Coplanar forces (Rigid bodies)

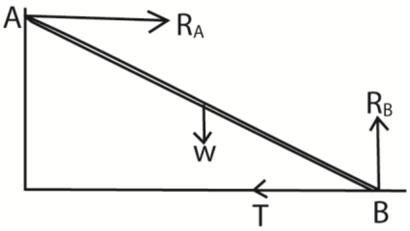
A rigid body is one in which distances between its various parts remain fixed.

Equilibrium of a rigid body

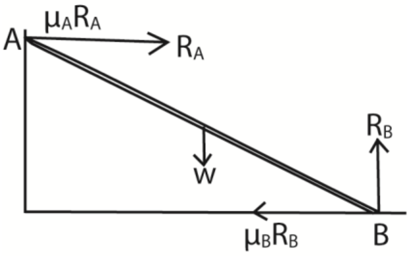
(i) Sum of forces acting in one direction is equal to the sum of force acting in opposite direction
(ii) Sum of clockwise moments about a point is equal to sum of anticlockwise moments about the same point.

Points to note

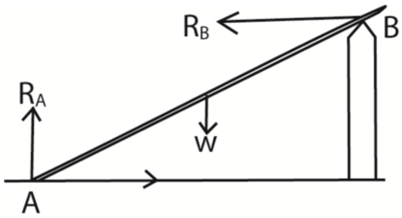
- When a rigid body rests in contact with a smooth wall and a string tied at the base the reaction on the body is perpendicular the surface



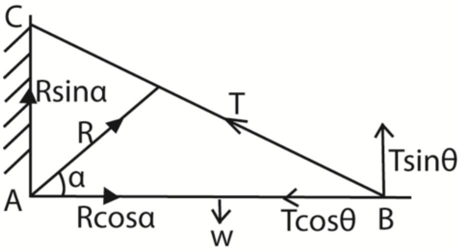
- When a rigid body rests in contact with a Rough wall and ground



- A rigid body resting against a smooth peg or bar, the reaction on the rigid body is perpendicular



- When a rod is hinged, the reaction at the hinge acts at an angle to either the horizontal or vertical



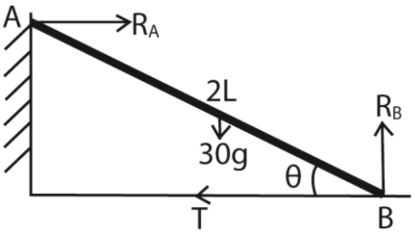
Smooth contacts at a ladder

Example 1

A uniform ladder AB of mass 30kg rests with its upper end A against a smooth vertical wall and the lower end B on a smooth horizontal ground. A light inextensible string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 60^0 to the horizontal. The vertical plane containing the ladder and the string is at right angles to the wall. Find

(i) Tension in the string (ii) Normal reaction at point A and B

Let the length of the ladder = 2L



(↑) R_B = 30g = 30 x 9.8 = 294

(→) RA = T ……… (i)

B↺) R_A x 2Lsinθ = 30g x Lcosθ ……. (ii)

R_A x 2Lsin60 = 30g x Lcos60

RA = 84.87N

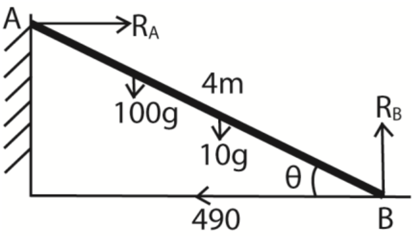
T = 84.87N

Example 2

A uniform ladder AB of mass 10kgand length 4m rests with its upper end A against a smooth vertical wall and lower end B on smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium at an angle $\tan^{-1} 2$ to the horizontal. The vertical plane containing the ladder and the string is at right angles to the wall. A man of mass 100kg ascends the ladder.

(i) If the string will break when the tension exceeds 490N, find how far up the ladder the man can go before this occurs

(ii) What tension must the string be capable of withstanding if the man is to reach the top of the ladder



Let the distance a man ascends be y

(→) R_A = 490N

B↺) R_A x 4sinθ = 10g x 2cosθ + 100gycosθ

R_A x 4tanθ = 20g + 100gy

490 x 4tanθ = 20g+ 100gy

y = $\frac{490\ x\ 4\ x\ 2 - 29\ x\ 9.8}{100\ x\ 9.8}$ = 3.8m

(ii)

B↺) R_A x 4sinθ = 10g x 2cosθ + 100g x 4cosθ

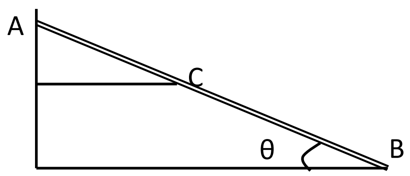
R_A x 4tanθ = 20g + 400g

RA = 514.5N

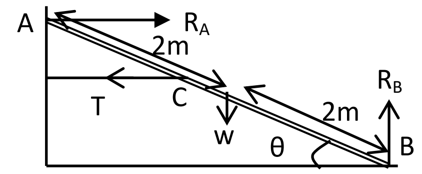
T= 514.5N

Example 3

A uniform ladder AB of weight W and length 4m rests with its upper end A against a smooth vertical wall and the lower end B on a smooth horizontal ground. The ladder is kept in equilibrium inclined at an angle θ to the ground by a light horizontal string attached to the wall and point C on the ladder. The vertical plane containing the ladder and the string is at right angle to the wall.



If tanθ = 2, find the tension in the string when BC is of length 3m



(↑) R_B = W ………. (i)

(→) R_A = T ………… (ii)

B↰ R_A x 4sinθ = w x 2cosθ + T x 3sinθ … (iii)

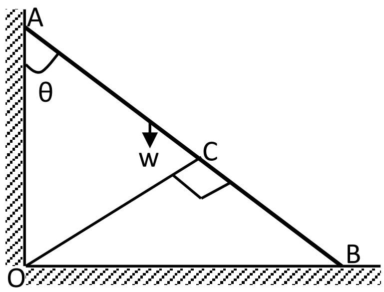
R_A x 4tanθ = 2w + T x 3tanθ

$$R_A = \frac{2w+T\ x\ 3\ x\ 2}{4\ x\ 2} = \frac{w+3T}{4}$$

$\frac{w+3T}{4} = T$; T= w

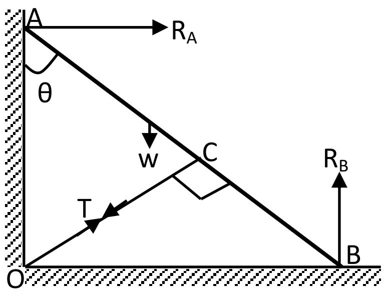
Example 4

The diagram below shows a uniform rod AB of weight w and length L resting at an angle θ against a smooth vertical wall at A. The other end B rests at a smooth horizontal table. The rod is prevented from slipping by an inelastic string OC. C being a point on AB such that AC is perpendicular to AB and O is the point of intersection of the wall and the table. Angle AOB is 90^0.



Find (i) tension in the string (ii) reaction at A and B in terms of θ and w.

Solution



(↑) R_B = w + Tsinθ ……………(i)

(→) R_A = Tcosθ ………………… (ii)

Taking moments at O

R_B x Lsinθ = $w\frac{L}{2}$sinθ + R_A x Lcosθ

(w + Tsinθ)Lsinθ = $w\frac{L}{2}$sinθ + Tsinθ x Lcosθ

(i) Tension in the string

$T(\cos^2\theta - \sin^2\theta) = \frac{w(2sin\theta - sin\theta)}{2}$

$T = \frac{wsin\theta}{2(cos^2\theta - sin^2\theta)} = \frac{w}{2}tan^2\theta$

(ii)Reaction at A and B in terms of θ and w

$$R_A = Tcos\theta = \left[\frac{wsin\theta}{2(cos^2\theta - sin^2\theta)}\right]cos\theta$$

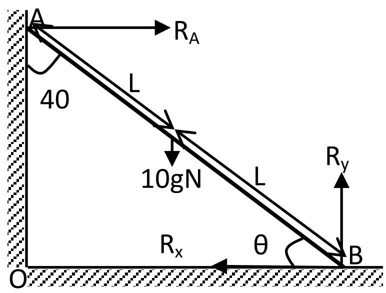
$$R_A = \frac{wsin\theta cos\theta}{2(cos^2\theta - sin^2\theta)} = \frac{wtan^2\theta}{4}$$

$$R_B = w + Tcos\theta = w + \left[\frac{wsin\theta}{2(cos^2\theta - sin^2\theta)}\right]sin\theta$$

$$R_B = \frac{w(2cos^2\theta - sin^2\theta)}{2(cos^2\theta - sin^2\theta)} = \frac{w}{2}tan^2\theta$$

Example 5

A uniform rod AB of mass 10kg is smoothly hinged at B and rests in a vertical plane with the end A against a smooth vertical wall. If the rod makes an angle 40^0 with the wall, find the reaction on the wall and the magnitude of the reaction at B



Let length of the ladder be 2L

$\theta = 90^0 - 40^0 = 50^0$

Taking moments at B

(→) $R_A \times 2L\sin\theta = 10g \times L\cos\theta$

$R_A \times 2L\sin\theta = 10 \times 9.8 \times L\cos\theta$

$R_A = 41.12N$

(↑) $R_y = 10gN = 10 \times 9.8 = 98N$

(→) $R_x = R_A = 41.12N$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$R = \sqrt{(41.12)^2 + 98^2}$ =106.28N

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{98}{41.12}\right) = 67.24^0$$

