Equations involving indices, logarithms and others

Equations involving indices

This involves expressing the values raised to the powers into simplest form and thereafter making appropriate substitutions.

Example 1

Solve the following equations

(a) $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$
(b) $2^{2x+1} - 2^{x+1} + 1 = 2^x$
(c) $3(3^{2x}) + 2(3^x) - 1 = 0$

Solution

(a) $3^{2x+1} - 3^{x+1} - 3^x + 1 = 0$
$3^{2x}.3^1 - 3^x.3^1 - 3^x + 1 = 0$
Let p = 3^x
$3p^2$ -4p + 1 = 0
(3p – 1)(p – 1) = 0
p = $\frac{1}{3}$ or p = 1
$\therefore 3^x = \frac{1}{3} = 3^{-1}; x = -1$
$\therefore 3^x = 1 = 3^0; x = 0$

(b) $2^{2x+1} - 2^{x+1} + 1 = 2^x$
$2(2^x)^2 - 2(2^x) + 1 = 2^x$
$Let\ 2^x = q$
$2q^2 - 2q + 1 = q$
$2q^2 - 3q + 1 = 0$
$(2q - 1)$(q – 1) = 0
$q = \frac{1}{2}\ or\ q = 1$
$\therefore 2^x = \frac{1}{2} = 2^{-1}; x = -1$
$\therefore 2^x = 1 = 2^0; x = 0$
Hence x = 0 or x = -1

(c) $3(3^{2x}) + 2(3^x) - 1 = 0$
$3(3^x)^2 + 2(3^x) - 1 = 0$
Let p = 3^x
$3p^2$ + 2p -1 = 0
(3p – 1)(p + 1) = 0
p = $\frac{1}{3}$ or p = -1
$\therefore 3^x = 3^{-1}$ or x = -1
$3^x = -1$(has no root)
$Hence\ x = -1$

Example 2

(a) Solve the simultaneous equations

$3^x = 2^{3y+1};\ 4^{x-1} = 12^{2y+1}$

Given $\frac{log3}{log2} = \frac{8}{5}$

Solution

$3^x = 2^{3y+1}$ .... (i)

$4^{x-1} = 12^{2y+1}$ ....(ii)

Introducing log to both sides of equation (i)
xlog 3 = 3y + 1log 2
$\frac{log3}{log2} = \frac{3y+1}{x} = \frac{8}{5}$
8x – 15y = 5 ....(iii)
From eqn. (ii)

$4^{x-1} = 12^{2y+1}$

$2^{2(x-1)} = (3^1.2^2)^{2y+1}$

$2^{2(x-1)} = 3^{2y+1}.2^{2(2y+1)}$

$3^{2y+1} = 2^{2x-4y-4}$

$\frac{log3}{log2} = \frac{2x-4y-4}{2y+1} = \frac{8}{5}$
10x – 36y = 28
5x – 18y = 14 ....(iv)
5eqn. (iii) – 8eqn (iv)

69y = -87

y = $\frac{-29}{23}$

substituting for y in equation (iii)

8x = 5+ 15$\left(\frac{-29}{23}\right)$

$x = \frac{320}{23 \: x \: 8} = \frac{-40}{23}$

∴ x = $\frac{-40}{23}$ and y = $\frac{-29}{23}$

(b) Find the value of x to 2 decimal places if $2^{3x+1} = 3^{x+2}$

Solution

Introducing log

(3x + 1)log2 = (x +2)log3

x(3Log2 – log3) = 2log3 – log2

$x = \frac{2\log3 - \log2}{(3\text{Log}2 - \log3)} = 1.53$

(c) Evaluate

$3^{2x+1} - 26(3^x) = 9$

Solution

$3^{2x+1} - 26(3^x) = 9$

$3^{2x+1} - 26(3^x) = 9$

$3^1.3^{2x} - 26(3^x) = 9$

$3^1.(3^x)^2 - 26(3^x) = 9$

Let q = 3^x

$3q^2 – 26q – 9 = 0$

(q – 9)(3q + 1) = 0

q = 9 0r q = $-\frac{1}{3}$

$\therefore 3^x = 9 = 3^2; x = 2$

$\therefore 3^x = -\frac{1}{3}(has\ no\ roots)$

Hence x = 2

Logarithmic equations

It convenient to convert logarithms to the same base before the calculations

Example 3

Solve the equations

(a) $\log_x 5 + 4\log_5 x = 4$

Expressing terms on LHS to $\log_5$.

