Linear momentum

This is the product of mass of a body and its velocity.
Momentum = mass x velocity

Collisions

Case 1: bodies separate after collision

Consider two bodies A and B with body A having a mass of M_A, initial velocity U_A, and body B having a mass f M_B, initial velocity U_B, after collision body A has a final velocity V_A and body B has a final velocity V_B.

$M_AU_A + M_BU_B = M_AV_A + M_BV_B$

Loss in k.e = $\left(\frac{1}{2}M_AU_A{}^2 + \frac{1}{2}M_BU_B{}^2\right) - \left(\frac{1}{2}M_AV_A{}^2 + \frac{1}{2}M_BV_B{}^2\right)$

%loss in k.e = $\frac{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)-\left(\frac{1}{2}M_AV_A{}^2+\frac{1}{2}M_BV_B{}^2\right)}{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)}x100\%$

Case2: bodies stick together and move with a common velocity after collision

Consider two bodies A and B with body A having a mass of M_A, initial velocity U_A, and body B having a mass f M_B, initial velocity U_B, after collision body A and body B stick together and move with common velocity V.

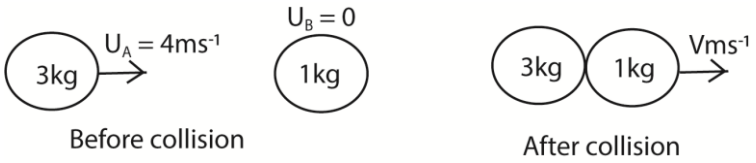
$M_AU_A + M_BU_B = (M_A + M_B)V$

Loss in k.e = $\left(\frac{1}{2}M_AU_A{}^2 + \frac{1}{2}M_BU_B{}^2\right) - \left(\frac{1}{2}(M_A + M_B)V^2\right)$

%loss in k.e = $\frac{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)-\left(\frac{1}{2}(M_A+M_B)V^2\right)}{\left(\frac{1}{2}M_AU_A{}^2+\frac{1}{2}M_BU_B{}^2\right)}x100\%$

Example 1

A trolley of mass 3kg travelling at a velocity of 4ms^{-1} collide with another trolley of mass 1kg which is at rest. At what velocity do the two bodies move together after collision?



$M_AU_A + M_BU_B = (M_A + M_B)V$

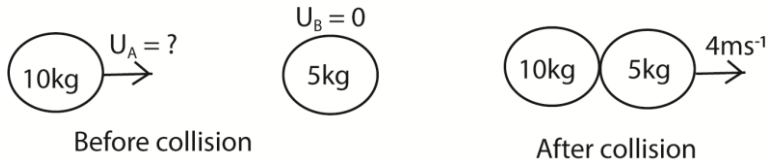
(3 x 4) + (1 x 0) = (3 + 1)V

12 = 4V

V = 3ms^{-1}

Example 2

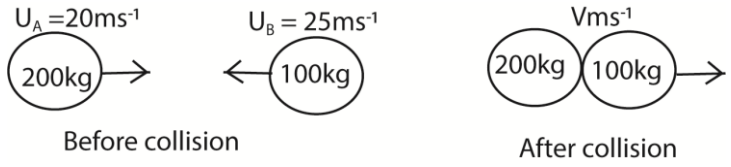
An object of mass 10kg collides with a stationary object of mass 5kg. If the objects stick together and move forward with a velocity of 4ms^{-1}. What was original velocity of the moving objects?



$M_AU_A + M_BU_B = (M_A + M_B)V$
$(10 \times U) + (5 \times 0) = (10 + 5) \times 4$
$U = 60ms^{-1}$

Example 3

Two bodies of masses 200kg and 100kg travel towards each other with velocities of $20ms^{-1}$ and $25ms^{-1}$ respectively and join to form one body on collision. Find the common velocity.



$M_AU_A + M_BU_B = (M_A + M_B)V$
$(200 \times 20) + (100 \times -25) = (200 + 100) \times v$
$V = 5ms^{-1}$

Example 4

A particle of mass 2kg moving with a speed $10ms^{-1}$ collides with a stationary particle of mass 7kg. Immediately after impact, the particle moves with the same speed but in opposite directions. Find the loss in kinetic energy