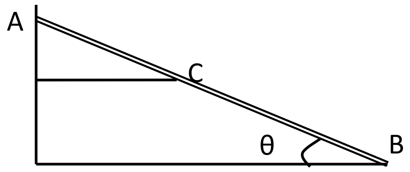
Reaction at B is 106.24 at 67.24^0 to the beam

Revision exercise A

1. A uniform ladder AB of mass 10kg and length 4m rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 40^0 to the horizontal. The vertical plan containing the ladder and the string is at the right angles to the wall. Find
   (i) Tension in the string [58.4]
   (ii) Normal reaction at points A and B [R_A= 58.4N, R_B= 98N]
2. A uniform ladder AB of mass 30kg and length 6m rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 70^0 to the horizontal. The vertical plan containing the ladder and the string is at the right angles to the wall. Find
   (i) Tension in the string [53.3N]
   (ii) Normal reaction at points A and B [R_A= 53.3, R_B= 294N]

3. A uniform ladder AB of mass 30kg rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium inclined at 80^0 to the horizontal. The vertical plan containing the ladder and the string is at the right angles to the wall. Find
   (i) Tension in the string [25.92N]
   (ii) Normal reaction at points A and B [R_A= 25.92, R_B= 294N]
4. A uniform ladder AB of mass 8kg and length 6m rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. The base of the ladder is 1m from the floor and the top of the ladder is 2m from the floor. A light horizontal string, which has one end attached to B and the other end attached to the wall vertically below the top of the ladder and 1m above the floor, keeps the ladder in equilibrium. Find the tension in the string [27.7N]
5. A uniform ladder AB of weight w and length 2L rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. A light horizontal string, which has one end attached to B and the other end attached to the wall, keeps the ladder in equilibrium The vertical plan containing the ladder and the string is at the right angles to the wall. Find the tension in the string $\left[\frac{w}{2\sqrt{3}}\right]$

6. A uniform ladder AB of weight w and length 4m rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. The ladder is kept in equilibrium inclined at an angle θ to the ground by a light inextensible string attached to the wall and to a point C on the ladder. The vertical plane containing the ladder and the string is at right angles to the wall

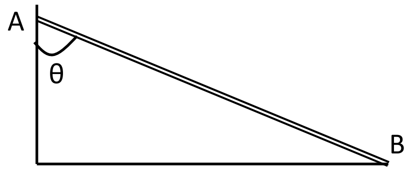


If tanθ = 2, find the tension in the string when BC is of length

(i) 1m $\left[\frac{w}{3}N\right]$

(ii) 2m$\left[\frac{w}{2}N\right]$

7. A uniform ladder AB of mass mkg and length 2L rests with its upper end A against a smooth vertical wall and lower end B on a smooth horizontal ground. The ladder is kept in equilibrium inclined at an angle θ to the vertical by a light inextensible string attached B and to the wall vertically below the top of the ladder. The vertical plane containing the ladder and the string is at right angles to the wall



(a) A man of mass 4m kg stands on the ladder at a distance $\frac{L}{2}$ from the bottom of the ladder, find

(i) The tension in the string $\left[\left(\frac{3mgtan\theta}{2}\right)N\right]$

(ii) Normal reaction at the bottom of the ladder [5mg N)

(iii) Normal reaction at the top of the ladder $\left[\left(\frac{3mgtan\theta}{2}\right)N\right]$

(b) If the maximum tension which the string can bear without breaking is 4mgtanθ, find how far up the ladder the man can safely climb. $\left[\frac{7L}{4}\right]$

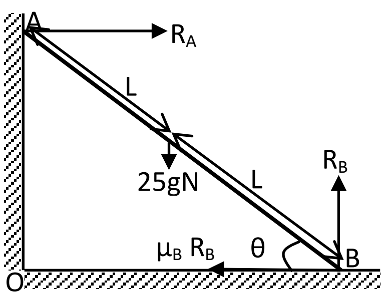
Rough contact at the foot and smooth contact at the top

Example 6

A uniform ladder of mass 25kg rests in limiting equilibrium with the top end against a smooth vertical wall and its base on a rough horizontal floor. If the ladder makes an angle 75^0 with the horizontal, find:

(i) Magnitude of the normal reaction at the floor

(ii) Coefficient of friction between the floor and the ladder



(i) (↑) $R_B = 25g = 25 \times 9.8 = 245N$

(→) $R_A = \mu_B R_B = 245\ \mu_B$

Taking moments about B

$R_A \times 2L\sin\theta = 25g \times L\cos\theta$

R_A x 2Lsin75 =25g x Lcos75

R_A = 32.824N

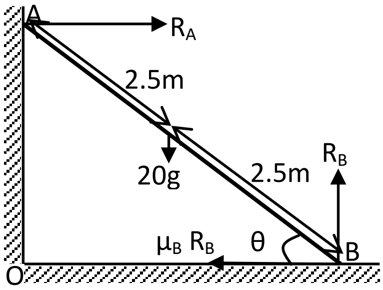
R_A =32.824 = 245 μ_B

μ_B =0.134

Example 7

A uniform ladder which is 5m long and mass of 20kg leans with its upper end against a smooth vertical wall and its lower end on a rough ground. The bottom of the ladder is 3m from the wall. Calculate

(i) The frictional force between the ladder and the ground
(ii) The coefficient of friction



$\theta = \cos^{-1}\frac{3}{5} = 53.13^0$

(i) (↑) R_B = 20g = 20 x 9.8 = 196N

(→) $R_A = \mu_B R_B = 196\mu_B$

Taking moments about B

R_A x 5sinθ =20g x 2.5cosθ

R_A x 5sin53.13 =20g x 2.5cos53.13

R_A = 73.5N

Frictional force = 73.5N

(ii) $R_A = \mu_B R_B = 196\mu_B$

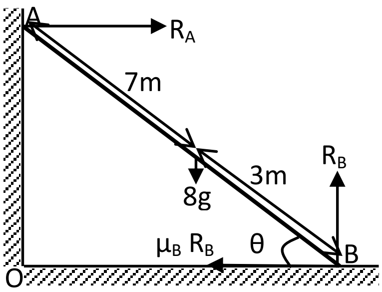
73.5 = 196 μ_B

μ_B = 0.375

Example 8

A non-uniform ladder AB 10m long and 8kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground and the upper end resting against a smooth vertical wall. If the centre of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30^0 with the horizontal, find the

(i) Coefficient of friction between the ladder and the ground
(ii) Reaction at the wall



(i) (↑) R_B = 8g = 8 x 9.8 = 78.4N

(→) $R_A = \mu_B R_B = 78.4\mu_B$

Taking moments about B

R_A x 10sinθ =8g x 3cosθ

R_A x 10sin30 =8g x 3cos 30

R_A = 40.738N

Reaction at the wall = 40.738N

$R_A = \mu_B R_B = 78.4\mu_B$

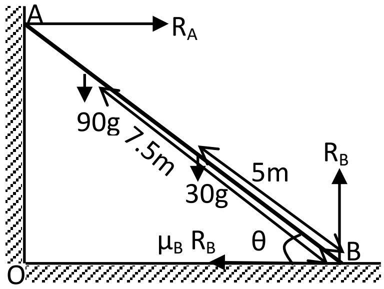
78.4 μ_B = 40.738

μ_B = 0.5196

Example 9

A uniform ladder AB 10m long and mass 30kg lies in a limiting equilibrium with its lower end resting on a rough horizontal ground and its upper end resting against a smooth vertical wall. If the ladder makes an angle of 60^0 with the horizontal, with a man of mass 90kg standing on the ladder at a point 7.5m from its base, find

(i) Magnitude of normal reaction and friction force at the ground
(ii) The minimum value of the coefficient of friction between the ladder and the ground that would enable the man to climb to the top of the ladder



(i) (↑) $R_B = 30g + 90g = 120 \times 9.8 = 1176N$

(→) $R_A = \mu_B R_B = 1176\,\mu_B$

Taking moments about B

$R_A \times 10\sin 60 = 30g \times 5\cos 60 + 90g \times 7.5\cos 60$

$R_A = 466.788N$

$\mu_B R_B = R_A$

$1176\,\mu_B = 466.788$

$\mu_B = 0.397$

(ii) $R_A \times 10\sin 60 = 30g \times 5\cos 60 + 90g \times 10\cos 60$

$R_A = 594.093N$

$\mu_B R_B = R_A$

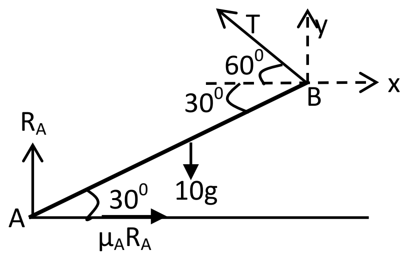
$1176\,\mu_B = 594.093$

$\mu_B = 0.5052$

Rough contact at the root of the ladder

Example 10

A uniform pole AB of mass 10kg has its lower end A on a rough horizontal ground and being raised to a vertical position by a rope attached to B. The rope and the pole lie in the same vertical plane and A does not slip across the ground. If the rope is at right angles to the pole and the pole is 30^0 to the horizontal, find (i) tension in the rope (ii) coefficient of friction on the ground