$\frac{\log_5 5}{\log_5 x} + 4\log_5 x = 4$

$\frac{1}{\log_5 x} + 4\log_5 x = 4$

Let $\log_5 x = y$

$\frac{1}{y} + 4y = 4$

$4y^2 - 4y + 1 = 0$

(2y – 1)(2y – 1) = 0

2y = 1

y = $\frac{1}{2}$

⇨ $\log_5 x = \frac{1}{2}$

x = $5^{\frac{1}{2}} = \sqrt{5}$

(b) Show that:

2log4 + $\frac{1}{2}\log 25 - \log 20 = 2\log 2$.

2log4 + $\frac{1}{2}\log 25 - \log 20$

$2\log2^2 + \frac{1}{2}\log 5^2 - (\log 4 + \log 5)$

$2\log2^2 + \frac{1}{2}\log 5^2 - \log 4 - \log 5$

4log2 + $\log 5 - 2\log 2 - \log 5$

2log 2

(c) Express $\log_{25}(xy)$in terms of $\log_5 x\ and\ \log_5 y$. Hence or otherwise solve the simultaneous equations:

$\log_{25}(xy) = 4\frac{1}{2}$

$\frac{\log_5 x}{\log_5 y} = -10$

Solution

$$\log_{25} xy = \frac{\log_5 xy}{\log_5 25} = \frac{\log_5 x + \log_5 y}{\log_5 5^2}$$

$$= \frac{\log_5 x + \log_5 y}{2}$$

$$\therefore \log_{25} xy = \frac{\log_5 x + \log_5 y}{2}$$

Hence solving

$\log_{25} xy = 4\frac{1}{2}$

$\frac{\log_5 x + \log_5 y}{2} = \frac{9}{2}$

$\log_5 x + \log_5 y = 9$ ……………….. (i)

$\frac{\log_5 x}{\log_5 y}$ = -10

$\log_5 x = -10\log_5 y$ …………….(ii)

Substituting eqn. (ii) into eqn. (i)

$-10\log_5 y + \log_5 y = 9$

$\log_5 y = -1$

y = $5^{-1} = \frac{1}{5}$

Substituting $\log_5 y$ into equation (ii)

$\log_5 x = 10$

$x = 5^{10}$

$\therefore\ x = 5^{10}$ and y = $\frac{1}{5}$

Equations with repeated terms (s)

Whenever a term or terms appear repeated in an equation make use of appropriate substitution.

Example 4

(a) Solve the simultaneous equations

$2^x + 4^y = 12$

$3(2^x) - 2(2^y)$=16

$Solution$

$2^x + 4^y = 12$

$2^x = 12 - 4^y$……………(i)

$3(2^x) - 2(2^y)$=16 ……..(ii)

Substituting eqn.(i) into eqn. (ii)

$3(12 - 4^y) - 2(2^y)$=16

$36 - 3(2^{2y}) - 2(2^y) = 16$

$5(2^{2y}) = 20$

$(2^{2y}) = 4 = 2^2$

$2y = 2; \gg y = 1$

Substituting y into eqn. (i)

$2^x = 12 - 4^1$ = 8 = 2^3

$x = 3$

Hence x = 3 and y = 1

(b) Solve the equations

(i) $9x^{\frac{2}{3}} + 5x^{-\frac{2}{3}}$ = 37

Solution

$9x^{\frac{2}{3}} + \frac{4}{x^{\frac{2}{3}}}$ = 37

Let q = $x^{\frac{2}{3}}$

9q+ $\frac{5}{q}$ = 37

$9q^2 - 37q + 4 = 0$

(9q – 1)(q – 4) = 0

q = $\frac{1}{9}$ or q = 4

When q = $\frac{1}{9}$;

