Line of action of the resultant force

The equation of line of action is given by

$$G = \begin{vmatrix} x & F_x \\ y & F_y \end{vmatrix} = xF_y - yF_x$$

$G - xF_y + yF_x = 0$

Note

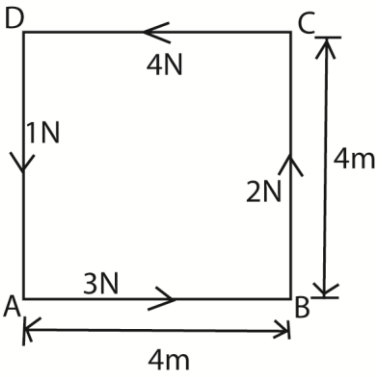
The line of action cuts the horizontal when y = o and the cuts the vertical axis when x = 0

Example1

Forces of 3N, 2N, 4N and 1N act along the sides of a square ABCD of side 4m in the direction AB, BC, CD and DA respectively, the direction of force in each case being the order of the letters. Find the;

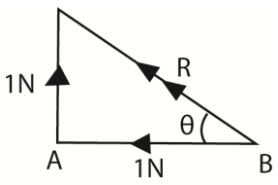
(i) Magnitude and direction of the resultant force
(ii) equation of the line of action
(iii) point where the line of action of the resultant of forces cuts AB

Solution



$$R = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$|R| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}N$



Direction, θ =$\tan^{-1}\left(\frac{1}{1}\right)$ = 45^0 to AB

A: G = 4 x 4 + 2 x 4 = 24Nm anticlockwise

Equation is 24 – x + y = 0

(ii) Line cuts AB when y = 0; 24 – x + 0 = 0

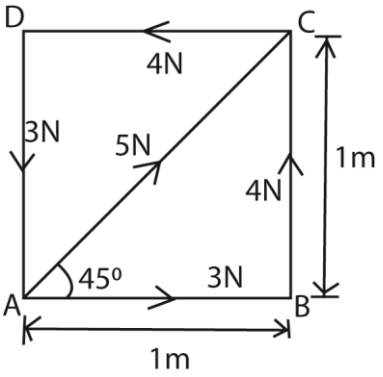
x = 24m from A

Example 2

Five forces of magnitude 3N, 4N, 4N, 3N and 5N act along AB, BC, CD, DA and AC respectively of squares of side 1m. The directions of the forces being given in the order of the letters Taking AB and AD as horizontal and vertical respectively. Find the

(a) magnitude and direction of the resultant force
(b) equation of the line of action

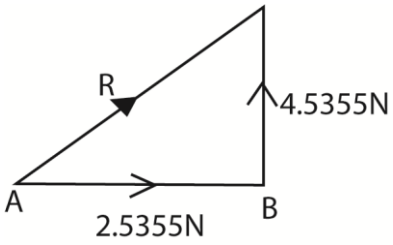
Solution



$R = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}4\\0\end{pmatrix} + \begin{pmatrix}-4\\0\end{pmatrix} = \begin{pmatrix}0\\-3\end{pmatrix} + \begin{pmatrix}5cos45\\5sin45\end{pmatrix}$

$= \begin{pmatrix}2.5355\\4.5355\end{pmatrix}$

$|R| = \sqrt{(2.5355)^2 + (4.5355)^2} = 5.1961N$



Direction, θ =$\tan^{-1}\left(\frac{4.5355}{2.5355}\right)$ = 60.8^0 to AB

A: G = 4 x 1 + 4 x 1 = 8Nm anticlockwise

Equation is 8 – 4.5355x + 2.5355y = 0

(ii) Line cuts AB when y = 0; 8 – 4.5355x + 0 = 0

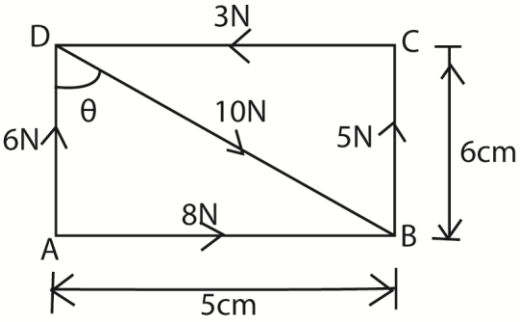
x = $\frac{8}{4.5355} = 1.769$m from A

Example 3

ABCD is a rectangle with AB = DC = 5m and AD = BC = 6m. Forces of magnitude 8N, 5N, 3N, 6N and 10N act along AB, BC, CD, AD and DB of the rectangle respectively. The directions of the forces being given by the order of the letters. Find