(i) Taking moments about A

$T \times 2L = 10g \times L\cos 30$

$T = 42.44N$

(ii) (↑) $R_A + T\sin 60 = 10g$

$RA + 42.44\sin 60 = 10 \times 9.8$

$RA = 61.25N$

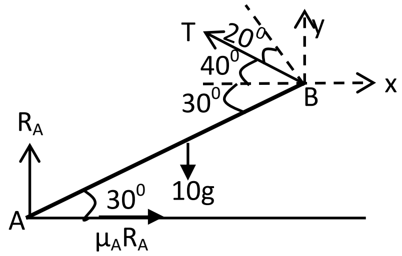
(→) $T\cos 60 = \mu_A R_A$

$42.44\cos 60 = 61.25\mu_A$

$\mu_A = 0.346$

Example 11

A uniform pole AB of mass 10kg has its lower end A on a rough horizontal ground and being raised to a vertical position by a rope attached to B. The rope and the pole lie in the same vertical plane and A does not slip across the ground. If the rope is makes angle of 70^0 the pole and the pole is 30^0 to horizontal, find (i) tension in the rope (ii) coefficient of friction on the ground



(i) Taking moments about A

2L x Tcos20 = 10g x Lcos30

T = 45.157N

(ii) (↑) R_A + Tsin40 = 10g

R_A + 45.157sin40 = 10 x 9.8

R_A = 68.974N

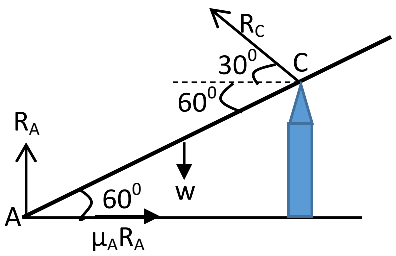
(→) Tcos40 = $\mu_A R_A$

45.157cos40 = 68.974μ_A

μ_A = 0.5015

Example 12

A uniform ladder AB of length 2L rests in limiting equilibrium with its lower end A resting on a rough horizontal ground. A point C on the beam rests against a smooth support. AC is of length $\frac{3L}{2}$ with C higher than A and AC makes an angle of 60^0 with the horizontal. Find the coefficient of friction between the ladder and the ground



Taking moments about A

$R_C = \frac{2wcos60}{3}$ ……………….(i)

(↑) R_A + $R_C$sin30 = w ……(ii)

$R_A + \frac{2wcos60}{3} sin30 = w$

$R_A = \frac{(3-2sin30cos60)w}{3}$

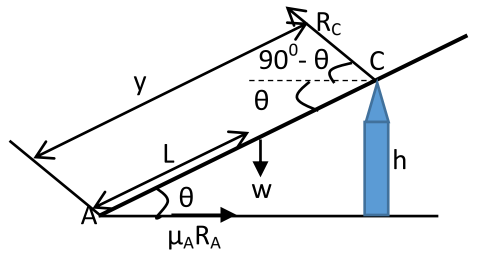
(→)$R_C$cos30 = $\mu_A R_A$

$\frac{2wcos60}{3}$cos30 = $\frac{(3-2sin30cos60)w}{3}\mu_A$

$\mu_A = \frac{2cos30cos60}{(3-2sin30cos60)} = 0.346$

Example 13

A uniform rod of length 2L inclined at an angle θ to the horizontal rests in vertical plane against a smooth horizontal bar at a height h above the ground. Given that the lower end of the rod is on a rough ground and the rod is about to slip; show that the coefficient of friction between the rod and the ground is $\frac{Lsin^2\theta cos\theta}{h-Lcos^2\theta sin\theta}$.



Taking moments about A

R_C x y = wLcosθ

$R_C = \frac{wLcos\theta}{y}$ ……………….(i)

(↑) $R_A + R_C\sin(90-\theta)$ = w ……(ii)

$R_A = \frac{(y - lcos^2\theta)w}{y}$

(→) $R_C\cos(90 - \theta) = \mu_A R_A$ …. (iii)

$\frac{wLcos\theta}{y}sin\theta = \mu_A \frac{(y - lcos^2\theta)w}{y}$

$\mu_A = \frac{Lcos\theta sin\theta}{(y - lcos^2\theta)}$

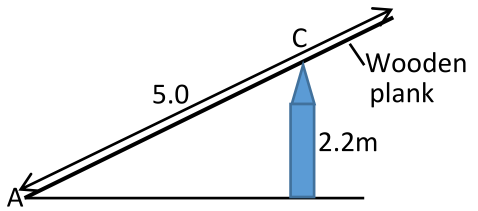
But sinθ = $\frac{h}{y}$ => y =$\frac{h}{sin\theta}$

$\mu_A = \frac{Lcos\theta sin^2\theta}{\left(\frac{h}{sin\theta} - lcos^2\theta\right)}$

$\mu_A = \frac{Lcos\theta sin\theta}{(h - lcos^2\theta sin\theta)}$

Example 14

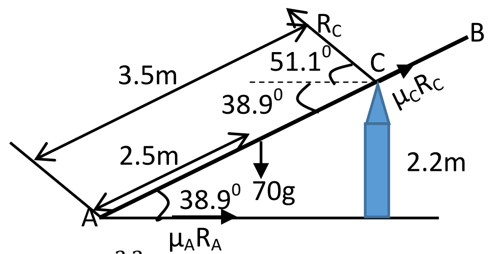
The diagram below shows a uniform wooden plank of mass 70kg and length 5m. The end A rests on a rough horizontal ground. The plank is in contact with the top of a rough pillar at C. the height of the pillar is 2.2m and AC = 3.5m



Given that the coefficient of friction at the ground id 0.6 and the plank is just about to slip, find

(i) Angle the plank makes with the ground at A
(ii) Normal reaction at A and normal reaction at C.
(iii) Coefficient of friction at C

Solution



$\theta = \sin^{-1}\frac{2.2}{3.5} = 38.9^0$

Taking moments about A

R_C x 3.5 = 70g x 2.5cos38.9

R_C = 381.34N

(↑) $R_A + \mu_C R_C\sin38.9 + R_C\sin51.1$= 70g

$R_A + \mu_C381.34\sin38.9 + 381.34\sin51.1$= 70g

$R_A = 389.2248 - 239.4674\ \mu_C$

(→) $R_C\cos51.1 = 0.6R_A + \mu_C R_C\sin38.9$

381.34cos51.1 =0.6(389.2248 – 239.4674 μ_C) +

μ_C x 381.34 x sin38.9

$\mu_C = \frac{5.93252}{153.0948} = 0.0388$

R_A = 389.2248 – 239.4674 x 0.0388 = 379.933N

Revision Exercise 2

1. A non- uniform ladder AB 10m long and mass 8kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground of angle of friction 17^0 and the upper end resting against a smooth vertical wall. if the centre of gravity is at point C and the ladder makes an angle of 63^0 with horizontal, find the length AC. [6m]

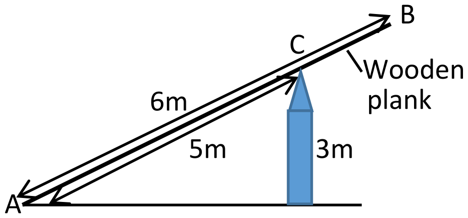
2. A uniform ladder of mass 30kg lies in limiting equilibrium with its lower end resting on a rough horizontal ground of coefficient of friction 0.4 and the upper end resting against a smooth vertical wall. If the ladder makes an angle of 60^0 with the horizontal, find the magnitude of the frictional force at the ground. [158.4N]

3. A uniform pole AB of mass 100kg has its lower end A on a rough horizontal ground and being raised to vertical position by a rope attached to B. The rope and the pole lie in the same vertical plane and A does not slip across the ground. If the rope is at right angles to the pole and the pole is 20^0 to the horizontal, find

   (i) Normal reaction on the ground [157N)
   (ii) Frictional force on the ground [547N]

4. A ladder 12m long and weighing 200N is placed 60^0 to the horizontal with one end B leaning against a smooth vertical wall and the other end A on a rough horizontal ground. Find:
   (a) Reaction at the wall [57.7N]
   (b) Reaction at the ground [208.2N at 73.9^0 to the horizontal]
5. A uniform pole AB has its lower end A on a rough horizontal ground of friction λ and being raised to vertical position by a rope attached to B. The rope and the pole lie in the same vertical plane and A does not slip across the ground. If the rope is at right angles to the pole and the pole is at an angle θ to the horizontal. Show than $tan\lambda = \frac{sin2\theta}{3-cos2\theta}$.
6. A uniform ladder of length 10m and weight w lies in limiting equilibrium with its lower end resting on a rough horizontal ground of coefficient of friction $\frac{1}{3}$ and the upper end resting against a smooth vertical wall. The ladder makes an angle of θ with the horizontal where tanθ = 1.7. A man of weight 2w starts to climb the ladder. Find
   (i) How far up the ladder the man can climb before slipping can occur. [6m]
   (ii) Find in terms of w, the magnitude of the frictional force at the ground to enable the man reach the top.$\left[\frac{8w}{17}\right]$
7. A uniform ladder of length 5m and weight 80N lies in limiting equilibrium with its lower end resting on a rough horizontal ground and the upper end resting against a smooth horizontal rail fixed 4m vertically above the ground. If the ladder makes an angle of θ with the vertical where tanθ ≤ 0.75 with the horizontal.
   (a) Find expressions in terms of θ for
      (i) Vertical reaction R of the ground [$80-50\sin^2\theta\cos\theta$]
      (ii) Friction F at the ground [$50\cos^2\theta\sin\theta$]
      (iii) Normal reaction N at the rail [$50\sin\theta\cos\theta$]
   (b) Given that the ladder does not slip, show that F is maximum when tanθ = $\frac{1}{\sqrt{2}}$ and find its maximum value $\left[\frac{100}{3\sqrt{3}}N\right]$