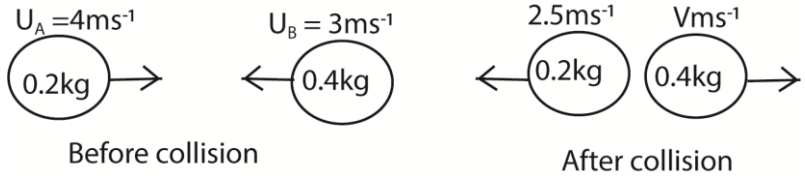
$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
$2 \times 10 + 7 \times 0 = 2 \times -v + 7 \times v$
$v = 4ms^{-1}$
k.e before = $\frac{1}{2}\ x\ 2\ x\ 10^2 + \frac{1}{2}\ x\ 7\ x\ 0^2 = 100J$

k.e after = $\frac{1}{2}\ x\ 2\ x\ 4^2 + \frac{1}{2}\ x\ 7\ x\ 4^2$ = 72J

lossin k.e = 100 – 72 = 28J

Example 5

Two particles are moving towards each other along a straight line. The first particle has mass of 0.2kg and moving with velocity $4ms^{-1}$ and then the second has a mass of 0.4kg moving with a velocity of $3ms^{-1}$. On collision, the first particle reverses its direction and moves with a velocity of $2.5ms^{-1}$. Find the percentage loss in kinetic energy.



$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
$0.2 \times 4 + 0.4 \times -3 = 0.2 \times -2.5 + 0.4V$
$V = 0.25ms^{-1}$
k.e before = $\frac{1}{2}\ x\ 0.2\ x\ 4^2 + \frac{1}{2}\ x\ 0.4\ x\ 3^2$
= 3.4J

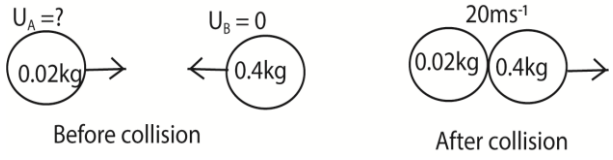
k.e after = $\frac{1}{2}\ x\ 0.2\ x\ -2.5^2 + \frac{1}{2}\ x\ 0.4\ x\ 0.25^2$
= 0.6375J

% loss in k.e = $\frac{(3.4-0.6375)}{3.4}\ x\ 100\%$ = 81.25%

Example 6

A bullet of mass 20g is fired into a block of wood of mass 400g lying on a smooth horizontal surface. If the bullet and the wood move together with the speed of $20ms^{-1}$. Calculate

(a) the speed with which the bullet hits the wood



$M_AU_A + M_BU_B = (M_A + M_B)V$
0.02 x UA + 0.4 x 0 = 0.42 x 20
UA = 420ms^{-1}

(b) The kinetic energy loss
k.e energy before = $\frac{1}{2}\ x\ 0.02\ x\ 420^2 = 1764J$
k.e after = $\frac{1}{2}\ x\ 0.42\ x\ 20^2 = 84J$
Loss in k.e = 1764 - 84 = 1680J

Example 7

Two bodies A and B of mass 4kg and 3kg moving with velocities (2i + 3j)ms^{-1} and (5i - 6j)ms^{-1} respectively collide. After collision A moves with a velocity (5i)ms^{-1}, Find the

(i) velocity of B after collision
$M_AU_A + M_BU_B = M_AV_A + M_BV_B$
$4(2i + 3j) + 3(5i – 6j) = 4\ (5i) + 3V_B$
$V_B = (i - 2j)\ ms^{-1}$

(ii) loss in kinetic energy
k.e before = $\frac{1}{2}\ x\ 4(2^2 + \ 3^2) + \frac{1}{2}\ x\ 3(5^2 + \ (-6)^2)$ = 117.5J
k.e after = $\frac{1}{2}\ x\ 4(5^2) + \frac{1}{2}\ x\ 3(1^2 + \ (-2)^2)$= 57.5J
Loss in k.e = 117.5 – 57.5 = 60J

Example 8

Two bodies A and B of mass 7.5kg and 5.0kg moving with velocities of (–i – 2j)ms^{-1} and (9i + 8j)ms^{-1} respectively. After collision the bodies stick together and move with a common velocity, find the

(i) common velocity
$M_AU_A + M_BU_B = (M_A + M_B)V$
$7.5(–i – 2j) + 5.0(9i + 8j) = 12.5V$
$V = (3i + 2j)ms^{-1}$

(ii) percentage loss in kinetic energy
k.e before = $\frac{1}{2}\ x\ 7.5((-1)^2 + \ (-2)^2) + \frac{1}{2}\ x\ 5(9^2 + \ 8^2)$ = 381.25J
k.e after = $\frac{1}{2}\ x\ 12.5(3^2 + \ 2^2)$= 81.25J
Loss in k.e = 381.25 – 81.25 = 300J
%loss in k.e = $\frac{300}{381.25}\ x\ 100\% = 78.69\%$

Example 9

Two bodies A and B of masses 3kg and 2kg respectively are 7m a part on a smooth horizontal surface. A moving directly towards B with a speed of 2ms^{-1} and acceleration of 0.3ms^{-2}. B is moving in the in the same direction as A with a speed of 5ms^{-1} and retardation of 0.2ms^{-2}. If the bodies collide and coalesce, calculate