(i) the magnitude and the direction of a single force that could replace this system of force
(ii) equation of the line of action
(iii) where the line of action cuts AB

Solution



(i) $\theta = \tan^{-1}\left(\frac{5}{6}\right) = 398^0$

$R = \begin{pmatrix}8\\0\end{pmatrix} + \begin{pmatrix}0\\5\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}0\\6\end{pmatrix} + \begin{pmatrix}cos39.8\\-sin39.8\end{pmatrix}$

$= \begin{pmatrix}11.4011\\3.3172\end{pmatrix}$

$|R| = \sqrt{11.4011^2 + 3.3172^2}$ =11.8739N

Direction, α =$\tan^{-1}\left(\frac{3.3172}{11.4011}\right)$ = 16.2^0 to AB

(ii) A G= (5 x 0.05) + (3 x 0.06) + (10 sin39.8 x 0.06) = 0.046Nm anticlockwise

0.046 - 0.3172x + 11.4011y = 0

(iii) line cuts AB where y = 0

0.046 - 0.3172x =0

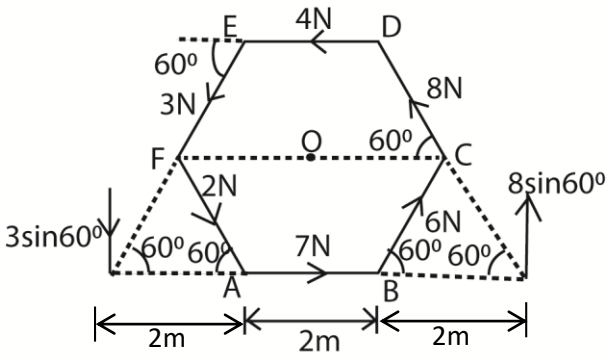
x = 0.0139m from A

Example 4

ABCDEF is a regular hexagon 2m. Forces of magnitude 7N, 6N, 8N, 4N, 3N and 2N act along AB, BC, CD, DE, EF and FA respectively in each case the direction of force being given by the order of letters. Given that AB is horizontal, find the

(a) the magnitude and direction of resultant force
(b) where its line of action of the resultant cuts AB

Solution



$R = \begin{pmatrix}7\\0\end{pmatrix} + \begin{pmatrix}6cos60\\6sin60\end{pmatrix} + \begin{pmatrix}-8cos60\\8sin60\end{pmatrix} + \begin{pmatrix}-4\\0\end{pmatrix} + \begin{pmatrix}-3cos60\\-3sin60\end{pmatrix} + \begin{pmatrix}2cos20\\-2sin60\end{pmatrix} = \begin{pmatrix}1.5\\7.794\end{pmatrix}$

$|R| = \sqrt{1.5^5 + 7.794^2} = 7.94N$

Direction, α = $\tan^{-1}\left(\frac{7.794}{1.5}\right)$= 79.11⁰ to AB

A: G =(6sin60⁰ x 2) + (8sin60⁰ x 4) + (4 x 4sin60⁰) + (3sin60⁰ x 2) = $33\sqrt{3}$ = 57.158Nm

$G = xF_y - yF_x$

57.158 = 7.794x - 1.5y

Line cuts AB when y = 0

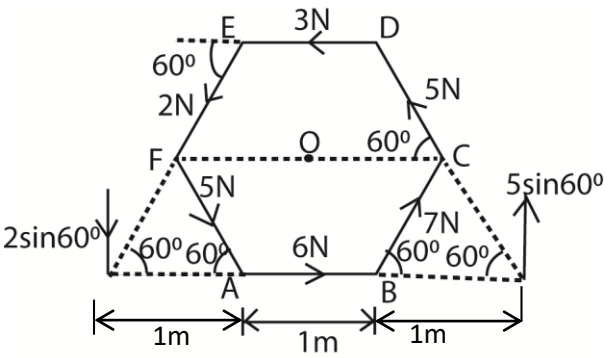
x = $\frac{57.158}{7.794}$ = 7.334m from A

Example 5

ABCDEF is a regular hexagon 1m. Forces of magnitude 6N, 7N, 5N, 3N, 2N, and 52N act along AB, BC, CD, DE, EF and FA respectively in each case the direction of force being given by the order of letters. Given that AB is horizontal, find the

(a) the magnitude and direction of resultant force
(b) where its line of action of the resultant cuts AB