8. A uniform ladder AB of weight w rests in limiting equilibrium with its lower end A resting on a rough horizontal ground, coefficient of friction μ. A point C on the ladder rests against a smooth peg. AC is of length $\frac{1}{4}AB$ from end B and a height h from the ground. If AB makes an angle of θ with the ground , show that the frictional force is $\frac{\mu w}{3}(2-3cos^2\theta)$
9. A smooth horizontal rail is fixed at a height of 3m above the horizontal rough ground. A straight uniform pole AB of length 6m and mass 20kg rests in limiting equilibrium at a point Con the rail with its lower end A resting on the ground. The vertical plane containing the pole is at right angles to the rail. The distance AC is 5m



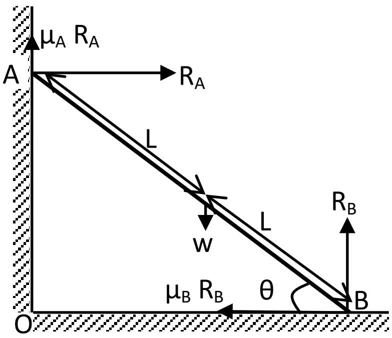
Calculate the
(i) Magnitude of the force exerted by the rail on the pole [94N]
(ii) Coefficient of friction between the pole and the ground [0.47]
(iii) Magnitude of the force exerted by the ground on the pole [133N]

10. A uniform ladder lies in limiting equilibrium with its lower end resting on a rough horizontal ground of coefficient of friction $\frac{5}{8}$ and the upper end resting against a smooth vertical wall. If the ladder makes an angle of 45^0 with the vertical.
(i) Show that a man whose weight is equal to that of the ladder can ascend $\frac{3}{4}$ of the length of the ladder
(ii) Find what weight must be placed on the bottom of the ladder to enable the man ascend to the top $\left[\frac{2w}{5}\right]$

Rough contacts both at the top and foot of a ladder

Example 15

A uniform ladder rests with one end on a rough horizontal ground and the other against a rough vertical wall, the coefficient of friction being respectively $\frac{3}{5}$ and $\frac{1}{3}$. Find the inclination of the ladder to the vertical when it is about to slip



(↑) $R_B + \frac{1}{3}R_A = w$ …….. (i)

(→) $R_A = \frac{3}{5}R_B \Rightarrow R_B = \frac{5}{3}R_A$

Substituting for R_B in equation (i)

$\frac{5}{3}R_A + \frac{1}{3}R_A = w$

$R_A = \frac{w}{2}$

Taking moments about B

$R_A \times 2L\sin\theta + \frac{1}{3}R_A \times 2L\cos\theta = w \times L\cos\theta$ ...(ii)

Substituting for R_A in eqn. (ii)

$\frac{w}{2}$ x 2Lsinθ + $\frac{1}{3}x$ $\frac{w}{2}$ x 2Lcosθ = w x Lcosθ

Sinθ = $\frac{2}{3}cos\theta$

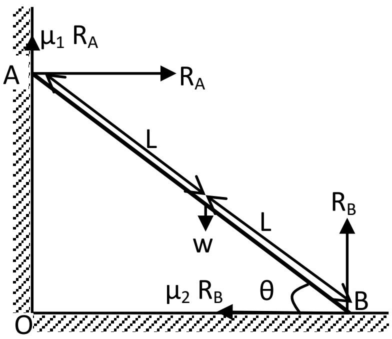
tanθ = $\frac{2}{3}$

θ = 33.7^0

Angle to the vertical = 90 - 33.7 = 56.3^0

Example 16

A uniform ladder rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction μ_1 and its base on a rough horizontal floor with coefficient of friction μ_2.If the ladder makes an angle of θ with the floor, prove that tanθ = $\frac{1-\mu_1\mu_2}{2\mu_2}$



(↑) $R_B + \mu_1 R_A = w$ …….. (i)

(→) $R_A = \mu_2 R_B$ ……….(ii)

$R_B = \frac{1}{\mu_2}R_A$

Substituting R_B in eqn. (i)

$\frac{1}{\mu_2}R_A + \mu_1 R_A = w$

$R_A = \frac{\mu_2 w}{1+\mu_1\mu_2}$

Taking moments about B

R_A x 2Lsinθ + $\mu_1 R_A$ x 2Lcosθ = w x Lcosθ …….(iii)

$\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lsinθ + μ_1x $\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lcosθ = w x Lcosθ

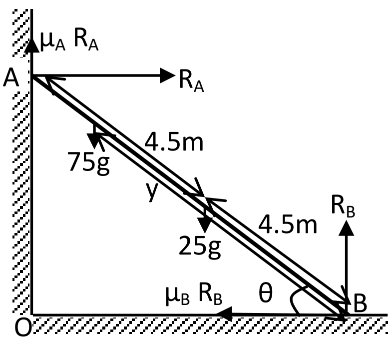
$\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lsinθ = w x Lcosθ - μ_1x $\frac{\mu_2 w}{1+\mu_1\mu_2}$ x 2Lcosθ

$\frac{2\mu_2}{1+\mu_1\mu_2}$ sinθ = $\frac{1+\mu_1\mu_2-2\mu_1\mu_2}{1+\mu_1\mu_2}$ cosθ

tanθ = $\frac{1-\mu_1\mu_2}{2\mu_2}$

Example 17

The foot of a ladder length of 9m and mass 25kg rests on a rough horizontal surface while the upper end rests in contact with a rough vertical wall. The ladder is in vertical plane perpendicular to the wall. If the first rug is 30cm from the foot and the rest at the interval of 30cm, find the highest rug to which a man of mass 75kg can climb without causing the ladder to slip, when the ladder is inclined at 600 to the horizontal and the coefficient of friction at each end is 0.25.



Let the man ascend a distance = ym

(↑) $R_B + \frac{1}{4}R_A = 25g + 75g$ …… (i)

(→) $R_A = \frac{1}{4}R_B$ => $R_B = 4R_A$ ….. (ii)

Substituting for R_B in eqn. (i)

$4R_A + \frac{1}{4}R_A = 25g + 75g$

$R_A = 226.154N$

Taking moments about B

R_A x 9sinθ + $\frac{1}{4}R_A$ x 9cosθ = 25 x 4.5cosθ +75 x 4.5cosθ

$226.154N$ x 2Lsinθ + μ_1x $226.154N$ x 2Lcosθ

= 25 x 4.5cos60 +75 x 4.5cos60

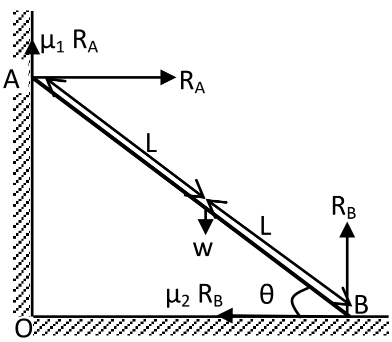
Y = 4m

Number of rugs = $\frac{4}{0.3}$ = 13

Example 18

A uniform ladder of length 2L and weight w rests in a vertical plane with one end on a rough horizontal ground and the other against a rough vertical wall, the angle of friction being respectively $\tan^{-1}\left(\frac{1}{3}\right)$ and $\tan^{-1}\left(\frac{1}{2}\right)$.

(a) Find the inclination of the ladder to the horizontal when it is in limiting equilibrium at either end



$\mu_A = \frac{1}{3}$ and $\mu_B = 2$

(↑) $R_B + \frac{1}{3} R_A = w$ .... (i)

(→) $R_A = \frac{1}{2} R_B$

$R_B = 2R_A$ ....(ii)

Substituting R_B in eqn. (i)

$2R_A + \frac{1}{3}R_A = w$

$R_A = \frac{3w}{7}$

Taking moments about B

$R_A \times 2L\sin\theta + \frac{1}{3}R_A \times 2L\cos\theta = w \times L\cos\theta$ ..(iii)

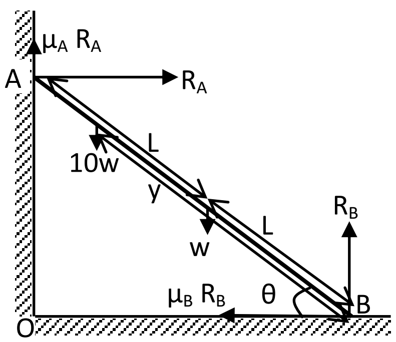
$\frac{3w}{7} \times 2L\sin\theta + \frac{1}{3} \times \frac{3w}{7} \times 2L\cos\theta = w \times L\cos\theta$

$\frac{6}{7}\sin\theta = \frac{5}{7}\cos\theta$

$\tan\theta = \frac{5}{6}$

$\theta = 39.8^0$

(b) A man of weight 10times that of the ladder begins to ascend it. How far will he climb before the ladder slips



Let the man ascend a distance = ym

(↑) $R_B + \frac{1}{3}R_A = 10w + w$ ...... (i)

(→) $R_A = \frac{1}{2}R_B \Rightarrow R_B = 2R_A$ ..... (ii)

Substituting for R_B in eqn. (i)

$2R_A + \frac{1}{3}R_A = 11w$

$R_A = \frac{33}{7}w$

Taking moments about B

$R_A \times 2L\sin\theta + \frac{1}{3}R_A \times 2L\cos\theta = wL\cos\theta + 10wy\cos\theta$

$\frac{33}{7}w \times 2L\sin\theta + \frac{1}{3} \times \frac{33}{7}w \times 2L\cos\theta$

