{3}$ > find a horizontal force that should be applied to make the carton just about to move up the plane. [33.155N]
6. A particle of weight 20N is placed on a rough plane inclined at an angle of 400 to the horizontal. the coefficient of friction between the plane and the particle is 0.25. When a horizontal force P is applied on the particle it rests in equilibrium. Calculate the value of P. [9.739N]
7. The diagram below shows the three strings AB = 15cm, BC = 10cm and CD = 13cm, A and D are fixed to small rings each of mass 2kg which can slide on a rough horizontal rail AD. Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance 12cm below AD.



   (a) Show that $9m_1 = 5m_2$.
   (b) If the coefficient of friction between each ring and the rail is 0.25 and the ring A is on the pint of slipping, determine the value of m_1. [m_1 = 1kg]
8. A 2kg body lies on a plane of inclination 600. The coefficient of friction between the body and the plane is 0.25. Find the least horizontal force which prevents the body from sliding down the plane. [20.27N]
9. A particle od mass 12kg slides from rest down a plane inclined at 50^0 to the horizontal. If the coefficient of friction between the particle and the plane is 0.4, calculate the acceleration of the particle. [4.99ms^{-2}]
10. A body of mass 3kg is released from a rough surface which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If after 2.5s the body has acquired a velocity of 4.9ms-1 down the surface. Find the coefficient of friction between the body and the surface.

Thank You

Dr. Bbosa Science