## Moment of a force

This is the product of a force and perpendicular distance from the pivot to the line of action of the force. The unit of moments is Nm.


## Matrix approach of finding sum of moments about the origin

If forces $\left(a_{1} \mathrm{i}+b_{1} \mathrm{j}\right) \mathrm{N},\left(a_{2} \mathrm{i}+b_{2} \mathrm{j}\right) \mathrm{N}, \ldots \ldots \ldots . .\left(a_{n} \mathrm{i}+b_{n} \mathrm{j}\right)$ act on the body at point $\left(x_{1}+y_{1}\right),\left(x_{2}+y_{2}\right), \ldots .$. $\left(\left(x_{n}+y_{n}\right)\right.$. The sum of the moments about the origin is
$G=\left|\begin{array}{ll}x_{1} & a_{1} \\ y_{1} & b_{1}\end{array}\right|+\left|\begin{array}{ll}x_{2} & a_{2} \\ y_{2} & b_{2}\end{array}\right|+\cdots+\left|\begin{array}{ll}x_{n} & a_{n} \\ y_{n} & b_{n}\end{array}\right|$
$G=\left(b_{1} x_{1}-a_{1} y_{1}\right)+\left(b_{2} x_{2}-a_{2} y_{2}\right)+\cdots+\left(b_{n} x_{n}-a_{n} y_{n}\right)$
Note
If $G$ is positive, the sum of moments will be anticlockwise and if $G$ is negative the sum of moments will be clockwise.

Example 1
Find the moment about the origin of a force of 4 jN acting at a point which has position vector -5 iN
Solution
$G=\left|\begin{array}{cc}-5 & 0 \\ 0 & 4\end{array}\right|=-5 \times 4-0 \times 0=-20 \mathrm{Nm}$ clockwise
Example 2
Find the moment about the origin of a force of 4 jN acting at a point which has position vector 5 iN
$G=\left|\begin{array}{ll}5 & 0 \\ 0 & 4\end{array}\right|=5 \times 4-0 \times 0=20 \mathrm{Nm}$ anticlockwise
Example 3
Forces of $(2 \mathrm{i}-3 \mathrm{j}) \mathrm{N},(4 \mathrm{i}+\mathrm{j}) \mathrm{N}$ and $(5 \mathrm{i}-3 \mathrm{j}) \mathrm{N}$ act on a body at points with Cartesian co-ordinates $(1,1)$, $(2,4)$, and $(-1,3)$ respectively. Find the sum of moments of the forces about the origin.

Solution

$$
\begin{aligned}
G & =\left|\begin{array}{cc}
1 & 2 \\
1 & -3
\end{array}\right|+\left|\begin{array}{ll}
2 & 4 \\
4 & 1
\end{array}\right|+\left|\begin{array}{cc}
-1 & 5 \\
3 & -3
\end{array}\right|=(1 \times-3-2 \times 1)+(2 \times 1-4 \times 4)+(-1 \times-3-3 \times 5)=-31 \mathrm{Nm} \\
& =31 \mathrm{Nm} \text { clockwise }
\end{aligned}
$$

Example 4
Forces $(2 \mathrm{i}-3 \mathrm{j}) \mathrm{N},(5 \mathrm{i}+\mathrm{j}) \mathrm{N}$ and $(-4 \mathrm{i}+4 \mathrm{j})$ act on a body at points with position vector $(\mathrm{i}+\mathrm{j}),(-2 \mathrm{i}+2 \mathrm{j})$ and $(3 i-4 j)$ respectively. Find the sum of moments of forces about the
(i) origin

$$
\begin{aligned}
G & =\left|\begin{array}{cc}
1 & 2 \\
1 & -3
\end{array}\right|+\left|\begin{array}{cc}
-2 & 5 \\
2 & 1
\end{array}\right|+\left|\begin{array}{cc}
3 & -4 \\
-4 & 4
\end{array}\right|=(1 \times-3-2 \times 1)+(-2 \times 1-2 \times 5)+(3 \times 4--4 \times-4) \\
& =-21 \mathrm{Nm}=21 \mathrm{Nm} \text { clockwise }
\end{aligned}
$$

(ii) point with position vector ( $\mathrm{i}-\mathrm{j}$ )

$\mathrm{G}=(5 \times 3)+(1 \times 3)+(2 \times 0)+(2 \times 2)+(4 \times 3)-(4 \times 2)=26 \mathrm{Nm}$ clockwise

## Revision exercise

1. Find the moment about the origin of a force of $3 i$ acting at a point which has position vector ( $2 \mathrm{i}+3 \mathrm{j}$ )m. [9Nm clockwise]
2. Find the moment about the origin of force $(4 i+2 j) N$ acting at a point which has position vector $(3 i+2 j) m$. [2Nm clockwise]
3. A force of $(3 \mathrm{i}-2 \mathrm{j}) \mathrm{N}$ act at a point which has position vector $(5 i+j) \mathrm{m}$. Find the moment about the point which has a position vector $(\mathrm{i}+2 \mathrm{j}) \mathrm{m}$. [5Nm clockwise]
4. A force of $(2 i+j) N$ act at a point which has position vector $(2 i+2 j) m$ and a force of $5 i N$ at a point which has position vector $(-2 i+j) m$. Find the sum of moments of these forces about the origin. [7Nm clockwise]
5. A force of $(3 i+2 j) N$ act at a point which has position vector $(5 i+j) m$ and a force of $(1+j) N$ act at a point which has position vector $(2 i+j) m$. Find the sum of moments of these forces about the point which has position vector $(\mathrm{i}+3 \mathrm{j}) \mathrm{m}$. [17Nm anticlockwise]
