Curve sketching (A-level)

The procedures of curve sketching depend on the nature of the curve to be sketched

Graphs of y = f(x) (Non rational functions)

For any graph of the form y = f(x) where f(x) is not linear, some or all the following steps are followed.

(a) Determine if the curve is symmetrical about either or both axes of coordinates.
- Symmetry about the x-axis occurs if the equation contains only even powers of y. here equation will be uncharged when (-y) is substituted for y. this applies to graphs of the type $y^2=f(x)$
- Symmetry about the y-axis occurs if the equation contains only even powers of x. Here the equation will be uncharged when (-x) is substituted for x. Here the graph is said to even i.e. f(x) =f(x). For example the graph of $y = x^2$. **Note** if there are odd powers of x and y then there will be no symmetry.

(b) Determine if there is symmetry about the origin. Here symmetry occurs when a change in the sign of x causes a change in the sign of y without altering its numerical value.

(c) Find the intercepts i.e. the curve cuts the x-axis at a point when y = 0 and cuts the y-axis at the point when x = 0.

(d) The curve passes through the origin if (x, y)= (0, 0)

The behaviour of the neighbour of the curve through the origin is studied by considering the ratio of $\frac{y}{x}$.

- If this ratio is small, the curve keeps close to x-axis near the origin.
- If the ratio is unity, the direction of the curve bisects the angle between the axes.
- If the ratio is large, the curve keeps near the y-axis.

Or:

We consider the behaviour of $\frac{dy}{dx}$ near the origin.

- If $\frac{dy}{dx}$ is very small, then the curve lies near the x-axis.
- If $\frac{dy}{dx}$ is large, then the curve lies near the y-axis.
- If $\frac{dy}{dx}$ is near unity, then the direction of the curve (tangent at origin) bisects the angle between the axes.

(e) Examine the behaviour of the function as x→±∞ and y →±∞ (if any)

(f) Find the turning points and their nature as well as points if inflexion (if any)
Use the second derivative
- For min point, $\frac{d^2y}{dx^2} = +ve$
- For max point, $\frac{d^2y}{dx^2} = -ve$
- Point of inflexion, $\frac{d^2y}{dx^2} = 0$

Example 1

(a) Sketch the graph of $y= 5 + 4x – x^2$.
Steps taken
- Finding intercepts
x – intercept; y = 0
$0 = 5 + 4x – x^2$.
$5 + 5x –x – x^2 = 0$
5(1 + x) – x(1 + x) =0
(5 – x)(1 + x)

Either 5 – x = 0; x = 5

Or 1 + x = 0; x = -1

Hence the curve cuts the x-axis at point (-1, 0) and (0, 5)

y – intercept, when x = 0, y = 5

hence the curve cuts the y-axis at point (0, 5)

- As x→+∞, y →-∞ and x→-∞, y →+∞
- Finding turning point

$\frac{dy}{dx} = 4 - 2x$

At turning point $\frac{dy}{dx} = 0$

2x – 4 = 0; x = 2

When x = 2; y= 5 + 4(2) – $(2)^2$= 9

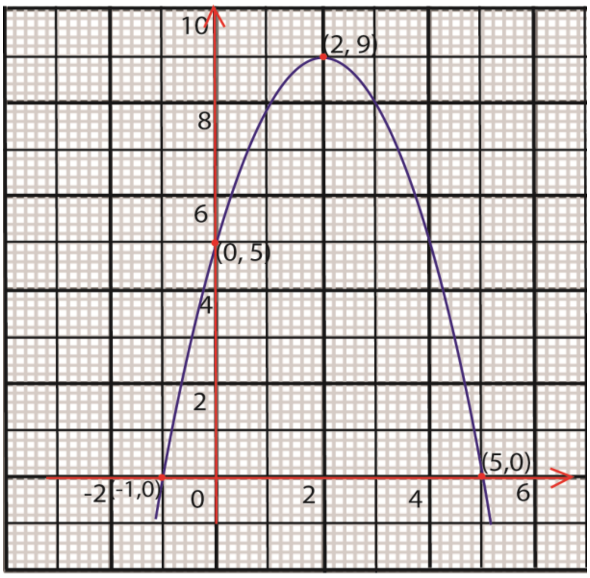
Hence turning point = (2, 9)

Finding the nature of turning point

$\frac{dy}{dx} = 4 - 2x$

$\frac{d^2y}{dx^2} = -2$

Since $\frac{d^2y}{dx^2} < 0$, hence the turning point is maximum.