$x^{\frac{2}{3}} = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$

$x = (3^{-2})^{\frac{3}{2}} = \frac{1}{27}$

When q = 4

$x^{\frac{2}{3}} = 4 = 2^2$

$x = (2^2)^{\frac{3}{2}} = 2^3 = 8$

Hence x = $\frac{1}{27}$ and x = 8

(ii) $(x^2 - 2x)^2 + 24 = 11(x^2 - 2x)$

Solution

Let t = $x^2 - 2x$

$t^2 - 11t + 24 = 0$

(t – 8)(t – 3) = 0

t = 8 or t = 3

When t = 8

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

(x – 4)(x + 2) = 0

x = 4 or x = -2

When t = 3

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

(x – 3)(x +1) = 0

x = 3 or x = -1

hence x = -2, -1, 3 or 4

Symmetrical equations

These are equations whose coefficients are symmetrical

Example 5

(a) (i) If $x + \frac{1}{x} = q, x^2 + \frac{1}{x^2}, x^3 + \frac{1}{x^3}$ and $x^4 + \frac{1}{x^4}$ in terms of q

Solution

$x + \frac{1}{x} = q$

Squaring both sides

$\left(x + \frac{1}{x}\right)^2 = q^2$

$x^2 + \frac{1}{x^2} + 2 = q^2$

$x^2 + \frac{1}{x^2} = q^2 - 2$

$x + \frac{1}{x} = q$

Cubing both sides

$\left(x + \frac{1}{x}\right)^3 = q^3$

$$x^3 + \frac{1}{x^3} + 3x^2\left(\frac{1}{x}\right) + 3x\left(\frac{1}{x}\right) = q^3$$
$$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = q^3$$
$$x^3 + \frac{1}{x^3} = q^3 - 3q = q(q^2 - 3)$$

$x + \frac{1}{x} = q$

Raising both sides to power four

$$\left(x + \frac{1}{x}\right)^4 = q^4$$
$$x^4 + \frac{1}{x^4} + 4x^3\left(\frac{1}{x}\right) + 6x^2\left(\frac{1}{x}\right)^2 + 4x\left(\frac{1}{x}\right)^3 = q^4$$
$$x^4 + \frac{1}{x^4} + 4\left(x^2 - \frac{1}{x^2}\right) + 6 = q^4$$
$$x^4 + \frac{1}{x^4} = q^4 - 4(q^2 - 2) - 6$$
$$x^4 + \frac{1}{x^4} = q^4 - 4q^2 + 2$$

(a)(ii) Solve the equation

$2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

Solution

Dividing through by x^2

$2x^2 - 9x + 14 - \frac{9}{x} + \frac{2}{x^2} = 0$

$2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0$

Let q = $x + \frac{1}{x}$

$2(q^2 - 2) - 9q + 14 = 0$

$2q^2 - 9q + 10 = 0$

(q – 2)(2q – 5) = 0

q = 2 or q = $\frac{5}{2}$

When q = 2

$x + \frac{1}{x} = 2$

$x^2 - 2x + 1 = 0$

(x – 1)(x – 1) = 0

x = 1

When q =$\frac{5}{2}$

$x + \frac{1}{x} = \frac{5}{2}$

$2x^2 - 5x + 2 = 0$

(2x – 1)(x – 2) = 0

x = $\frac{1}{2}$ or x = 2

Hence [x:x = $\frac{1}{2}$, 1, or 2]

(b) By using the substitution q = $x + \frac{1}{x}$ solve the equation

$4x^4 + 17x^3 + 8x^2 + 17x + 4 = 0$

$4x^2 + 17x + 8 + \frac{17}{x} + \frac{4}{x^2} = 0$

$4\left(x^2 + \frac{1}{x^2}\right) + 17\left(x + \frac{1}{x}\right) + 8 = 0$

$4(q^2 - 2)$ 17q + 8 = 0

$4q^2$ + 17q = 0

q(4q + 17) = 0

q = 0 or q =$-\frac{17}{4}$

When q = 0

$x + \frac{1}{x} = 0$

$x^2 + 1 = 0$ (no real roots)