(i) Time taken before collision occurs
let the distance travelled by B before collision = x
$s = ut + ½\ at^2$

For A: 7 + x = 2t + $\frac{1}{2}$ x $0.3t^2$ …… (i)

For B: x = 5t – $\frac{1}{2}$ x 0.2t^2 ……(ii)

Subtract (ii) from (i)

7 = -3t + $0.25t^2$

$t^2 - 12t - 28 = 0$

$t = 14s$

(ii) Common velocity immediately after collision

Initial velocity of A before collision V_A = 2 + 0.3 x 14 = 6.2ms^{-1}

Initial velocity of B before collision VB = 5 – 0.2 x 14 = 2.2ms^{-1}

Let the common velocity be V

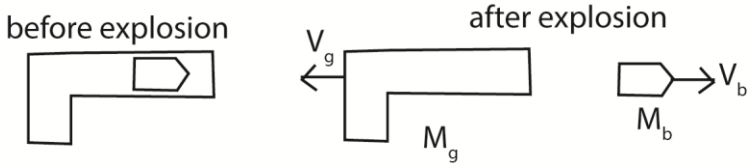
$M_AU_A + M_BU_B = (M_A + M_B)V$

3 x 6.2 + 2 x 2.2 = 5V

V = 4.6ms^{-1}

Recoil velocity of a gun and muzzle velocity of a bullet

When a bullet of mass Mb is fired with a muzzle vel0city of Vb, the gun of mass Mg jerks backward with a recoil velocity of Vg.



M_g x 0 + M_b x 0 = M_g x -V_g + M_bV_b

M_g x V_g = M_bV_b

Example 10

A bullet of mass 60g is fired from a gun of mass 3kg. The bullet leaves the gun with velocity of 400ms^{-1}. Find the initial speed of recoil of the gun and gain in kinetic energy of the system.

0.06 x 400 = 3 x V

V = 8ms^{-1}

Gain in k.e = k.e after = $\frac{1}{2}$ x 0.06 x 400^2 + $\frac{1}{2}$ x 3 x 8^2 =4896J

Example 11

A gun of mass 3000kg fires horizontally a shell at initial velocity of 300ms^{-1}. If the recoil of the gun is brought to rest by a constant opposing force of 9000N in 2 seconds, find the

(a)(i) Initial velocity of the recoil gun

F = ma	v = u + at
-9000 = 3000a	0 = u – 3 x 2
a = -3ms^{-2}	u = 6ms^{-1}

(ii) Gain in kinetic energy of the shell just after firing

$M_gV_g = M_bV_b$

$3000 \times 6 = M_b \times 300$

$M_b = 60kg$

(b)(i) displacement of the gun

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 6^2}{2\,x - 3} = 6m$$

(ii) work done against the opposing force

$W = Fs = 9000 \times 6 = 54{,}000J$

Revision exercise

1. A bullet of mass 50g travelling horizontally at 80ms-1 hits a block of wood of mass 10kg resting on a smooth horizontal plane. If the bullet emerges with a speed of $50ms^{-1}$, find the speed with which the block moves. [$0.15ms^{-1}$]
2. A bullet of mass 0.1kg travelling at 420ms-1 hits a block of wood of mass 2kg resting on a smooth horizontal plane. If the bullet becomes embedded on the block, find the speed with which the block moves after impact. [20ms-1]
3. A 2kg object moving with a velocity of $8ms^{-1}$ collides with a 3kg object moving with a velocity $6ms^{-1}$ along the same direction. If the collision is completely inelastic, calculate the decrease in kinetic energy.[2.4J]
4. A particle A of mass 150g lies at rest in a smooth horizontal surface. A second particle B of mass 100g is projected along the surface with the speed ums^{-1} and collides directly with A. On collision the masses coalesce and move with speed $4ms^{-1}$. Find the value of u and loss in the kinetic energy of the system during impact. [$10ms^{-1}$, 3J]
5. Two bodies A and B of masses 2kg and 4kg moving with velocities of $8ms^{-1}$ and $5ms^{-1}$ respectively collide and move in the same direction. Object A's new velocity is $6ms^{-1}$.
   (i) find the velocity of B after collision [$6ms^{-1}$]
   (ii) calculate the percentage loss in kinetic energy [5.26%]
6. Two bodies A and B of mass 2kg and 3kg moving with velocities of $4ms^{-1}$ and $3ms^{-1}$ respectively in the same direction collide and coalesce, find he
   (i) common speed after collision [$3.4ms^{-1}$]
   (ii) loss in kinetic energy [0.6J]
   (iii) percentage loss in kinetic energy [2.03%]
7. A bullet of mass 30g is fired horizontally at $200ms^{-1}$ and hits a block of wood of mass 2kg resting on smooth horizontal plane. If the bullet becomes embedded on the block, find the
   (i) common velocity of bullet and wood. [$2.96ms^{-1}$]
   (ii) percentage loss in kinetic energy. [98.52%]
8. Two smooth sphere A and B of equal radii and mass 3kg and 1.5kg respectively are travelling along the same horizontal line in opposite direction. The speeds of A and B are $6ms^{-1}$ and $2ms^{-1}$ respectively. The sphere A collides and after collision B reverses its direction and moves with speed of $4ms^{-1}$. Find the velocity of A after collision. [$3ms^{-1}$]
9. Two smooth spheres A and B of equal radii and masses 180g and 100g respectively travelling along the same horizontal line. The initial speeds of A and B are $2ms^{-1}$ and $6ms^{-1}$ respectively. The spheres collide and after collision both spheres reverse their directions and B moves with speed of $3ms^{-1}$. Find the speed of A after collision and loss in kinetic energy of the system. [$3ms^{-1}$, 0.9J]

