Vertical motion under gravity

When a body is projected **vertically downwards**, it is subjected to an acceleration of $9.8ms^{-2}$. i.e. $a = g = 9.8ms^{-2}$

Equations of motion become

$v = u + gt$; **$h = ut + \frac{1}{2}gt^2$;** **$v^2 = u^2 + 2gh$**

When a body is projected **vertically upwards**, it is subjected to a retardation of $9.8ms^{-2}$. i.e. $a = g = 9.8ms^{-2}$

Equations of motion become

$v = u - gt$; **$h = ut - \frac{1}{2}gt^2$;** **$v^2 = u^2 - 2gh$**

Maximum /greatest height

When a particle is projected vertically upwards, the final velocity is 0ms-1 at its maximum height

$v^2 = u^2 - 2gh$

$0 = u^2 - 2gh_{max}$ | $h_{max} = \frac{u^2}{2g}$

Time to reach maximum height

$v = u - gt$

$0 = u - gt$ | $t = \frac{u}{g}$

Time of flight

$T = \frac{2u}{g}$

Example 1

A stone is dropped from a point which is 40m above the ground. Find the time taen for the stone to reach the ground

$h = ut + \frac{1}{2}gt^2$

$40 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$ | $t = \sqrt{\frac{40}{9.8}} = 2.857s$

Example 2

A ball is thrown vertically upwards with an initial speed of $30ms^{-1}$. Calculate

(i) Time taken to reach thrower

$T = \frac{2u}{g} = \frac{2\ x\ 30}{9.8} = 6.12s$

(ii) maximum height reached

$h_{max} = \frac{u^2}{2g} = \frac{30^2}{2\ x\ 9.8} = 45.92m$

Example 3

A particle is projected from the ground level vertically upwards with velocity of $19.6ms^{-1}$. Find

(i) greatest height reached

$h_{max} = \frac{u^2}{2g} = \frac{19.6^2}{2\ x\ 9.8} = 19.6m$

(ii) time taken by the particle to reach maximum height

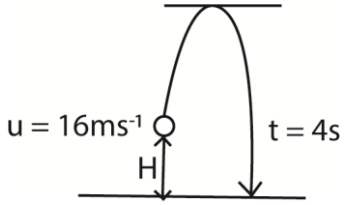
$t = \frac{u}{g} = \frac{19.6}{9.8} 2s$

(iii) Time of flight

T = 2t = 2 x 2 = 4s

Example 4

1. A stone is thrown vertically upwards with velocity $16ms^{-1}$ from a point H meters above the ground level. The stone hits the ground 4 seconds later.
   Calculate the
   (a) Value of H (03marks)



Using s = ut + ½ at^2; s = -H(below point of projection), u = $16ms^{-1}$, a = -g, t = 4s

$-H = 16 \times 4 - \frac{1}{2}\ x\ 9.8\ x\ 4^2$

H = 14.4m

   (b) Velocity of the stone as it hits the ground (02marks)

Using v = u + at; v = -v(below point of projection), a = -g, t = 4s

-v = 16 -9.8 x 4

v = $23.2ms^{-1}$

∴ the velocity of the stone as it hits the ground is $23.2ms^{-1}$

Example 6

A stone is thrown vertically upwards with a velocity of $21ms^{-1}$. Calculate the

(a) Maximum height attained by the stone (03marks)

$H = \frac{u^2}{2g} = \frac{21^2}{2\ x\ 9.8} = 225m$

(b) Time the stone takes to reach the maximum height. (02marks)

$t = \frac{u}{g} = \frac{21}{9.8} = 2.143s$

Example 7

A particle is projected vertically upwards with velocity ums-1. After t seconds another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Prove that the particles collide after $\left(\frac{2}{2}+\frac{u}{g}\right)s$. Hence show that they meet at a height of $\frac{u^2-(gt)^2}{8g}$.