Solution



$R = \begin{pmatrix}6\\0\end{pmatrix} + \begin{pmatrix}7cos60\\7sin60\end{pmatrix} + \begin{pmatrix}-5cos60\\5sin60\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}-2cos60\\-2sin60\end{pmatrix} + \begin{pmatrix}5cos20\\-5sin60\end{pmatrix} = \begin{pmatrix}5.5\\4.33\end{pmatrix}$

$|R| = \sqrt{5.5^5 + 433^2} = 7.0N$

Direction, α = $\tan^{-1}\left(\frac{4.33}{5.5}\right)$= 38.21^0 to AB

A: G = (7sin60^0 x 1) + (5sin60^0 x 2) + (4 x 4sin60^0) + (3 x 2sin60^0)+ (2sin60 x 1= 12.5$\sqrt{3}$

= 21.65Nm anticlockwise

$G = xF_y - yF_x$

21.65 = 4.33x - 5.5y

Line cuts AB when y = 0

x = $\frac{21.65}{4.33}$ = 5m from A

Example 6

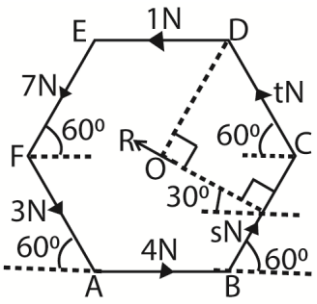
The center of a regular hexagon ABCDEF of side 2a is O. Forces of magnitude 4N, sN, tN, 1N, 7N and 3N act along the sides AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters.

(i) Given that the resultant of these six forces is of magnitude $2\sqrt{3}N$ acting in the direction perpendicular to BC, determine the value of s and t.

(ii) show that the sum of moments of forces about O is 27a$\sqrt{3}Nm$

(iii) If the midpoint of BC is M, find the equation f the line of action of the resultant, refer to OM as x-axis and OD as y-axis

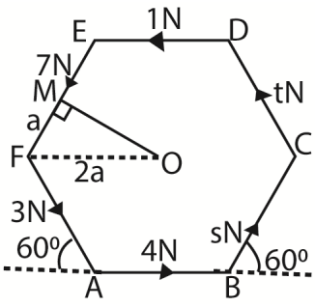
Solution



$$\begin{pmatrix} -2\sqrt{3}cos30 \\ 2\sqrt{3}sn30 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} + \begin{pmatrix} s\ cos60 \\ s\ sin60 \end{pmatrix} + \begin{pmatrix} -tcos60 \\ tsin60 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} -7cos60 \\ -7sin60 \end{pmatrix} + \begin{pmatrix} 3cos60 \\ -3sin60 \end{pmatrix}$$

$s = 2\ and\ t = 10$

(ii)



OM = $\sqrt{(2a)^2 - a^2}$ =a$\sqrt{3}$

O: G =(4 + 2 + 10 + 1 + 7 + 3) a$\sqrt{3}$ = 27 a$\sqrt{3}$Nm

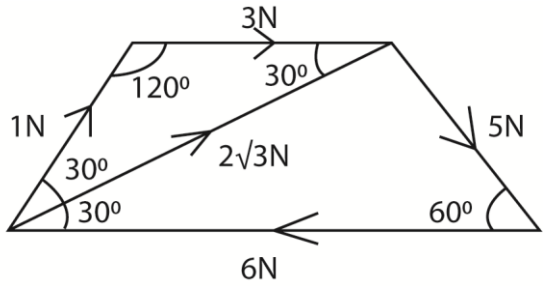
Since the resultant is in direction OM, => x = 2$\sqrt{3}$, y =0

G = xY + yX; resultant cuts the x-axis when Y = 0

27 a$\sqrt{3}$ = -2$\sqrt{3}$y; y = -13.5a

Example 7

The diagram below shows a trapezium rule ABCD, AD = DC = CB = 1 and AB = 2meters. Forces of magnitude 1N, 3N, 5N, 6N and $2\sqrt{3}$ N respectively.