$= wL\cos\theta + 10wy\cos\theta$

$\frac{66}{7}L\sin\theta + \frac{15}{7}L\cos\theta = 10y\cos\theta$

$\frac{66}{7}L\tan\theta + \frac{15}{7}L = 10y$

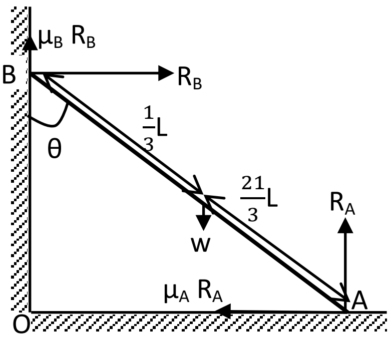
10L = 10y => y = Lm

Example 19

A non-uniform ladder AB is in limiting equilibrium with its lower end A resting on a rough horizontal ground and the upper end B resting against a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is t a G = $\frac{2}{3}AB$. The coefficient of friction between the ladder and the wall is twice that between the ladder and the floor. If the ladder makes an angle θ with the wall and the angle of friction between the ladder and the floor is λ;

(i) Show that 4tanθ = 3tan2λ

(ii) How far can a man of equal mass as the ladder ascend without the ladder slipping given that θ = 45^0 and coefficient of friction between the ladder and the floor is $\frac{1}{2}$.$\left[\frac{2}{3}AB\right]$



$\mu_A = tan\lambda \ and \ \mu_B = 2tan\lambda$

(↑) $R_A + 2tan\lambda = w$……………..(i)

(→) $R_B = tan\lambda$ R_A……………… (ii)

Substituting R_B in eqn. (i)

R_A + 2tanλtanλ$R_A = w$

$R_A = \frac{w}{1+ tan^2\lambda}$

Taking moments about B

R_A x Lsinθ = w$\frac{L}{3}$sinθ + tanλR_ALcosθ

$\frac{w}{1+ tan^2\lambda}$ x Lsinθ - w$\frac{L}{3}$sinθ = tanλ$\left(\frac{w}{1+ tan^2\lambda}\right)$ Lcosθ

$\frac{2-2tan^2\lambda}{3(1+2tan^2\lambda)}sin\theta = \frac{tan\lambda}{1+2tan^2\lambda}cos\theta$

$tan\theta = \frac{3tan\lambda}{2-2tan^2\lambda} =$

$tan\theta = \frac{3}{2}x\frac{2}{2}\left[\frac{3tan\lambda}{2(1-tan^2\lambda)}\right] = \frac{3}{4}\left[\frac{2tan\lambda}{(1-tan^2\lambda)}\right] = \frac{3}{4}tan2\lambda$

4tanθ = 3tan2λ

Revision exercise 3

1. A uniform ladder rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction 0.25 and its base on a rough horizontal floor with coefficient of friction μ. If the ladder makes an angle of 300 with the vertical; find the value of μ [0.269]
2. A non-uniform ladder AB of length 6m is in limiting equilibrium with its lower end A resting on rough horizontal ground with coefficient of friction $\frac{1}{3}$ and the upper end B resting against a rough vertical wall with coefficient of friction $\frac{1}{4}$. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is at C where AC = 4m. If the ladder makes an acute angle θ with the ground. Show that tanθ =$\frac{23}{12}$.
3. A uniform ladder AB is of weight 2w and length 10m rests in limiting equilibrium with the top end against a rough vertical wall with coefficient of friction $\frac{1}{3}$and its base on a rough horizontal floor with coefficient of friction $\frac{1}{3}$. If the ladder makes an angle an angle of θ with horizontal, such that tanθ = $\frac{16}{17}$. A man of weight 5w starts to climb the ladder.
   (a) How far up the ladder can a man climb before slipping [9m]
   (b) When a boy of weight Y stands on the bottom rung of the ladder at A, the man is just able to climb to the top safely. Find Y in terms of W $\left[\frac{7w}{11}\right]$

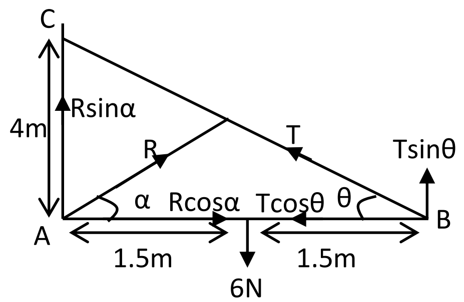
4. A non-uniform ladder AB of length 12m and mass 30 is in limiting equilibrium with its lower end A resting on a rough horizontal ground with coefficient of friction 0.25 and upper end B resting against a rough vertical wall with coefficient of friction 0.2. The ladder is in a vertical plane perpendicular to the wall. The centre of gravity of the ladder is at its trisection of the length nearer to A. The ladder makes an angle θ with the horizontal such that tanθ =$\frac{9}{4}$. A straight horizontal string connects A to a point at the base of the wall vertically below B. A man of mass 9okg begins to climb the ladder
   (i) How far up the ladder can the man climb without causing tension in the string [8m]
   (ii) What tension must the string be capable of withstanding if the man is to reach the tip of the ladder safely [126N]

Beams hinged and maintained in a horizontal position

Example 20

A uniform beam AB, 3.0m long and weight 6N is hinged at a wall at A and is held stationary in a horizontal position by a rope attached to B and joined to a point C on the wall, 4.0m vertically above A. find

(i) The tension T in the rope
(ii) The magnitude and direction of the reaction R at the hinge.



$\theta = \tan^{-1}\frac{4}{3} = 53.13^0$

Taking moments at A at equilibrium

(Tsin53.13) x 3 = 6 x 1.5

T = 3.75N

(↑) $R_A$sinα + Tsionθ = 6

$R_A$sinα = 6 – 3.75sin53.13

$R_A$sinα = 3……………………………(i)

(→)Rcosα = Tcosθ

Rcosα = 3.75cos 53.13

Rcosα = 2.238 ……………………(ii)

$(i) \div (ii)$

Tanα $\frac{3}{2.238}$ α = 53.3^0

Substituting α into eqn. (i)

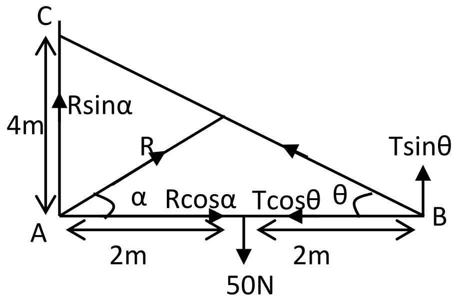
$R_A$sin53.3 = 3;

R_A = 3.74N at 53.3^0 to the beam

Example 21

A uniform beam AB of length 4m and weight 50N is freely hinged at A to a vertical wall and is held horizontally in equilibrium by a string which has one end attached to B and the other end attached to a point C on the wall 4m above A. Find

(i) The tension in the rope
(ii) The magnitude and direction of the reaction R at the hinge



$\theta = \tan^{-1}\frac{4}{4} = 45^0$

Taking moments at A at equilibrium

(Tsin45) x 3 = 50 x 2

T = 35.36N

(↑) $R_A$sinα + Tsionθ = 50

Rsinα = 50 – 35.36sin45

Rsinα = 24.997……………………………..(i)

(→)Rcosα = Tcosθ

Rcosα = 35.36cos 45

Rcosα = 25 ……………………(ii)

$(i) \div (ii)$

Tanα $\frac{24.997}{25}$ α = 45^0

Substituting α into eqn. (ii)

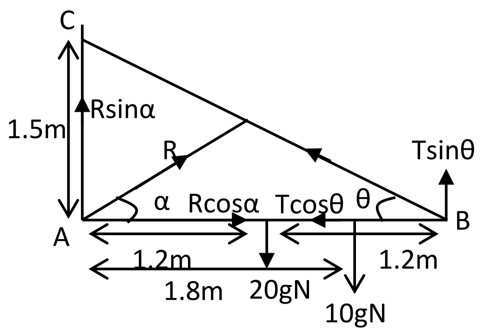
Rcos45 = 25;

R = 35.36N at 45^0 to the horizontal

Example 22

A uniform beam AB of length 2.4m and weight 20N is freely hinged at A to a vertical wall and is maintained in horizontally in position by a chain which has one end attached to B and the other end attached to a point C on the wall 1.5m above A. If the beam carries a load of 10N at a point 1.8m from A. Calculate

(i) The tension in the chain
(ii) The magnitude and direction of the reaction R at the hinge



$\theta = \tan^{-1}\frac{1.5}{2.4} = 32.01^0$

Taking moments at A at equilibrium

(Tsin32.01) x 2.4 = 20g x 1.2 + 10g x 1,8

Tension in chain, T = 323.87N

(↑) $R_A$sinα + Tsionθ = 20gN +10gN

Rsinα = 30g – 323.87sin32.01

Rsinα = 122.63……………………………..(i)

(→)Rcosα = Tcosθ

Rcosα = 323.87cos 32.01

Rcosα = 274.63 ……………………(ii)

$(i) \div (ii)$

Tanα $\frac{122.63}{274.63}$ α = 24.06^0

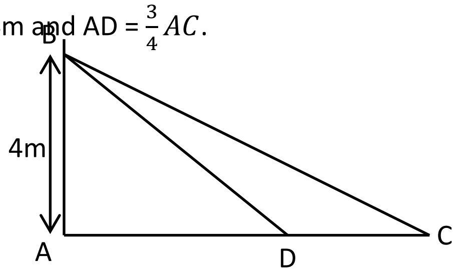
Substituting α into eqn. (ii)

Rcos24.06 = 274.63;

R = 300.85N at 24.07^0 to the horizontal

Example 23

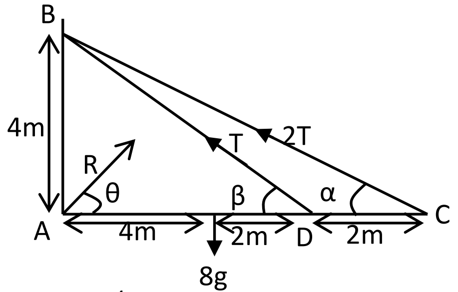
A uniform beam AC of mass 8kg and length 8m is hinged at A and maintained in equilibrium by two strings attached to it at points A and D as shown below. The tension in BC is twice that in AB. AB = 4m and AD = $\frac{3}{4}AC$.



