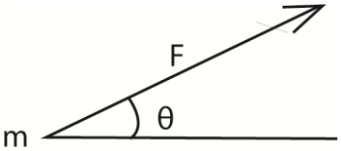
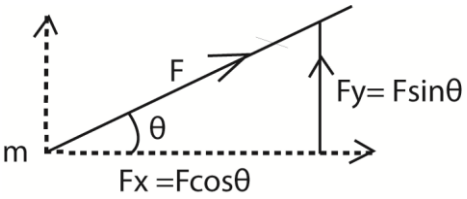
Resolution of forces

The component of a vector is the effective value of a vector along a particular direction. The component along any direction is the magnitude of a vector multiplied by the cosine of the angle between its direction and the direction of the component.

Suppose a force F pulls a body of mass m along a truck at an angle θ to the horizontal as shown below



The effective force that makes the body move along the horizontal is the component of F along the horizontal



cosθ =$\frac{F_x}{F}$;

$F_x = Fcos\theta$

sinθ =$\frac{F_y}{F}$;

$F_y = Fsin\theta$

Resultant $F_R = \sqrt{{F_x}^2 + {F_y}^2}$

Direction, α =$\tan^{-1}\left(\frac{F_y}{F_x}\right)$

Note

When a vector is inclined at an angle θ to the horizontal then;

- along the horizontal, the component of the vector is cosθ
- along the vertical, the component of the vector is sinθ

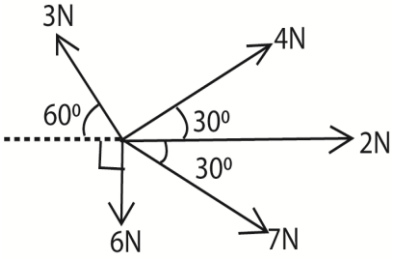
When a vector is inclined at an angle θ to the horizontal then;

- along the horizontal, the component of the vector is sinθ
- along the vertical, the component of the vector is cos θ

Example 1

Find the resultant of the system of forces below

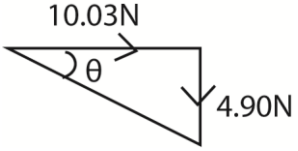
(a)



$F_R = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}4cos30\\4sin30\end{pmatrix} + \begin{pmatrix}7cos30\\-7sin30\end{pmatrix} + \begin{pmatrix}-3cos60\\3sin60\end{pmatrix} + \begin{pmatrix}0\\-6\end{pmatrix}$

$= \begin{pmatrix}10.03\\-4.90\end{pmatrix}$

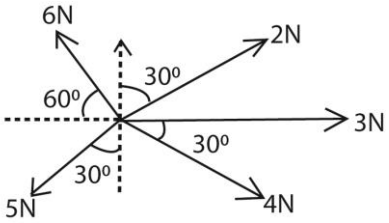
$F_R = \sqrt{(10.03)^2 + (-4.90)^2}$ = 11.16N



θ= $\tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{4.9}{10.03}\right)$ = 26.04⁰

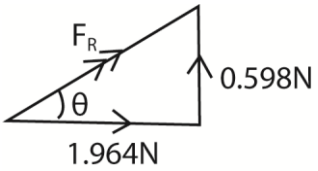
The resultant force is 11.16N at 26.04⁰ below the horizontal

(b)



$F_R = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}2sin30\\2cos30\end{pmatrix} + \begin{pmatrix}4cos30\\-4sin30\end{pmatrix} + \begin{pmatrix}-5sin30\\-5cos30\end{pmatrix} + \begin{pmatrix}-6\cos 60\\6\sin 60\end{pmatrix} = \begin{pmatrix}1.964\\0.598\end{pmatrix}$

$F_R = \sqrt{1.964^2 + 0.598^2}$ = 2.053N

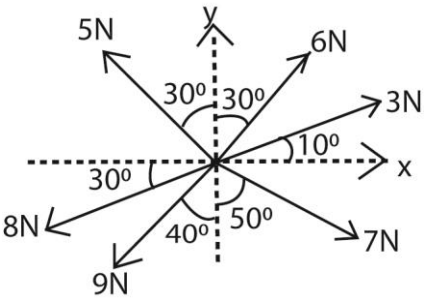


θ= $\tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{0.598}{1.964}\right)$ = 16.9⁰