When q =$\frac{-17}{4}$

$x + \frac{1}{x} = \frac{-17}{4}$

$4x^2 + 17x + 4 = 0$

(4x + 1)(x + 4) = 0

x = $\frac{-1}{4}$ or x = -4

Hence x = $\frac{-1}{4}$ or x = -4

Equations with ratios

The basis of these types of equations it the ration theorem

Given $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

Then $\frac{a+c+e}{b+d+f} = k$

This can

Then $\frac{a}{b} = k$ => a = bk

$\frac{c}{d} = k$ => c = dk
$\frac{e}{f} = k$ => e= fk

LHS: $\frac{bk+dk+fk}{b+d+f} = \frac{(b+d+f)k}{b+d+f} = k$

Example 6

Solve the equations

(a) $\frac{x+4z}{4} = \frac{y+z}{6} = \frac{3x+y}{5}; 4x + 2y + 5z = 30$

$Solution$
Let $\frac{x+4z}{4} = \frac{y+z}{6} = \frac{3x+y}{5} = k$
Then $\frac{x+4z+y+z+3x+y}{4+6+5} = k$
$\frac{4x+2y+5z}{15} = \frac{30}{15} = 2 = k$
$k = 2$
$\therefore \frac{x+4z}{4} = 2$; => x + 4z = 8 ……………….(i)
$\frac{y+z}{6} = 2$; => y + z = 12 ……………….(ii)
$\frac{3x+y}{5} = 2$; => 3x + y =10 ……………. (iii)
From eqn. (i): x = 8 – 4z
Substituting for x into eqn. (iii)
3(8 – 4z) + y = 10
y – 12z = -14 ……………………………..(iv)
eqn. (ii) – eqn. (iv)
13z = 26; z = 2
Substituting for z into eqn. (i)
x = 8 – 4 x 2 =0
substituting for x into eqn. (iii)
y = 10 – 3x = 10 – 0 = 10
Hence (x, y, z) = (0, 10, 2)

(b) $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5}; x + y + z = 2$

Solution

Let $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5} = k$

Then $\frac{3x+3y+3z}{6} = \frac{3(x+y+z)}{6} = \frac{3 \ x \ 2}{6} = 1 = k$

k = 1

$\therefore \frac{x+2y}{-3} = 1$; x + 2y = -3……………… (i)

$\frac{y+2z}{4} = 1$; y + 2z = 4 ………………(ii)

$\frac{2x+z}{5} = 1$; 2x + z = 5 ………………(iii)

From eqn. (i) x = -3 – 2y

Substituting x into eqn. (iii)

2(-3 – 2y) + z = 5

4y – z = -11…………………………….(iv)

From eqn. (ii): y = 4 – 2z

Substituting for y in eqn. (iv)

4(4 – 2z) –z = -11

9z = 27; z = 3

Substituting for z into eqn. (ii)

Y = 4 – 2 x 3 = -2

Substituting for y into eqn. (i)

x = -3 – 2(-2) = 1

Hence (x, y, z) = (1, -2, 3)

(c) 2x = 3y= -4z; $x^2 - 9y^2 - 4z = 0$

Solution
Let 2x = 3y= -4z =k, constant
$x = \frac{k}{2}; y = \frac{k}{3}; z = \frac{k}{-4}$
Substituting for x, y, z in $x^2 - 9y^2 - 4z = 0$
$\left(\frac{k}{2}\right)^2 - 9\left(\frac{k}{3}\right)^2 - 4\left(\frac{k}{-4}\right) = 0$
$3k^2 - 4k - 32 = 0$
(3k + 8)(k – 4) = 0
k = $-\frac{8}{3}$ or k = 4
When k =$-\frac{8}{3}$; x = 2, y = $\frac{4}{3}$, z = -1
When k = 4; x = $-\frac{4}{3}$, y = $-\frac{8}{9}$, z = $\frac{2}{3}$

$\therefore$ (x, y, z) = (2, $\frac{4}{3}$, -1) or ($-\frac{4}{3}$, $-\frac{8}{9}$, $\frac{2}{3}$)

Equations with repeated roots

If a function y = f(x) has a repeated root, then it is also a root of its derivative i.e. it is a root of $\frac{dy}{dx}f(x)$

In order to find other root, make use of the sum and product of roots

Note

Sum of roots = coefficient of the second term of the equation

Product of the roots= last term with the appropriate signs.