Find:

(i) Tension in the string BC

(ii) Magnitude and direction of the resultant force at the hinge



$\alpha = \tan^{-1}\frac{4}{8} = 26.6^0$

$\beta = \tan^{-1}\frac{4}{6} = 33.7^0$

Taking moments about A

6 x Tsinβ + 8 x Tsinα = 8g x 4

6 x Tsin33.7 + 8 x Tsin26.6 = 8g x 4

T= 29.886N

Tension in BC = 2 x 29.886 = 59.772N

(↑) Rsinθ + Tsinβ+ 2Tsinα = 8g

Rsinθ = 35.0545 ……………. (i)

(→) Rcosθ = Tcosβ+ 2Tcosα

Rcosθ = 78.3092 ………….(ii)

(i)÷ (ii) tanθ = $\frac{78.3092}{35.0545}$; θ = 24.1^0

Substituting θ in eqn. (ii)

R = $\frac{78.3092}{cos24.1}$ = 85.8N

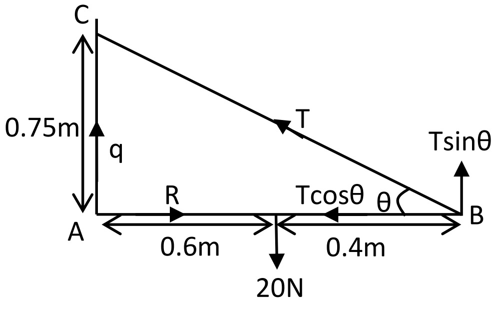
∴ R = 85.8N at 24.1^0 to horizontal

Example 24

A rod 1m long has weight of 20N and its centre of gravity 60cm from A. It rests horizontally with A against a rough vertical wall. A string BC is fastened to the wall at 75cm vertically above A. Find the

(i) Normal reaction and friction at A if friction is limiting the coefficient of friction.

(ii) Tension in the string



$\theta = \tan^{-1}\frac{0.75}{1} = 36.9^0$

taking moments about A

Tsinθ x 0.4 = 20 x 0.6

Tsin36.9 x 0.4 = 20 x 0.6

T = 19.99N

(↑) q + Tsinθ = 20

q + 19.99sin36.9 = 20

q =8N

(→) R = Tcosθ;

R= 19.99cos36.9 = 15.99N

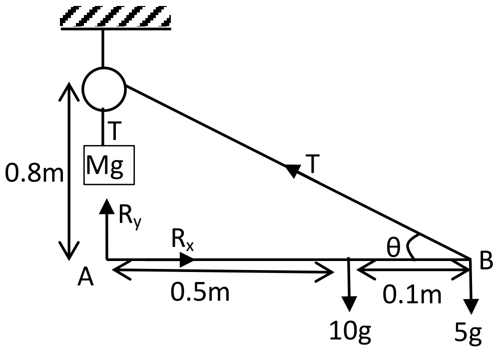
q =μR

μ = $\frac{8}{15.99} = 0.5$

Example 25

A rod of length 0.6m and mass 10kgis hinged at A. Its centre of gravity is 0.5m from A, a light inextensible string attached at B passes over a fixed pulley 0.8m above A and supports a mass M hanging freely. If a mass of 5kg is attached at B so as to keep the rod horizontal, find the

(i) Value of N
(ii) Reaction at the hinge



$tan\theta = \frac{0.8}{0.6} = \frac{4}{3}; sin\theta = \frac{4}{5}; cos\theta = \frac{3}{5}$

For Mkg mass: T = Mg…………….(i)

For the beam: taking moments a bout A

0.6 x Tsinθ = 5g x 0.6 + 10g x 0.5

0.6 x T x $\frac{4}{5}$ = 5g x 0.6 + 10g x 0.5

T = $\frac{50x\ 9.8}{3} = 163.33N$

From eqn. (i)

T = Mg

M = $\frac{163.33}{9.8} = 16.67kg$

(↑) Tsinθ + R_y = 10g + 5g

163.33 x $\frac{4}{5}$ + R_y = 15 x 9.8

R_y = 16.336N

(→) Tcosθ = Rx

Rx = 163.33 x $\frac{3}{5}$ =97.998N

R = $\sqrt{(R_x)^2 + (R_y)^2}$=$\sqrt{(97.998)^2 + (16.336)^2}$

R = 99.35N

$\alpha = \tan^{-1}\frac{16.336}{97.998} = 9.34^0$

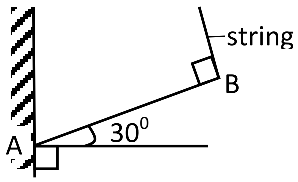
Reaction at B is 99.35N at 9.34^0 to horizontal

Beam Hinged and maintained at an angle

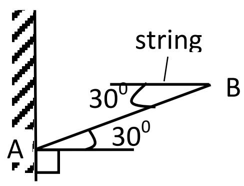
Example 26

Each of the following diagrams shows a uniform rod of mass 5kg and length 6m freely hinged at A to a vertical wall. A string attached to B keeps the rod in equilibrium. For each case, find the tension in the string and the magnitude and direction of the reaction at the hinge

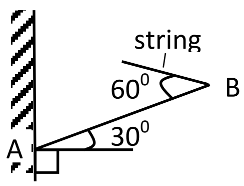
(a)



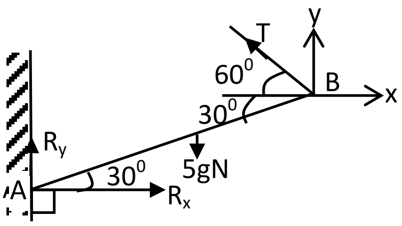
(b)



(c)



Solution



Taking moments about A

T x 2L = 5g x Lcos30

T = 21.2176N

(↑) Tsin60 + R_y = 5g

21.2176sin60 + R_y = 5g

R_y = 30.625N

(→) Rx = Tcos60 = 21.2176cos 60 = 10.6088N

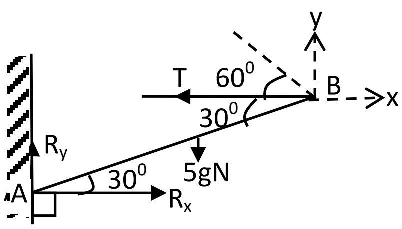
R = $\sqrt{(R_x)^2 + (R_y)^2}$=$\sqrt{(10.6088)^2 + (30.625)^2}$

R = 32.41N

$\alpha = \tan^{-1}\frac{30.625}{10.6088} = 70.9^0$

Reaction at A is 32.41N at 70.9^0 to horizontal

(ii)



Taking moments about A

T x 2L = 5g x Lcos60

T = 42.4352N

(↑)R_y = 5g = 49N

(→) Rx = T = 42.432N

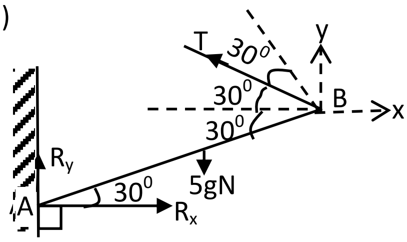
R = $\sqrt{(R_x)^2 + (R_y)^2}$=$\sqrt{(42.432)^2 + (49)^2}$

R = 64.8209N

$\alpha = \tan^{-1}\frac{49}{42.432} = 49.1^0$

Reaction at A is 64.8209N at 49.1^0 to horizontal

(iii)



Taking moments about A

Tcos30 x 2L = 5g x Lcos30

T = 24.9N

(↑)Tsin30 + R_y = 5g

24.5sin30 + Ry = 5g

Ry = 36.75N

(→) Rx = Tcos30 = 24.5cos30 =21.2176

R = $\sqrt{(R_x)^2 + (R_y)^2}$=$\sqrt{21.2176^2 + (24.5)^2}$

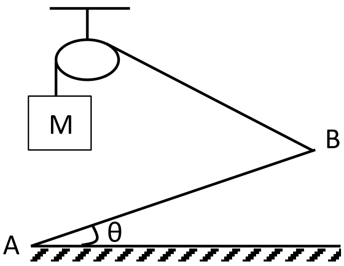
R = 42.4352N

$\alpha = \tan^{-1}\frac{36.75}{21.2176} = 60^0$

Reaction at A is 64.8209N at 49.1^0 to horizontal

Example 27

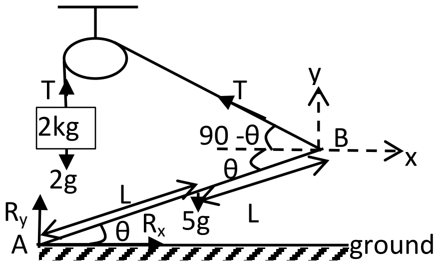
A uniform rod AB of mass 5kg is smoothly hinged on the ground at point A. The rod making an angle θ with the horizontal ground is kept in equilibrium by a light inelastic string attached to B. The string which makes 90^0 with the rod passes over a smooth fixed pulley and carries a stationary mass m of 2kg at the other end



Show that

(a) $\cos\theta = \frac{4}{5}$

(b) the magnitude of reaction at the hinge is $\frac{49}{5}\sqrt{13}N$



For 2kg mass: T = 2g .... (i)

For beam taking moments about A

T x 2L = 5g x Lcosθ .... (ii)

