Circular motion

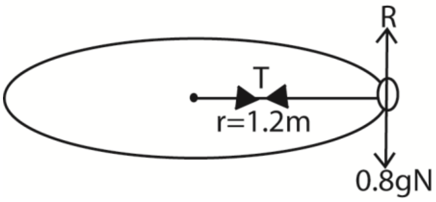
Circular motion on a smooth horizontal surface

Example 1

A particle of mass 0.8kg is attached to one end of a light inextensible string of length 1.2m. The other end is fixed to a point P on a smooth horizontal table. The particle is set moving in a circular path. If the speed of the particle is 16ms^{-1};

(i) Determine the tension in the string and the reaction on the table

(ii) If the string snaps when the tension in the string exceed 100N, find the greatest angular velocity at which the particle can travel.



(i) T = F= $\frac{mv^2}{r} = \frac{0.8\ x\ 16^2}{1.2}$ =170.667N

R = 0.8gN = 0.8 x 9.8 = 7.4N

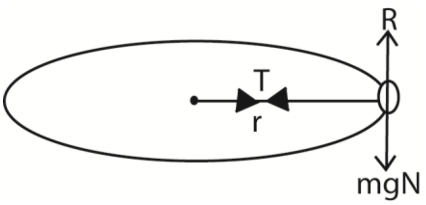
(ii) T = mω^{2}r

100 = 0.8 x ω^2 x 1.2

ω = 10.21rads^{-1}

Example 2

A ball is tied on an elastic sting of natural length 30m to a fixed point on a smooth horizontal table upon which a ball is describing a circle around a point at a constant speed. If the modulus of elasticity of the string is 100times the weight of the ball and the number of revolution per minute is 20. Show that extension in the string is approximately 4.7m



T= F = mω^{2}r

ω = 2πf = 2π$\left(\frac{20}{60}\right) = \frac{2}{3}\pi$ and r = 30 + e

$T = m\left(\frac{2}{3}\pi\right)^2(30+e)$ ………………(i)

T= $\frac{\lambda}{L}e = \frac{100mg}{30}e$ ………. (ii)

Eqn. (i) and (ii)

$\frac{100mg}{30}e = m\left(\frac{2}{3}\pi\right)^2(30+e)$

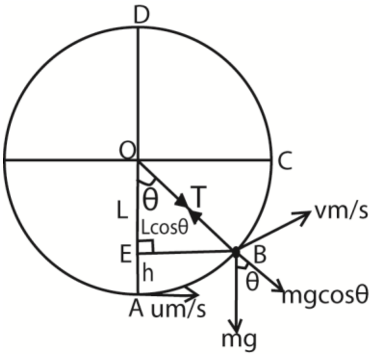
98 x 3e = $4\pi^2(30+e)$

$e = \frac{4\pi^2 x\ 30}{294-4\pi^2}$ = 4.6532m ≈ 4.7m

Motion in a vertical plane

Particle in fourth quadrant

Consider a body of mass m attached to a string (light rod) of length L and whirled in a vertical circle with a constant speed v. if there is no air resistance to the motion, then the net force towards the centre is the centripetal force. Tension in the string acts in the same way as the reaction R



At equilibrium at B: T- mgcosθ = $\frac{mv^2}{L}$

$T = \frac{mv^2}{L} + \text{ mgcos}\theta$ …………. (i)

But $v^2 = u^2 + 2as$

a = -g, s = h = L –Lcosθ

$v^2 = u^2 - 2g(L - L\cos\theta)$ ……… (ii)

when the particle comes momentarily to

rest at some point A, then v = 0

$o = u^2 - 2g(L - L\cos\theta)$

$cos\theta = L - \frac{u^2}{2gL}$ ……………… (iii)

If the particle is attached to a rod, it can it

can complete circle when v > 0 and θ= 180^0

$v^2 = u^2 - 2g(L - L\cos\theta)$

$u^2 - 2g(L - L\cos 180) > 0$

$u^2 > 2g(L + L)$

$u^2 > 4gL$ ……………(iv)

Put (ii) into (i)