Example 7

(a) Given that the following equations have repeated roots, solve them

(i) $X^3 - x^2 - 8x + 12 = 0$

A repeated root is also a root of

$\frac{d}{dx}(x^3 - x^2 - 8x + 12) = 0$

$3x^2 - 2x - 8 = 0$

(x – 2)(3x + 4) = 0

x = 2 or x = $-\frac{4}{3}$

Testing for repeated root

Sum of roots = 1

Products of root = -12

If x = 2 is the repeated root, then

Sum = 2 + 2 + n = 1

n = -3 where n is the third root

Product of roots = 2 x 2 x -3 = -12 which is correct

If x = $-\frac{4}{3}$ is the repeated root, then

Sum = $-\frac{4}{3} - \frac{4}{3}$ + n = 1

n = $\frac{11}{3}$ where n is the third root

Product of roots = $-\frac{4}{3}$ x $-\frac{4}{3}$ x $\frac{11}{3} \neq$ -12 which is not correct

Hence the roots are 2, 2, -3

(ii) $2x^3 - 11x^2 + 12x + 9 = 0$

A repeated root is also a root of

$\frac{d}{dx}(2x^3 - 11x^2 + 12x + 9) = 0$

$6x^2 - 22x + 12 = 0$

$3x^2 - 11x + 6 = 0$

(x – 3)(3x -2) = 0

x = 3 or x = $\frac{2}{3}$

Testing for repeated root;

$2x^3 - 11x^2 + 12x + 9 = 0$

$x^3 - \frac{11}{2}x^2 + 6x + \frac{9}{2} = 0$

Sum of roots = $\frac{11}{2}$

Product of roots = $-\frac{9}{2}$

If x = 3 is the repeated root, then

Sum = 3 + 3 + n = $\frac{11}{2}$

n = $-\frac{1}{2}$ where n is the third root

Product of roots = 3 x 3 x $-\frac{1}{2} = -\frac{9}{2}$ which is correct

Hence the roots are 3, 3, $-\frac{1}{2}$

(b) Find the value of k for which the equation $\frac{x^2-x+1}{x-1}$ = k has repeated roots. What are the repeated roots?

Solution

$\frac{x^2-x+1}{x-1}$ = k

$x^2 - x + 1 = kx - k$

$x^2 - (1 + k)x + (1 + k) = 0$

Since a quadratic equation has only two roots, therefore the roots will be equal if they are repeated

The condition for equal roots: $b^2 = 4ac$

⇨ $(1 + k)^2 = 4(1 + k)$

$k^2 + 2x + 1 = 4 + 4k$

$k^2 - 2k - 3 = 0$

(k + 1)(k – 3) = 0

k = -1 and k = 3

If k = -1; $x^2 - (0)x + 0 = 0$

x = 0

If k = 3; $x^2 - 4x + 4 = 0$

(x – 2)(x – 2) = 0

x = 2

Hence repeated roots are 0 and 2

Example 8

Show that if the equations

$x^2 + px + q = 0$ and $x^2 + mx + k = 0$ have a repeated root, then

$(q - k)^2 = (m - p)(pk - mq)$

Let the common root be x_1

The two equations become

$x_1^2 + px_1 + q = 0$ ……………… (i)

$x_1^2 + mx_1 + k = 0$……………….(ii)

Eqn. (i) – eqn. (ii)

$(p - m)x_1 + (q - k) = 0$

$x_1 = -\left(\frac{q-k}{p-m}\right)$

Substituting x^1 into equation (i)

$\left(\frac{q-k}{p-m}\right)^2 - p\left(\frac{q-k}{p-m}\right) + q = 0$

Thus, $(q-k)^2 = (m-p)(pk-mq)$

Equations with square roots

(i) In this context, $\sqrt{x}$ means the positive square root of x.
(ii) The solution should be checked to discard off the unwanted root might emerge due to squaring.