2g x 2L = 5g x Lcosθ

$\cos\theta = \frac{4}{5}$

(↑) Tsin(90-θ) + R_y = 5g

Tcosθ + R_y = 5g

2g x $\frac{4}{5}$ + Ry = 5g

Ry = $\frac{17}{5}g = \frac{17}{5}\,x\,9.8 = 33.32$ N

(→) Tcos(90 – θ) = Rx

Rx = Tsinθ = 2g x $\frac{3}{5}$ =11.76N

R = $\sqrt{(R_x)^2 + (R_y)^2}$=$\sqrt{(33.32\,)^2 + (11.76)^2}$

R = 35.3344N

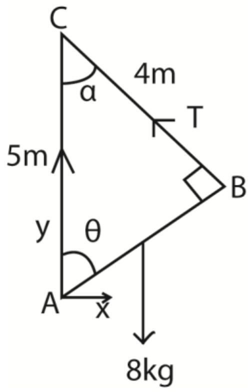
$\alpha = \tan^{-1}\frac{33.32}{11.76} = 70.56$

Reaction at A is 35.3344N at 70.56^0 to horizontal

Example 28

A uniform rod AB of length 3m and mass 8kg is freely hinged to a vertical wall at A. A string BC of length 4m attached at b and to point C on the wall, keeps the rod in equilibrium. If C is 5m vertically above A, find the

(a) Tension in the string (03marks)



$AB^2 + 4^2 = 5^2$

AB = $\sqrt{(25-16)}$ = 3

Let T be tension in the string, from the diagram

cos θ = $\frac{3}{5}$, cos α = $\frac{4}{5}$

Equation of moment about A

T x 3 = 8g x 1.5cosα

3T = 8 x 9.8 x $\frac{4}{5}$; T = 31.36N

∴ tension in the string is 31.36N

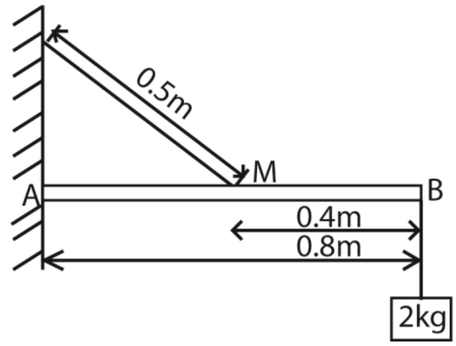
(b) Magnitude of the normal reaction at A. (02marks)

x = Tcosθ = 31.36 x $\frac{3}{5}$ = 18.816N

∴ the magnitude of normal reaction at A is 18.816N

Example 29

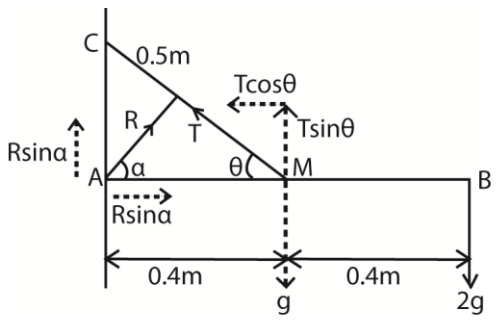
The figure below shows a uniform beam of length 0.8metres and mass 1kg. the beam is hinged at A and has a load of mass 2kg attached at B



The beam is held in a horizontal position by a light inextensible string of length 0.5metres. the string joins the mid-point M of the beam to a point C vertically above A.

Find the

(a) Tension in the string(08marks)



$AC^2 = (05)^2 – (0.4)^2$

AC = 0.3

cosθ = $\frac{0.4}{0.5} = 0.8$

sinθ = $\frac{0.3}{0.5} = 0.6$

Taking moments at A

(9.8 x 0.4) + (2 x 9.8 x 0.8) = T x 0.4sinθ

(9.8 x 0.4) + (2 x 9.8 x 0.8) = T x 0.4 x 0.6

T = 81.667N

(b) Magnitude and direction of the force exerted by the hinge. (04marks)

Resolving forces

(→); Rcosα = Tcosθ = $\frac{245}{3}$ x 0.

Rcosα = $\frac{196}{3}$ …………………………………… (i)

(↑); Rsinα + Tsinθ = g + 2g

Rsinα = 3 x 9.8 - $\frac{245}{3}$ x 0.6

Rsinα = -19.6 ………………………………… (ii)

Equn (ii) ÷ Eqn (i)

$\frac{Rsin\ \alpha}{Rcos\ \alpha} = \frac{-19.6}{\frac{196}{3}}$ = -0.3

tan α = -0.3

α = -16.7^0

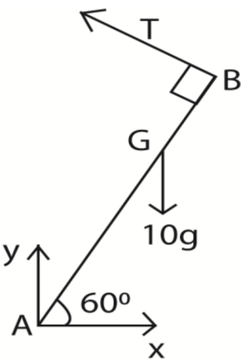
Hence the direction of force at the hinge is 16.7^0 with the beam

From eqn (i)

Rcos 16.7^0 = $\frac{196}{3}$; R = 68.21N

Example 30

A non-uniform rod AB of mass 10k has its centre of gravity a distance ¼ AB. The rod is smoothly hinged at A. it is maintained in equilibrium at 60^0 above the horizontal by a light inextensible string tied at B and at a right angle to AB. Calculate the magnitude and direction of the reaction at A. (12marks)



Taking moments about point A

T x (AB) = 10g$\left(\frac{3}{4}ABcos\ 60^0\right)$

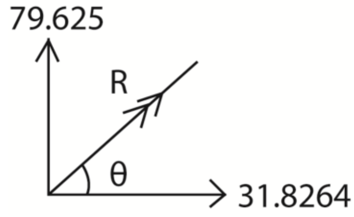
T = $\frac{15g}{4} = \frac{15\ x\ 9.8}{4}$ = 36.75N

Resolving forces horizontally

X = $T\cos30^0 = 36.75\cos30^0$ = 31.8264N

Y = 10g – 36.75sin 30^0 = 79.625N

$|R| = \sqrt{(31.8264)^2 + (79.625)^2}$ = 85.75N



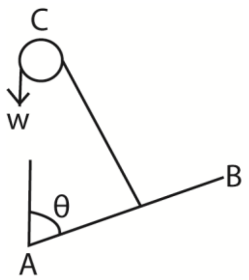
θ = $\tan^{-1}\left(\frac{79.625}{31.8264}\right)$= 68.2^0

The direction of resultant force is 68.2^0 or E68.2^0N or N21.8^0E

Revision Exercise 4

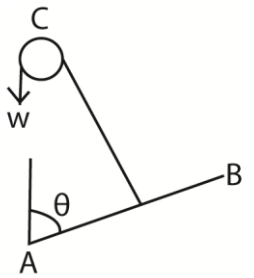
1. A uniform rod AB of mass 5kg is freely hinged at A to a vertical wall and held horizontal in equilibrium by a string which has one end attached to a point C on the wall above A. the string makes an angle of 30^0 with AB, find
   (a) The tension in the rope [49N]
   (b) The magnitude and direction of the reaction at A [49N at 30^0 with AB
2. A non-uniform beam AB of weight 20N and of length 4m is freely hinged at A to a vertical wall and is held horizontal in equilibrium by a string which has one end attached at B and the other end attached to point C on the wall above A. The string makes an angle of 600 with AB. If the tension in the string is 12N, find
   (a) The magnitude and direction of reaction at the hinge [11.3N at 58^0 with AB
   (b) Distance from A to the centre of gravity. [2.08m]
3. One end of a uniform plank of length 4m and weight 100N is hinged to a vertical wall. An inelastic rope, tied to the other end of the plank is fixed at a point 4m above the hinge. Find
   (a) tension in a rope [388.9N]
   (b) the reaction of the wall on the plank [302.1N at 24.4^0 to horizontal]
4. A uniform diving board AB, of length 4m and maintained in a horizontal position by means of a light strut DC. D is a point on the wall 1m below A and C is a point on the board where AC = 1m. An object of mass 60kg is placed at end B
   (a) Find the position of the centre of mass of the 60kg mass and the mass of the board combined. [80 cm from B]
   (b) Determine
      (i) Thrust in the strut [452.548N]
      (ii) Magnitude of reaction at A [388.33N]

5. Two light strings are perpendicular to each other and support a particle of weight 100N. The tension in one string is 40.0N. Calculate the angle this string with the vertical and the tension in the other string. [66.4^0, 91.7N]
6. A uniform pole AB of weight 5W and length 8a is suspended horizontally by two vertical strings attached to it at C and D where AC = BD = a.. A body of weight 9W hangs from the pole at E where ED = 2a. calculate the tension in each string [5.5W, 8.5W]
7. AB is a uniform rod of length 1.4m. it is pivoted at C where AC = 0.5m, and rests in horizontal equilibrium when weights of 16N and 8N are applied at A and B respectively. Calculate
   (a) Weight of the rod [4N]
   (b) The magnitude of the reaction at the pivot [28N]
8. A uniform rod AB of length 4a and weight W is smoothly hinged at its upper end A. the rod is held at 30^0 to the horizontal by a string which is at 90^0 to the rod and attached to it at C where AC = 3a. Find
   (a) The tension in the string [0.58W]
   (b) Reaction at A [0.578W]
9. A sphere of weight 40N and radius 30cm rests against a smooth vertical wall. The sphere is supported in this position by a string of length 20cm attached to a point on the sphere and to the point on the wall, find
   (a) Tension in the string [50N]
   (b) Reaction due to the wall[30N at 90^0 to the wall]
10. A smooth uniform rod AB of length 3a and eight 2w is pivoted at A so that it can rotate in a vertical plane. A light ring is free to slide over the rod. A light inextensible string is attached to the ring and passes over a fixed smooth peg at a point C a height 4a above A and carries a particle of weight w hanging freely



   (a) Show that the angle θ that the rod makes to the vertical in equilibrium is given by tanθ = $\frac{4}{3}$.
   (b) Find the magnitude of the force of the pivot on the rod at A in terms of w. $\left[\frac{3w\sqrt{5}}{3}\right]$.