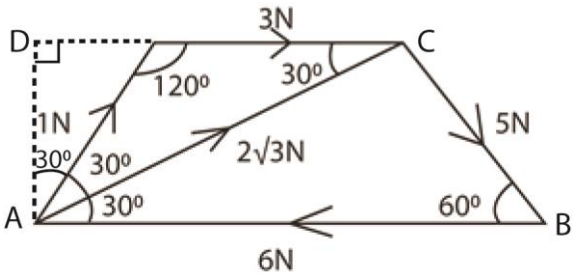
(a) Calculate the magnitude of the resultant force and the angle it makes with side AB (09marks)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2\sqrt{3}\cos 30^0 \\ 2\sqrt{3}\sin 30^0 \end{pmatrix} + \begin{pmatrix} \cos 60^0 \\ \sin 60^0 \end{pmatrix} + \begin{pmatrix} 5\cos 60^0 \\ -5\sin 60^0 \end{pmatrix} = \begin{pmatrix} 3 \\ -\sqrt{3} \end{pmatrix}$$

$Resultant\ force, R = \sqrt{(3)^2 + \left(-\sqrt{3}\right)^2}$ = 3.464N

$Direction, \alpha = tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ = 30⁰

(b) Given that the line of action of the resultant force meets AB at X, find AX. (03marks)



Equation of the line action of the resultant is given by G-xY + yX = 0

Taking moments about A

G= -3 x 1cos 30^0 – 5 x 2 cos 30^0

= -3 x $\frac{\sqrt{3}}{2}$ -10 x $\frac{\sqrt{3}}{2}$ = $\frac{-13\sqrt{3}}{2}$

By substitution

$\frac{-13\sqrt{3}}{2} + \sqrt{3}x + 3y = 0$

The line of action of the resultant cuts AB when y = 0

$\frac{-13\sqrt{3}}{2} + \sqrt{3}x + 3\ x\ 0 = 0$

$x = 6.5m$

Hence $\overline{AX}$= 6.5m

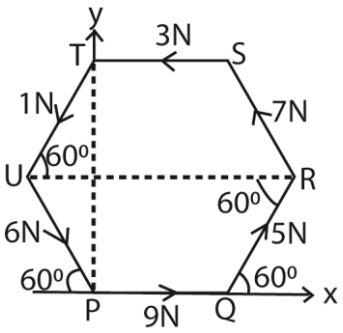
Example 8

Six forces, 9N, 5N, 7N, 3N, 1N and 4N act along the sides PQ, QR, RS, ST, TU and UP of a regular hexagon of side 2m, their direction being indicated by the order of letters. Taking PQ as the reference axis, express each of the forces in vector form. Hence find the

(i) magnitude and direction of the resultant of forces

(ii) distance from P, where the line of action of the resultant cuts PQ.

Solution



PQ = $\begin{pmatrix}9\\0\end{pmatrix}$;

QR = $\begin{pmatrix}5cos60^0\\5sin60^0\end{pmatrix} = \begin{pmatrix}\frac{5}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix}$

RS= $\begin{pmatrix}-7cos60^0\\7sin60^0\end{pmatrix} = \begin{pmatrix}\frac{7}{2}\\\frac{7\sqrt{3}}{2}\end{pmatrix}$

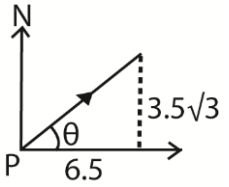
ST = $\begin{pmatrix}-3\\0\end{pmatrix}$

TU = $\begin{pmatrix}5cos60^0\\5sin60^0\end{pmatrix} = \begin{pmatrix}\frac{-1}{2}\\\frac{-\sqrt{3}}{2}\end{pmatrix}$

UP= $\begin{pmatrix}4cos60^0\\-4sin60^0\end{pmatrix} = \begin{pmatrix}2\\-2\sqrt{3}\end{pmatrix}$

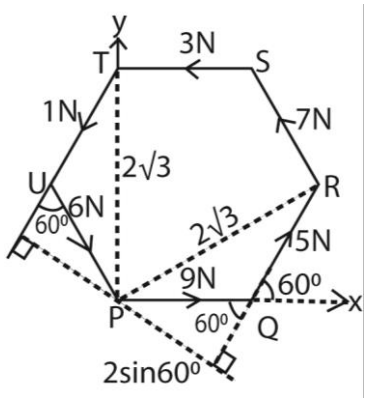
$$\mathrm{R} = \begin{pmatrix}9\\0\end{pmatrix} + \begin{pmatrix}\frac{5}{2}\\\frac{5\sqrt{3}}{2}\end{pmatrix} + \begin{pmatrix}\frac{7}{2}\\\frac{7\sqrt{3}}{2}\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}\frac{-1}{2}\\\frac{-\sqrt{3}}{2}\end{pmatrix} + \begin{pmatrix}2\\-2\sqrt{3}\end{pmatrix} = \begin{pmatrix}6.5\\3.5\sqrt{3}\end{pmatrix}$$