$T = \frac{m[u^2 - 2g(L - Lcos\theta)]}{L} + mgcos\theta$

$T = \frac{m[u^2 - 2g(L - Lcos\theta)] + mgLcos\theta}{L}$

$T = \frac{m}{L}(u^2 - 2gL + 3gLcos\theta)$ ….. (v)

If the particle attached to a string, it can complete circle when T > 0 and θ = 180^0

$T = \frac{m}{L}(u^2 - 2gL + 3gLcos\theta)$

$\frac{m}{L}(u^2 - 2gL + 3gLcos\theta) > 0$

$\frac{m}{L}(u^2 - 2gL + 3gLcos180) > 0$

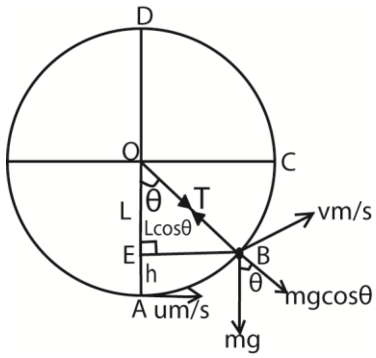
$(u^2 - 2gL - 3gL) > 0$

$u^2 > 5gL$ ……………… (vi)

Example 3

A particle P of mass 5kg is suspended from a fixed point O by light inextensible string of length 1m. The particle is projected from its lowest position at A, with horizontal speed of $4ms^{-1}$. When the angle AOB = 60^0, find

(a) Speed at P (b) the tension in the string at P



T.M.E_A = T.M.E_B

$5\left(\frac{1}{2}x\ 4^2 + 9.8\ x\ 0\right) = 5\left(\frac{1}{2}x\ 4^2 + 9.8\ x\ h\right)$

$5\left(\frac{1}{2}x\ 4^2 + 9.8\ x\ 0\right) = 5\left(\frac{1}{2}x\ 4^2 + 9.8\ (1 - cos60\right)$

v = 2.49ms^{-1}

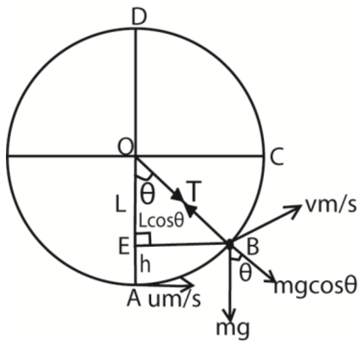
$T = \frac{mv^2}{L} + mgcos\theta$

$T = \frac{5(2.49)^2}{1} + 5\ x\ 9.8cos60$ = 55.5N

Example 4

A light rod of length 2m is pivoted at one end O and has a particle of mass 8kg attached at the other end. The rod is held vertically with the particle at A, directly below O and the particle is given an initial speed ums^{-1}; find

(a) Find an expression in terms of u and θ for the speed of the particle when at B where angle AOB = θ
(b) Restriction on u2 if the particle to perform complete oscillation



At B: $v^2 = u^2 + 2as$

a = -g, s = h = L –Lcosθ

$v^2 = u^2 – 2g(L –Lcos\theta)$

v = $\sqrt{u^2 - 2gL(1 - cos\theta)}$

If the particle to perform complete oscillation
v > 0 and θ = 180^0

$0^2 = u^2 – 2g(L –Lcos\theta)$

$u^2 – 2g(L –Lcos180) > 0$

$u^2 > 2g(L + L)$

$u^2 > 4gL$

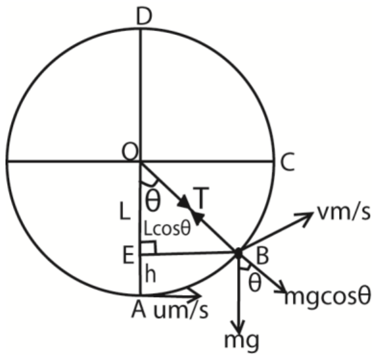
$u^2 > 4 \times 9.8 \times 2$

$u^2 > 78.4$

Example 5

A particle P of mass 8kg is suspended from a fixed point O by an inextensible string of length 2m. the particle is projected from its lowest position A with an initial speed ums^{-1}. Find

(a) An expression in terms of u and θ for the tension in the string when the particle is at B where angle AOB = θ
(b) Restriction on u2 if the particle is to perform complete oscillation