The resultant force is 2.053N at 16.9⁰ below the horizontal

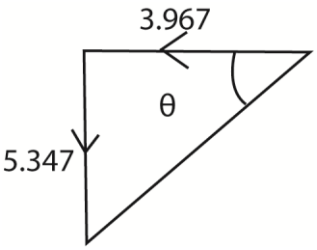
Example 2

Forces of 3N, 6N, 5N, 8N, 9N and 7N act on a particle as shown in the figure below. Find the resultant force.



$F_R = \begin{pmatrix}3cos10\\3sin10\end{pmatrix} + \begin{pmatrix}6sin30\\6cos30\end{pmatrix} + \begin{pmatrix}-5sin30\\5cos30\end{pmatrix} + \begin{pmatrix}-8cos30\\-8sin30\end{pmatrix} + \begin{pmatrix}-9sin40\\-9cos40\end{pmatrix} + \begin{pmatrix}7sin50\\-7cos50\end{pmatrix} = \begin{pmatrix}-3.967\\-5.347\end{pmatrix}$

$F_R = \sqrt{(-3.967)^2 + (-5.347)^2}$ =6.658N



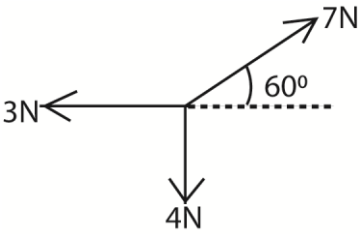
$\theta = \tan^{-1}\left(\frac{5.347}{3.967}\right) = 53.43^0$

Resultant force is 6.658N at 53.43^0 below horizontal

Example 3

A body of mass 1kg is acted on by the forces shown below. Find

(i) Magnitude of the resultant force
(ii) acceleration of the body
(iii) distance moved in 2s



$F_R = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + \begin{pmatrix} 7\cos 60 \\ 7sin60 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 2.06 \end{pmatrix}$

$F_R = \sqrt{(0.5)^2 + (2.06)^2}$ = 2.12N

but $F_R = ma$

2.12 =1a

a = 2.12ms^{-2}

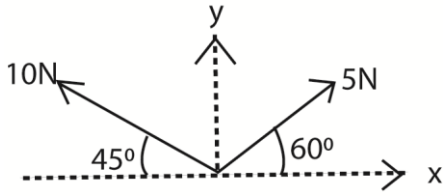
From s = ut + ½ at^2

u = 0, t = 2s, a =2.12ms^{-2}

s = 0 x 2 + ½ x 2.12 x 2^2 = 4.24m from the origin

Revision exercise

1. A force of 3N acts at 60^0 to a force of 5N. Find the magnitude and direction of their resultant. [7N at 21.8^0 to the 5N force]
2. A force of 3N at 90^0 to the force of 4N. Find the magnitude and direction of their resultant. [5N at 37^0 to the 4N force]
3. Two coplanar forces act on a point O as shown below



Calculate the magnitude and direction of the resultant force

[12.3N at 68.0 above the horizontal

4. The resultant of two forces pN and 3N is 7N. If the 3N is reversed, the resultant is $\sqrt{17}$N Find the value of p and the angle between the two forces.[$2\sqrt{6}N, 57.02^0$]

5. Forces of 2N, 3N, 5N and 5N act on a particle in the direction 030^0, 090^0, 120^0, 210^0, and 330^0 respectively. Find the magnitude and direction of the resultant force. [1.92N at 7.8^0 to the horizontal]
6. Forces of 7N, 2N, 2N and 5N act on a particle in the direction 060^0, 160^0, 200^0, and 315^0 respectively. Find the resultant force. [4.14N at 52.36^0 below the horizontal]
7. Find the magnitude and direction of the resultant of force 10N, 15N and 8N acting in the direction 030^0, 150^0 and 225^0. [12.1N at 55.6^0 below the positive x-axis]

Thank you

Dr. Bbosa Science