Illustration

Suppose the root of the equation is x = 5

By squaring x2 = 52 i.e. $x^2 - 5^2 = 0$

(x -5)(x + 5) = 0

Now solution is x = -5 and x = 5 where -5 is unwanted.

Example 9

Solve the equations

(a) $\sqrt{(x-5)} + \sqrt{x} = 5$

Solution

$\left(\sqrt{(x-5)}\right)^2 = \left(5 - \sqrt{x}\right)^2$

$x - 5 = 25 - 10\sqrt{x} + x$

=> $10\sqrt{x} = 30$

$\sqrt{x} = 3$; x = 9 (upon squaring)

(c) $\sqrt{(1-20x)} - 2\sqrt{x+1} = 3$

Solution

$\left(\sqrt{1-20x}\right)^2 = \left(3 + 2\sqrt{x+1}\right)^2$

$1 - 20x = 9 + 12\sqrt{x+1} + 4(x + 1)$

$24x + 12 = -12\sqrt{x+1}$

$2x + 1 = -\sqrt{x+1}$

Squaring both sides

$4x^2 + 4x + 1 = x + 1$

$4x^2 + 3x = 0$

x(4x + 3) = 0

x = 0 or $x = -\frac{3}{4}$

When x = 0; $\sqrt{1} - 2\sqrt{1} \neq 3$; ∴x = 0 is not a root

When $x = -\frac{3}{4}$; $\sqrt{16} - 2\sqrt{\frac{1}{4}} = 3$ which is consistent. ∴$x = -\frac{3}{4}$

Other equations

Example 10

Given that $x^2 + 2xy + y^2 - 8x - 8y + 15 = 0$, find x in terms of y

Hence or otherwise solve the pair of equations:

$x^2 + 2xy + y^2 - 8x - 8y + 15 = 0$ and $x^2 + y^2 = 17$

Solution

$x^2 + 2xy + y^2 - 8x - 8y + 15 = 0$ …….. (i)

$(x + y)^2 - 8(x + y) + 15 = 0$

Let q = (x + y)

$q^2 - 8q + 15 = 0$

(q – 5)(q – 3) = 0

q = 5 and q = 3

When q = 5; x = 5 – y ………………….(ii)

When q = 3; x = 3 – y …………………. (iii)

Now $x^2 + y^2 = 17$ ………………….(iv)

Eqn. (ii) into eqn. (iv)

$(5 – y)^2 + y^2 = 17$

$2y^2 - 10y + 8 = 0$

(y – 4)(y – 1) = 0; => y = 4 or y = 1

When y = 4: x = 5 – 4 = 1

When y = 1: x = 5 – 1 = 4

Eqn. (iii) into eqn. (iv)

$(3 – y)^2 + y^2 = 17$

$2y^2 - 6y – 8 = 0$

(y – 4)(y + 1)= 0; => y = 4, or y = -1

When y = 4: x = 3 – 4 = -1

When y = -1: x = 3 +1 = 4

∴ (x, y) = (1, 4), (4, 1), (-1, 4), (4, -1)

Example 11

Solve the simultaneous equations

$\frac{1}{x} - \frac{1}{y} = \frac{1}{6}$, x(5 – x) = 2y

Solution

From $\frac{1}{x} - \frac{1}{y} = \frac{1}{6}$

$\frac{1}{y} = \frac{1}{x} - \frac{1}{6} = \frac{6-x}{6x}$

y = $\frac{6x}{6-x}$ ……………………….(i)

x(5 – x) = 2y ………………(ii)

Substituting eqn. (i) into eqn. (ii)

x(5 – x) = $\frac{12x}{6-x}$

x(5 – x)(6 – x) – 12x = 0

x[(5 – x)(6 – x) – 12] =0

$x(x^2 – 11x + 18) = 0$

x(x – 2)(x – 9) = 0; x =0, x = 2, x = 9

Substituting for x in eqn. (i)

