



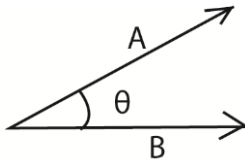
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## Resultant of forces

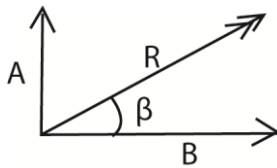
A force is anything which change a body's state of rest or uniform motion in a straight line. Examples are weight, tension, reaction, friction, resistance force.

Resultant of two forces

Consider two forces A and B inclined to each other at an angle  $\theta$



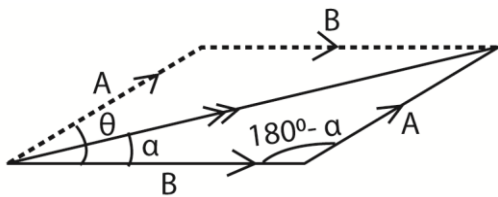
(i)  $\theta$  is right angle ( $\theta = 90^\circ$ )



$$\text{Resultant, } R = \sqrt{A^2 + B^2}$$

$$\text{Direction of resultant, } \beta = \tan^{-1} \left( \frac{A}{B} \right)$$

(ii)  $\theta$  is acute ( $0^\circ \leq \theta \leq 90^\circ$ )

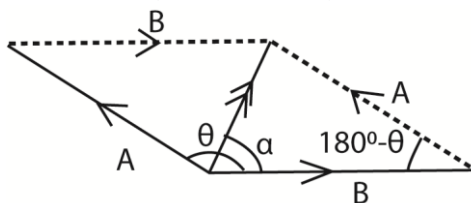


$$\text{Direction of resultant, } \frac{\sin \alpha}{A} = \frac{\sin(180-\theta)}{R}$$

$$\alpha = \sin^{-1} \left( \frac{A \sin(180-\theta)}{R} \right)$$

$$\text{Resultant, } R = \sqrt{A^2 + B^2 - 2AB \cos(180 - \theta)}$$

(iii)  $\theta$  is obtuse ( $90^\circ \leq \theta \leq 180^\circ$ )



$$\text{Direction of resultant, } \frac{\sin \alpha}{A} = \frac{\sin(180-\theta)}{R}$$

$$\alpha = \sin^{-1} \left( \frac{A \sin(180-\theta)}{R} \right)$$

$$\text{Resultant, } R = \sqrt{A^2 + B^2 - 2AB \cos(180 - \theta)}$$

**Example 1**

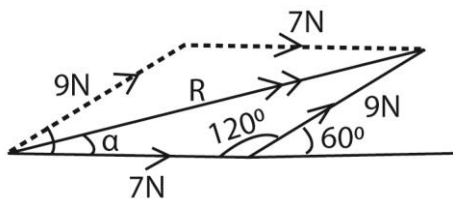
Two forces of magnitude 5N and 12N act on a particle with their direction inclined at  $90^\circ$ . Find the magnitude and direction of the resultant

$$R = \sqrt{5^2 + 12^2} = 13\text{N} \quad \alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^\circ$$

The resultant = 13N at  $22.6^\circ$  to 12N force

**Example 2**

Forces of magnitude 7N and 9N act on a particle at an angle of  $60^\circ$  between them. Find the magnitude and direction of the resultant.



$$\text{Direction of resultant, } \frac{\sin \alpha}{9} = \frac{\sin(180-\theta)}{13.89}$$

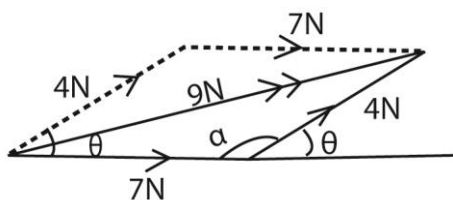
$$\alpha = \sin^{-1}\left(\frac{9\sin(180-60)}{13.89}\right) = 34.13^\circ$$

$$\begin{aligned} \text{Resultant, } R &= \sqrt{[A^2 + B^2 - 2AB\cos(180 - \theta)]} \\ &= \sqrt{[7^2 + 9^2 - 2 \times 7 \times 9 \cos(180 - 60)]} \\ &= 13.89\text{N} \end{aligned}$$

**Example 3**

Find the angle between a force of 7N and 4N their resultant has a magnitude of 9N

Solution



$$\alpha = \cos^{-1}\left(-\frac{2}{7}\right) = 106.6^\circ$$

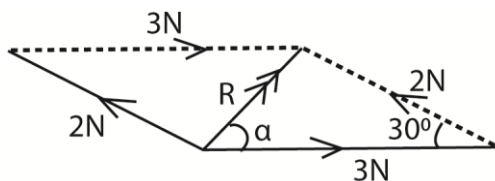
the angle  $\theta$  between the forces =  $180 - 106.6$

$$= 73.4^\circ$$

$$9^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \times \cos \alpha$$

**Example 4**

Forces of 3N and 2N act on a particle at an angle of  $150^\circ$  between them. Find the magnitude and direction of the resultant.



$$\text{Direction of resultant, } \frac{\sin \alpha}{2} = \frac{\sin(30)}{1.61}$$

$$\alpha = \sin^{-1}\left(\frac{2\sin(180-60)}{1.61}\right) = 38.3^\circ$$

$$R^2 = 2^2 + 3^2 - 2 \times 2 \times 3 \times \cos(30)$$

$$R = 1.61\text{N}$$

### Revision exercise

1. Two forces of magnitude 7N and 24N act on a particle with their direction at  $90^\circ$ . Find the magnitude and direction of the resultant. [25N,  $16.26^\circ$  with 24N force]
2. Forces of 5N and 8N act on a particle at an angle of  $50^\circ$  between them. Find the magnitude and direction of the resultant. [11.9N at  $19^\circ$  with 8N force]
3. Forces of 4N and 6N act on a particle at angle  $60^\circ$  between them. Find the magnitude and the direction of the resultant. [5.29N, at  $40.9^\circ$  with 6N force]
4. Forces of 9N and 10N act on a particle at angle  $40^\circ$  between them. Find the magnitude and the direction of the resultant. [17.9N, at  $18.9^\circ$  with 10N force]
5. Forces of 12N and 10N act on a particle at angle  $105^\circ$  between them. Find the magnitude and the direction of the resultant. [13.5N, at  $45.7^\circ$  with 12N force]
6. Forces of 8N and 3N act on a particle at angle  $160^\circ$  between them. Find the magnitude and the direction of the resultant. [5.28N, at  $11.2^\circ$  with 8N force]
7. Find the angle between a force of 10N and 4N their resultant has a magnitude of 8N. [ $130.5^\circ$ ]
8. The angle between a force  $\alpha$ N and a force of 3N is  $120^\circ$ . If the resultant of the two forces has magnitude 7N, find the value of  $\alpha$ . [8N]
9. The angle between a force  $\beta$ N and a force of 8N is  $45^\circ$ . If the resultant of the two forces has a magnitude 15N, find the value of  $\beta$ . [8.24N]

Thank you

Dr. Bbosa Science