$|R| = \sqrt{(6.5)^2 + \left(3.5\sqrt{3}\right)^2}$ = 8.9N



$\theta = \tan^{-1}\left(\frac{3.5\sqrt{3}}{6.5}\right) = 43^0$

(ii) Equation of the line of the resultant is given by: G - xY + yX



P: G = $5\sin60^0 + 7 \times 2\sqrt{3} + 3 \times 2\sqrt{3} + 1 \times 2\sin60^0$

$= 5\sqrt{3} + 14\sqrt{3} + 6\sqrt{3} + \sqrt{3} = 26\sqrt{3}$Nm

By substitution

$26\sqrt{3} - 3.5\sqrt{3}x + 6.5y = 0$

The line cuts PQ when y = 0

⇨ $26\sqrt{3} - 3.5\sqrt{3}x = 0$; x = 7.43m

∴The lines cuts PQ at a distance 7.43m from P

Revision exercise

1. ABCD is a square of side 5m. Forces of 4N, 6N, 8N and 10N act along BD, DC, CA and CB respectively. When a force P acts along AD and force Q acts along AB, the whole system is equivalent a couple. Find the magnitude of P and Q and the moment of the couple. [12.8N, 2.49N, 65,9Nm]
2. ABCD is a square of side am. Forces of 1N, 4N, 3N and 6N act along AB, CB, DC and AD respectively. Calculate the magnitude and direction of a single force that could replace this system of forces and find where its line of action cuts AB. [4.47N, 26.6^0 to AB, 3.5a from A]
3. ABCD is a rectangle with AB = 3m and angle CAB = 30^0. Forces of 10N, 20N and 20N act along AC, AD and DB respectively. Calculate the magnitude and direction of a single force that could replace this system of forces and where its line of action cuts AB. [30N, 30^0 to AB, 2m from A]
4. ABCD is a rectangle with AB = 5m and BC = 3m. Forces of magnitude 2N, 4N, 3N and 1N act along the lines AB, BC, DC and DA respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force [6N]
   (ii) equation of the line of action of the resultant
   (iii) distance from A where the line of action of the resultant force cuts AB. [1.57m]
5. ABCD is a rectangle with AB = 4m and BC = 3m. Forces of magnitude 3N, 5N, 6N, 4N and 7N act along the lines AB, BC, CD, DA and AC respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force. [5.81N]
   (ii) distance from A where the line of action of the resultant force cuts AB[7.31m]
6. ABCD is a square of side 2m. Forces of magnitude 10N, 9N, 8N, 7N and 5N act along the lines AB, BC, CD, DA and AC respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force [20.305N]

(ii) sum of moments about A [34Nm]

(iii) distance from A where the line of action of the resultant force cuts AB. [1.74m]

7. ABCDEF is a regular hexagon of side 4m. Forces of magnitude 6N, 4N, 7N, 8N, 4N and 2N act along the lines AB, BC, CD, DE, EF and FA respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force [7.55N]
   (ii) distance from A where the line of action of the resultant force cuts AB. [21.71m]
8. ABCDEF is a regular hexagon of side 34m. Forces of magnitude 5N, 6N, 2N, 3N, 6N and 1N act along the lines AB, BC, CD, DE, EF and FA respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force [4.583N]
   (ii) distance from A where the line of action of the resultant force cuts AB. [13.2m]
9. ABCDEF is a regular hexagon of side 2m. Forces of magnitude 2N, 3N, 4N and 5N act along the lines AC, AE, AF and ED respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
   (i) magnitude and direction of the resultant force [8.84N, 57.6^0 to AB]
   (ii) distance from A where the line of action of the resultant force cuts AB. [2.32m from A]
10. ABCDEF is a regular hexagon of side 2m. Forces of magnitude 3N, 4N, 2N, 1N, 2N and 6N act along the lines AB, BC, DC, ED, EF and AF respectively, in each case the direction of force being given by the order of the letters. Given AB is horizontal, find the
    (i) magnitude and direction of the resultant force [6N]
    (ii) Equation of the line of action of the resultant force [y = $\sqrt{3}x$]
11. ABCDE is a regular pentagon of side 2m. Forces of 5N act along AB, BC and AD. find the
    (i) magnitude and direction of the resultant force [12.5N,at 49.6^0 to AB]
    (ii) distance from A where the line of action of the resultant force cuts AB. [1m from A]

Thank You

Dr. Bbosa Science