When x = 0; y= 0

When x = 2; y = 3

When x = 9; y = - 18

∴(x, y) = (0, 0), (2, 3) or (9, - 18)

Example 12

Given that the equations $y^2 + py + q = 0$ and $y^2 + my + k = 0$ have common root. Show

$(q – k)^2 = (m – p)(pk – mq)$

Solution

Let α be the common root

For $y^2 + py + q = 0$

$\alpha^2 + p\alpha + q = 0$ ………………… (i)

For $y^2 + my + k = 0$

$\alpha^2 + m\alpha + k = 0$ ………………… (ii)

Eqn. (i) – eqn. (ii)

(p – m)k + (q – k) = 0

α = $\frac{q-k}{m-p}$

Substituting α into eqn. (i)

$$\left(\frac{q-k}{m-p}\right)^2 + \left(\frac{q-k}{m-p}\right)p + q = 0$$

$$\frac{(q-k)^2}{(m-p)^2} = -q - \left(\frac{q-k}{m-p}\right)p$$

$$(q-k)^2 = -q(m-p)^2 - (q-k)(m-p)p$$

$$(q-k)^2 = (m-p)[-q(m-p) - p(q-k)]$$

$$(q-k)^2 = (m-p)[-qm + qp - pq + pk]$$

$(q-k)^2 = (m-p)(pk-qm)$ as required

Revision questions

1. Solve the equations
   (a) $\log_x 8 - \log_x 2 = 16$[x = 2]

(b) $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{16}$

$\left[x = 8^{\frac{1}{4}} = 1.6818\right]$

2. (a) Given that $\log_2 x + 2\log_4 y = 4$, show that xy = 16. Hence solve the simultaneous equations
   $\log_{10}(x + y) = 1$
   $\log_2 x + 2\log_4 y = 4$
   [(x, y) = (8, 2) or (2, 8)]

(b) Show that $\log_a b = \frac{1}{\log_b a}$. Hence solve the simultaneous equations
$\log_a b + 2\log_b a = 3$
$\log_9 a + 2\log_9 b = 3$
$\therefore (x, y) = (27, 27) or\ (9, 81)$

3. Show that if the expressions
   (i) $x^2 + px + q$ and $3x^2 + q$ have a common root, then $3p^2 + 4q = 0$
   (ii) $x^2 + bx + c$ and $x^2 + px + q$ have a common root, then
   $(c - q)^2 = (b - p)(cp - bq)$

4. Solve the simultaneous equations
   (a) 2x+ y = 1

   $5x^2 + 2xy = y + 2x - 1$[(x, y)=(0,1), (-2, 5)]

   (b) x + 2y = 1
   3x2 + 5xy – 2y2 = 10 [(x, y) = (3, -1)

5. Solve the simultaneous equation
   $2^x + 4^y = 12$
   $3(2)^x - 2(2)^{2y} = 16$ [x= 2, y = 1]
   Hence show that $(4)^x + 4(3)^{2y} = 100$

6. Solve $4^x - 2^{x+1} - 15 = 0$ [x=2.322]

7. Show that if the expressions:
   $x^2 + bx + c$ and $x^2 + px + q$ have a common factor. Then $(c - q)^2 = (b - p)(cp - bq)$

8. Solve $2\sqrt{(x-1)} - \sqrt{(x+4)} = 1$
   [x = 5 or x =$\frac{13}{9}$]

9. Solve the simultaneous equations
   x + y + z = 2
   $\frac{x+2y}{-3} = \frac{y+2z}{4} = \frac{2x+z}{5}$ [x = 1, y = -2, z = 3]

10. Solve the simultaneous equations
    $x^2 - 10x + y^2 = 25$
    y – x = 1 [(x, y) =(6, 7) or $(-2, -1)$

11. Solve for x in the equation
    $\log_4(6 - x) = \log_2 x$ [x = 2]

Thank you

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