10. A particle of mass 2kg moving with speed $10ms^{-1}$ collides with stationary particles of mass 7kg. Immediately after impact the particles move with the same speeds but in opposite direction. Find the loss in kinetic energy during collision. [28J].
11. Two identical rail way truck are travelling in the same direction along the same straight piece of track with constant speed of $6ms^{-1}$ and $2ms^{-1}$. The faster truck catches up with the other one on collision, the two trucks couple together. Find the common speed of the trucks after collision. [$4ms^{-1}$]
12. A 2kg object moving with a velocity of $6ms^{-1}$ collides with a stationary object of mass 1kg. If the collision is perfectly elastic, calculate the velocity of each object after collision. [$2ms^{-1}$, $8ms^{-1}$]
13. A van of mass of mass 1200kg and a lorry of mass 3200kg collide. Just before the crash they are moving directly towards each other and each has a speed of $12ms^{-1}$. Immediately after the crash they move with the same velocity. Find the loss in kinetic energy. [251kJ]
14. A body of mass 6kg moving with velocity $(8i - 4j)ms^{-1}$ collides with a body of mass 2kg which is at rest. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(6i - 3j)ms^{-1}$]
    (ii) loss in kinetic energy [960J]
15. A body of mass 2kg moving with velocity $(-2i + 4j)ms^{-1}$ collides with a body of mass 3kg moving with velocity $(3i + 4j)ms^{-1}$. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(i + 4j)\ ms^{-1}$]
    (ii) loss in kinetic energy [15J]
16. A body of mass 500g moving with velocity $(2i - 4j)ms^{-1}$ collides with a body of mass 1500g moving with velocity $(6i + 8j)ms^{-1}$. On collision the two bodies coalesce, find the
    (i) common velocity after collision [$(5i + 5j)ms^{-1}$]
    (ii) loss in kinetic energy [30J]
17. Two bodies A and B of mass 2kg and 5kg moving with velocities of $(-2i + 3j)ms^{-1}$ and $(6i - 10j)ms^{-1}$ respectively collide. After collision A moves with velocity $(3i - 2j)ms^{-1}$, find the
    (i) velocity of B after collision [$(4i - 8j)ms^{-1}$]
    (ii) loss in kinetic energy [140J]
18. Two bodies A and B of mass 5kg and 2kg moving with velocities of $(-4i + 3j)ms^{-1}$ and $(3i - 10j)ms^{-1}$ respectively collide. After collision A moves with a velocity $(-2i + j)ms^{-1}$, find the
    (i) velocity of B after collision [$(-2i + 4j)ms^{-1}$]
    (ii) loss in kinetic energy [40J]
19. A body X, of mass 250g moving with velocity $(-2i + 3j)ms^{-1}$ collide with a body Y of mass 750g moving with velocity $(5i + 8j)ms^{-1}$. After collision X moves with a velocity (-2i + j)ms-1, find the
    (i) velocity of Y after collision [$(4i + 6j)ms^{-1}$]
    (ii) loss in kinetic energy [5.25J]
20. A shell of mass 5kg is fired from a gun of mass 2000kg. The shell leaves the gun with a velocity of $400ms^{-1}$. find the initial speed of recoil of the rifle and gain in kinetic energy of the system [$1ms^{-1}$, 2520J)
21. A bullet of mass 20g is fired from a rifle of mass 2.5kg. The bullet leaves the gun with a velocity of $500ms^{-1}$. Find the initial of the recoil of rifle and gain in kinetic energy of the system. [$4ms^{-1}$, 2520J]
22. A bullet of mass 5kg is fired from a rifle of mass 2000g. The bullet leaves the gun with a velocity of $400ms^{-1}$. Find the recoil velocity of the rifle. [$1ms^{-1}$]

Thank you

Dr. Bbosa Science