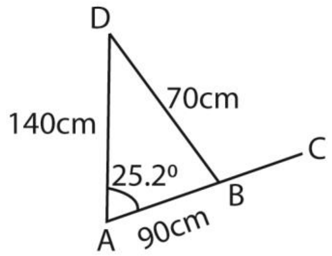
11. A smooth uniform rod AB of length 3a and weight W is pivoted at A so that it can rotate in a vertical plane. A light ring is free to slide over the rod. A light inextensible string is attached to the ring and passes over a fixed smooth peg at point C a height 4a above A and carries a particle of weight w hanging freely.



(a) Show that the angle θ that the rod makes to the vertical in equilibrium is given by tanθ= $\frac{8w}{3W}$.

(b) Find the smallest value of the ratio $\frac{w}{W}$ for which equilibrium is possible. $\left[\frac{\sqrt{7}}{8}\right]$

12. A uniform rod AC of mass 2kg and length 120cm hangs at rest in a vertical plane with end A in contact with a vertical wall. An inelastic string of length 70cm is attached to a point B on AC such that AB is 90cm. The other end of the string is attached to the wall at a point D, 140cm vertically above A



If the string is taut and angle DAC is 25.2^0, find

(a) Angle DBA [121.6^0]

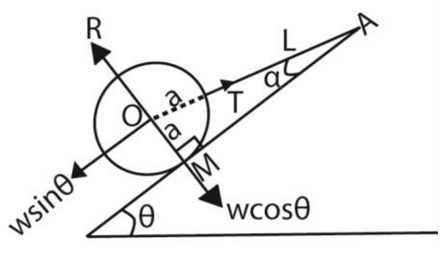
(b) The tension in the string [6.53N]

Beams on inclined plane

Example 31

A sphere of radius a and weight w rests on a smooth inclined plane supported by a string of length with one end attached to appoint on the surface of the sphere and the other end fastened to a point on the plane. If the angle of inclination of the plane to the horizontal is θ. Prove that the tension of the string is $\frac{w(a+L)sin\theta}{\sqrt{L^2+2aL}}$

Solution



$AM^2 = OA^2 - OM^2$

AM = $\sqrt{(a+L)^2 - a^2}$

AM = $\sqrt{L^2 + 2aL}$

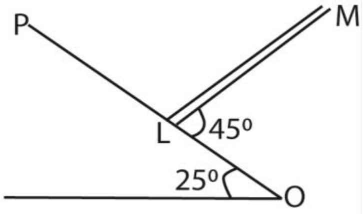
$cos\beta = \frac{\sqrt{L^2+2aL}}{(L+a)}$

Also wsinθ = Tcosα

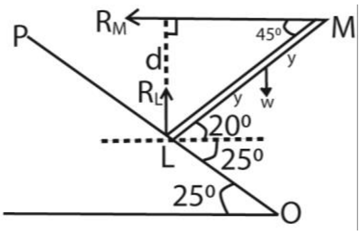
$T = \frac{wsin\theta}{cos\alpha} = \frac{w(a+L)sin\theta}{\sqrt{L^2+2aL}}$

Example 32

A uniform rod LM weight w rests with L on a smooth plane PO of inclination 250 as shown in the diagram below



The angle between LM and the plane is 450. What force parallel to PO applied at M will keep the rod in equilibrium?



R_M x d = w x ycons20

But d = 2ysin45

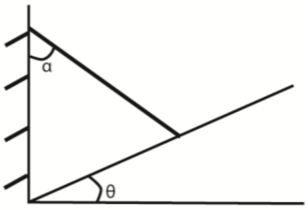
R_M x 2ysin45 = w x ycos20

R_M = 0.6645w

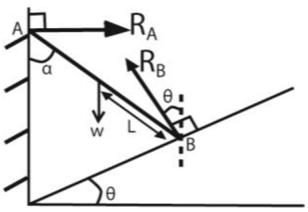
Taking moments about L

Example 33

The diagram shows a uniform ladder resting in equilibrium with its top end against a smooth vertical wall and its base on a smooth inclined plane. The plane makes an angle of θ with the horizontal and the ladder makes an angle α with the wall.



Prove that tanα = 2tanθ



Taking moments about B

R_A x 2Lcosα = w x Lsinα

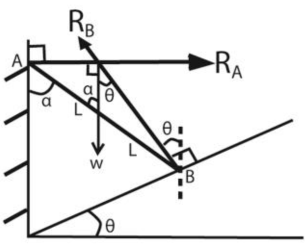
$R_A = \frac{wtan\alpha}{2}$ .... (i)

(↑) $R_B$cosθ = w ……. (ii)

(→)$R_B$sinθ = R_A ……..(iii)

(iii)÷ (ii)$\frac{R_B sin\theta}{R_B cos\theta} = \frac{w}{R_A}$; $R_A = wtan\theta$

$wtan\theta = \frac{wtan\alpha}{2}$; tanα = 2tanθ



Using cotangent rule for triangle

Lcotθ – Lcot90^0 = 2Lcotα

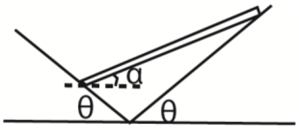
cot θ =2cotα

$\frac{1}{tan\theta} = \frac{2}{tan\alpha}$

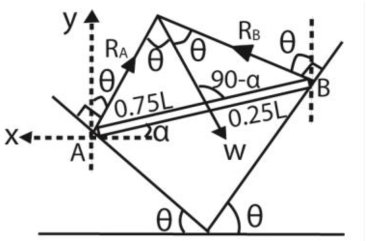
tanα = 2tanθ

Example 34

A rod AB of length L has its centre of gravity at a point G where AG = $\frac{1}{4}L$. The rod rests in equilibrium in a vertical plane at an angle α to the horizontal with its ends in contact with two inclined planes whose line of intersection is perpendicular to the rod. If the planes are smooth and equally inclined at an angle θ to the horizontal



Show that 2tanθtanα = 1 and the reaction on each plane is $\frac{w}{1+cos\theta}$



Using cotangent rule for triangle

$\frac{3Lcot\theta}{4} - \frac{L}{4cot\theta} = \left(\frac{3L}{4} + \frac{L}{4}\right)\cot(90^0 - \alpha)$

0.5cotθ = tanα

2tanθtanα = 1

(↑)$R_A cos\theta + R_B cos\theta$= w

$R_B = w - R_A cos\theta$ ………….(i)

(→)$R_A cos\alpha + R_B cos\alpha$

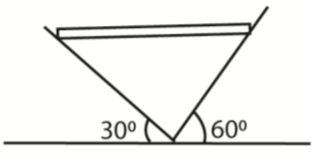
$R_B = R_A$ ……………………….(ii)

$R_B = w - R_B cos\theta$

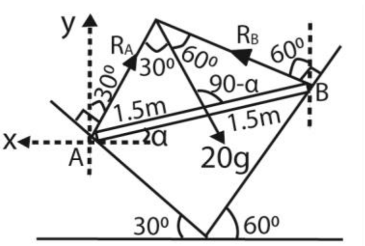
$R_B = R_A = \frac{w}{1+cos\theta}$

Example 35

A uniform road 3m long and of mass 29kg is placed on two smooth planes inclined at 30^0 and 60^0 to the horizontal



Find the reaction on each plane and the inclination of the rod to horizontal when it is in equilibrium.



(↑) $R_A cos30 + R_B \cos 60 = 20g$

$R_B = 40g - R_A\sqrt{3}$……………………..(i)

(→)$R_B \sin 60 = R_A \cos 30$

$R_{B\sqrt{3}} = R_A$ ……………………………………….(ii)

$R_B = 40g - R_B\sqrt{3}\ x\ \sqrt{3}$

$R_B = 98N$

$R_A = 98\sqrt{3} = 169.74N$

Using cotangent rule for triangles

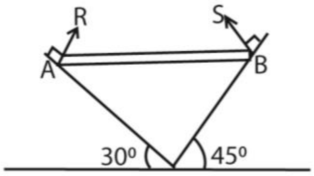
1.5cot30 – 1.5cot60 = 3cot(90 – α)

$1.5\sqrt{3} - 1.5\frac{1}{\sqrt{3}}$ = 3tanα

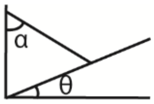
α = 30^0

Revision exercise 5

1. A uniform rod of length 2a and mass m is placed on two smooth inclined planes at 30^0 and 60^0 to the horizontal. The normal reactions at the ends of the rod have magnitude R and S.



   (a) Show that R = S$\sqrt{2}$
   (b) Prove that the inclination of the rod to horizontal is $\cot^{-1}(1+\sqrt{3})$
2. A uniform rod rests with its end on two smooth planes inclined at 30^0 and 60^0 to the horizontal. A weight equal to twice that of the beam can slide along its length. Find the position of the sliding weight when the beam rests in horizontal position $\left[\frac{1}{8} of\ thelength\ from\ 30^0 plane\right]$
3. The diagram shows a uniform ladder resting with its top end against a smooth vertical wall and its base on a smooth inclined plane. The plane makes an angle of θ with the horizontal and the ladder makes an angle of α with the wall.



   Find α when θ is
   (i) 10^0 [19.4^0]
   (ii) 30^0 [49.1^0]

Thank you

Dr. Bbosa Science