Solution

t_1 = time taken by 1st particle

t_2 = time taken by 2nd particle

$t_1 - t_2$ = t …….. (i)

t_1 and t_2 are roots of the equation

h = ut - $\frac{1}{2}gt^2$ or $gt^2 - 2ut + 2h = 0$

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$

$t_2 = \frac{2u-\sqrt{4u^2-8gh}}{2g}$

$\frac{2u+\sqrt{4u^2-8gh}}{2g} - \frac{2u-\sqrt{4u^2-8gh}}{2g}$ = t

$\sqrt{4u^2-8gh} = gt$ …… (ii)

From eqn (ii)

$h = \frac{4u^2-(gt)^2}{8g}$

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$ putting eqn. (ii)

$$t_1 = \frac{2u+gt}{2g} = \left(\frac{2}{2}+\frac{u}{g}\right)s$$

Example 8

A particle is projected upwards with velocity of 10ms-**1**. After 2s another particle is projected vertically upwards from the same point of projection with the same initial velocity. Find the height above the level of projection where the particle meet and time taken by the first particle before they meet.

t_1 = time taken by 1st particle

t_2 = time taken by 2nd particle

$t_1 - t_2$ = t …….. (i)

t_1 and t_2 are roots of the equation

h = ut - $\frac{1}{2}gt^2$ or $gt^2 - 2ut + 2h = 0$

$t_1 = \frac{20+\sqrt{400-8gh}}{2g}$

$t_2 = \frac{20-\sqrt{400-8gh}}{2g}$

$\frac{20+\sqrt{400-8gh}}{2g} - \frac{20-\sqrt{400-8gh}}{2g}$ = t

$\sqrt{400-8gh} = gt$ …… (ii)

From eqn (ii)

$h = \frac{400-(2\ x\ 9.8)^2}{8\ x\ 9.8} = 0.202m$

$t_1 = \frac{2u+\sqrt{4u^2-8gh}}{2g}$ putting eqn. (ii)

$t_1 = \frac{2u+gt}{2g} = \left(\frac{2}{2}+\frac{u}{g}\right) = \frac{2\ x10+9.8\ x\ 2}{2\ x\ 9.8} = 2.02\ s$

Revision exercise

1. A particle is projected vertically upwards with a velocity of 21ms^{-1}. How long it takes to reach a point 280m below the point of projection. [10s]
2. A particle is projected vertically upwards with a velocity of 17.5ms^{-1}. Find
   (i) how high the particle goes. [15.6m]

(ii) what time elapse before it's at a height of 10m [$\frac{5}{7}s$; $\frac{22}{7}s$

3. A particle is projected vertically upwards with velocity of 24.5ms^{-1}. Find
   (a) when its velocity is 4.9ms^{-1} [2s]
   (b) how long it takes to return to the point of projection.[5s]
   (c) at what time it will be 19.6m above the point of projection. [1s and 4s]
4. A particle is projected vertically upwards with a velocity of 35ms^{-1}. find
   (a) how long it takes to reach the greatest height.[3.57s]
   (b) distance it ascends during the 3rd second of motion. [10.5m]
5. Two objects are dropped from a cliff of height H. the second is dropped when the first has travelled a distanc e d. Prove that the instant when the first object reaches the bottom, the second is a distance $2\sqrt{DH} - D$ from the top of the cliff.
6. A particle is projected vertically upwards from point O with a speed of $\frac{4}{3}v\ ms^{-1}$. After it has travelled a distance of $\frac{2}{5}X\ m$ above O on its upward motion, another particle is projected vertically upwards from the same point with the same initial speed. Given that the particles collide at a height $\frac{2}{5}X\ m$ above O, prove that
   (i) the maximum height, H is given by $8v^2 = 9gH$
   (ii) when the particle collide 9X = 20H.
7. A particle is projected vertically upwards with velocity um/s. After t seconds another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Prove that the particles collide each other having a velocity of $\frac{1}{2}gt$.
8. A particle is projected vertically upwards with velocity 28m/s. After 2s another particle is projected vertically upwards from the same point of projection and with an initial velocity of 21m/s. Find when the two particles are at the same height and the velocity of each body at that instant. [4.9s after the first particle is projected, 20m/s, 7.4m/s]
9. A particle is projected vertically upwards with velocity 25m/s. After 4s another particle is projected vertically upwards from the same point of projection and with the same initial velocity. Find the time and height when the two particles meet.[4.55s after the first particle is projected, 12.288m]
10. A stone is dropped from the top of a tower. In the last second of its motion, it falls through a distance which is a fifth of the height of the tower. Find the height of the tower. [439.6m]

Thank You

Dr. Bbosa Science