At equilibrium at B:

T – mgcosθ = $\frac{mv^2}{L}$

$T = \frac{mv^2}{L} + mgcos\theta$ ………… (i)

$v^2 = u^2 + 2ah$

a = -g, s = h = L –Lcosθ

$v^2 = u^2 - 2g(L - Lcos\theta)$

v = $\sqrt{u^2 - 2gL(1 - cos\theta)}$………(ii)

Put (ii) into (i)

$T = \frac{m[u^2 - 2g(L - Lcos\theta)]}{L} + mgcos\theta$

$T = \frac{m[u^2 - 2g(L - Lcos\theta)] + mgLcos\theta}{L}$

$T = \frac{m}{L}(u^2 - 2gL + 3gLcos\theta)$

for the particle attached to circle when T > 0 and θ = 180^0

$T = \frac{m}{L}(u^2 - 2gL + 3gLcos\theta)$

$\frac{m}{L}(u^2 - 2gL + 3gLcos\theta) > 0$

$\frac{m}{L}(u^2 - 2gL + 3gLcos180) > 0$

$(u^2 - 2gL - 3gL) > 0$

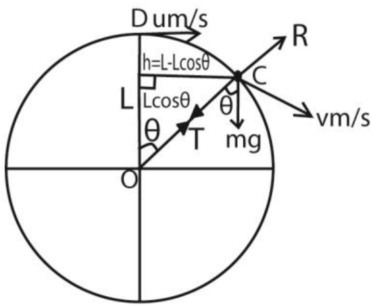
$u^2 > 5gL$

$u^2 > 5\ x\ 9.8\ x\ 2$

$u^2 > 98$

Particle in first quadrant

Consider a body of mass m rolled from the top of a sphere of radius L. The normal reaction R acts outwards



At equilibrium at C

mgcosθ – R = $\frac{mv^2}{L}$

R = mgcosθ - $\frac{mv^2}{L}$ ……….(i)

$v^2 = u^2 + 2ah$

a = g, s = h = L –Lcosθ

$v^2 = u^2 + 2g(L - Lcos\theta)$ ……. (ii)

Putting (ii) into (i)

R = mgcosθ - $\frac{m[u^2 + 2g(L - Lcos\theta)]}{L}$

R = $\frac{mgLcos\theta - mu^2 - 2mgL + 2mgLcos\theta}{L}$

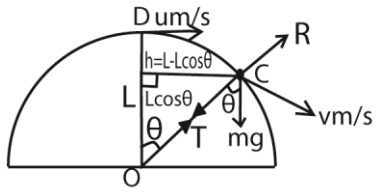
$R = \frac{m}{L}(3gLcos\theta - 2gL - u^2)$……..(iii)

Example 6.

A particle of mass 5kg is slightly disturbed from rest on the top of a smooth hemisphere, radius 4m and centre O, resting with its plane face on horizontal ground.

(a) Show that the particle leaves the surface of hemisphere at point C, where the angle between the radius OC and the upward vertical is $\cos^{-1}\frac{2}{3}$

(b) Find the speed at C



At equilibrium at C

mgcosθ – R = $\frac{mv^2}{L}$

R = mgcosθ - $\frac{mv^2}{L}$ ……….(i)

$v^2 = u^2 + 2ah$

u = 0, a = g, s = h = L –Lcosθ

v^2 = 2g(L –Lcosθ) ……. (ii)

Putting (ii) into (i)

R = mgcosθ - $\frac{m[2g(L - L\cos\theta)]}{L}$

$R = \frac{3mgL\cos\theta - 2mgL}{L}$

When the particle leaves the surface of the sphere R = 0

$0 = \frac{3mgL\cos\theta - 2mgL}{L}$

3cosθ =2

θ =$\cos^{-1}\frac{2}{3}$

From eqn. (ii)

v^2 = 2g(L –Lcosθ) = 2g(L –L x $\frac{2}{3}$)

$v^2 = \frac{2}{3}gL = \frac{2}{3}\ x\ 9.8\ x\ 4$ = 26.1333

v = 5.1121ms^{-1}

Thank you

Dr. Bbosa Science