Couple of forces

These are equal forces acting in opposite direction

Conditions for forces to form a couple

Forces reduce to a couple if;

- resultant force is zero
- the sum of moments about a point is not zero

Example 1

Forces of (-5i – j)N, -3j and (5i + 4j) act on a body a point with position vectors (i – j)m, (2i + j)m and (4i – 5j)m respectively. Show that these forces reduce to a couple

Solution

$$R = \begin{pmatrix}-5\\-1\end{pmatrix} + \begin{pmatrix}0\\-1\end{pmatrix} + \begin{pmatrix}5\\4\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

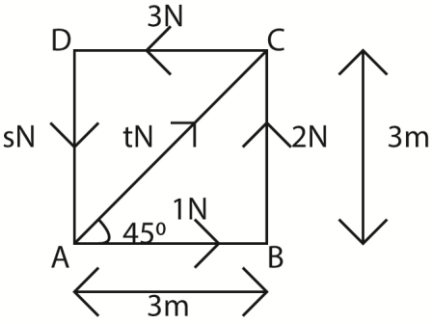
$$G = \begin{vmatrix}1 & -5\\-1 & -1\end{vmatrix} + \begin{vmatrix}2 & 0\\1 & -3\end{vmatrix} + \begin{vmatrix}4 & 5\\-5 & 4\end{vmatrix}$$

= [(1 x -1) – (-5 x -1)]+[(3 x -3) – (1 x 0)] + [(4 x 4) – (5 x -5)] = 29Nm

Since the resultant force is zero an the sum of moment G is not zero, the forces reduce to a couple.

Example 2

ABCD is a square of side 3m. Forces of magnitude 1N, 2N, 3N, sN and tN act along the line AB, BC, CD, DA and AC respectively, in each case the direction of the force being given by the order of letters. Taking AB as horizontal and BC as vertical, find the values of s and t so that the resultant of the forces is a couple.



$$R = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\2\end{pmatrix} + \begin{pmatrix}-3\\0\end{pmatrix} + \begin{pmatrix}0\\-s\end{pmatrix} + \begin{pmatrix}tcos45\\tsin45\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$$

(→) tcos45 = 2; t = $\frac{2}{cos45} = 2\sqrt{2}$

(↑) 2 – s + tsin45 = 0; s = 2 + $2\sqrt{2}$ sin45 = 4N

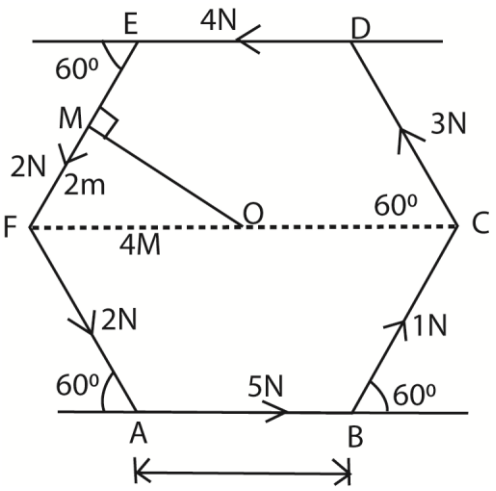
It must also be shown the G≠0

A G = 2 x 3 + 3 x 3 = 15Nm

Example 3

ABCDEF is a regular hexagon of side 4m. Forces of magnitude 5N, 1N, 3N, 4N, 2N and 2N act along the lines AB, BC, CD, DE, EF and FA respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal, show that these forces reduce to a couple.

Solution



OM = $\sqrt{4^2 - 2^2} = 2\sqrt{3}m$

(→) 5 – 4 + (1 – 3 – 2 +2)cos60 = 0

(↑) (1 + 3 – 2 – 2) sin60 = 0

↺ O G = $2\sqrt{3}$ (5 + 1 + 3 + 4 + 2)

= $34\sqrt{3}$N anticlockwise

Since R = 0 and G≠ 0 then is a couple.

Revision exercise

1. Forces of 6N, 8N, 6N and 8N act along sides of a rectangle ABCD where AB =8m and BC = 6m in the direction AB, BC, CD and DA respectively.
   (a) show that the forces reduce to couple
   (b) find the moment of the couple about A. (100Nm)
2. Forces of 5N, 3N, 5N and 3N act along the side of a square ABCD of side 4m in the directions AB, BC, CD and DA respectively.
   (a) show that the forces reduce to couple
   (b) find the moment of the couple about A. (100Nm
3. ABCD is a rectangle with AB = 6m and BC = 2m. A force of 3N acts along each of the four sides AB, BC, CD and DA in the directions indicated by the order of the letters. Show that the forces form a couple and find its moment. [21Nm]
4. ABCD is a rectangle with AB = 6m and BC = 2m. A forces of 5N, 5N, xN, and xN acts along each of the four sides CB, AD, AB and CD in the directions indicated by the order of the letters. If the system is in equilibrium, find the value of x. [15N]
5. ABCD is a square of side 40cm. Forces of magnitude 20N, 15N, 20N and Y act along the line AB, BC, CD and DA respectively in each case the direction of the force being given by the order of letters. If the system is equivalent to a couple, find the magnitude of Y and the moment of the couple. [15N, 14Nm]
6. A force of (3i – 5j)N acts at a point which has position vector (6i + j)m and a force of (-3i + 5j)N acts at the point which has position vector (4i + j)m. Show that the forces reduce to a couple and find the moment of the couple [10Nm]
7. A force of (4i + 3j)N acts at a point which has position vector (6i + 3j)m and a force of (-4i -3j)N acts at the point which has position vector (3i – j)m. Show that the forces reduce to a couple and find the moment of the couple [7Nm]

8. Force of (i + j)N, (-4i + j)N and (3i – 2j)N act at the points having position vectors (2i + 2j)m, (-1 + 4j)m and (4i – 2j)m respectively. Show that the forces reduce to a couple and find the moment of the couple [13Nm]
9. Forces of (ai + bj)N and (6i – 4j)N act at points having position vectors (-2i – 2j)m and (3i – j)m respectively. If these forces reduce to a couple, find a and b and the moment of the couple. [-6, 4 and 26Nm]

Thank You

Dr. Bbosa Science