(b) Sketch the curve y = $x^3 - x^2 - 5x + 6$

Steps taken

For y – intercept; x = 0, y = 0

Hence the y – intercept is (0, 6)

For x – intercept, y = 0

$x^3 - x^2 - 5x + 6 = 0$

error approach is used to find the first factor i.e. (x-2), then other factor is found by long division

$x^2 + x - 3$

(x – 2)) $x^3 - x^2 - 5x + 6$

$- x^3 - 2x^2$

$x^2 - 5x + 6$

$- \; x^2 - 2x$

3x + 6

$-$ 3x + 6

0 + 0

⇨ $x^3 - x^2 - 5x + 6 = (x - 2)(x^2 + x - 3) = 0$

Solving $x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1+12}}{2} = \frac{-1 \pm 3.6}{2}$

x = 1.3 or -2.6

Hence the x- intercepts are (2, 0), (1.3, 0) and (-2.3, 0)

Finding turning points

$y = x^3 - x^2 - 5x + 6$

$\frac{dy}{dx} = 3x^2$-2x-5

At turning point $\frac{dy}{dx} = 0$

$3x^2$-2x-5 = (3x – 5)(x+1) = 0

Either 3x -5 = 0 x = $\frac{5}{3}$

Or x + 1 = 0; x =-1

When x = $\frac{5}{3}$;

$y = \left(\frac{5}{3}\right)^3 - \left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right) + 6 = \frac{-13}{27}$

When x = -1

$y = (-1)^3 - (-1)^2 - 5(-1) + 6 = 9$

Hence turning points are $\left(\frac{5}{3}; \frac{-13}{27}\right)$ and(-1, 9)

Finding the nature of turning points

$\frac{dy}{dx} = 3x^2$-2x-5

$\frac{d^2y}{dx^2} = 6x - 2$

For$\left(\frac{5}{3}; \frac{-13}{27}\right)$

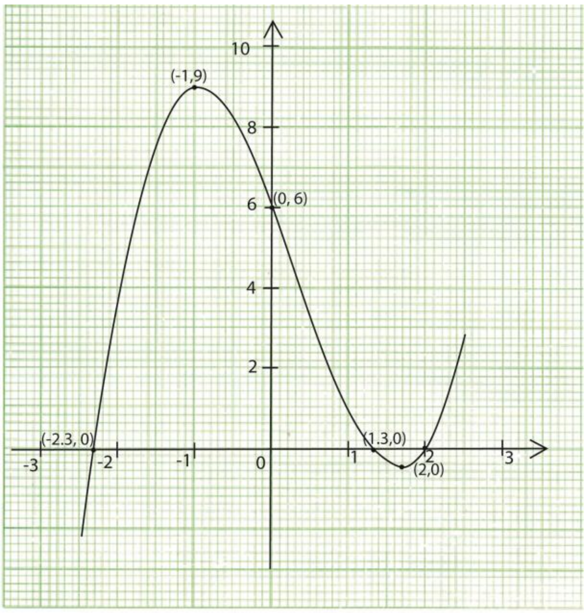
$\frac{d^2y}{dx^2} = 6\left(\frac{5}{3}\right) - 2 = 8\ (> 0)$

$\therefore \left(\frac{5}{3}; \frac{-13}{27}\right)$is minimum

For (-1, 9)

$\frac{d^2y}{dx^2} = 6(-1) - 2 = -8\ (< 0)$

$\therefore (-1; 9\ )$is maximum



(c) sketch the curve y = $x^3 - 8$

y = $x^3 - 8$

Intercepts

When x = 0, y = -8

When y = 0, x = 2

(x, y) = (2, 0)

Turning point: $\frac{dy}{dx} = 3x^2$

$3x^2 = 0$

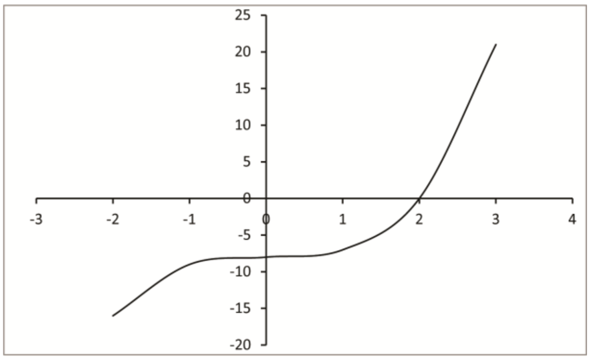
x = 0

$\frac{d^2y}{dx^2} = 6x$

$\frac{d^2y}{dx^2} = 0, x = 0$

Point of reflection= (0, 8)

	x < 2	x > 2
y	-	+



(d) Sketch the curve $y = x^2(x - 4)$

Steps taken

- Finding the intercepts

y – intercept, (0,0)

hence y – intercept is (0, 0)

For x – intercept, y= 0

=>$x^2(x - 4) = 0$

Either x = 0 or x = 4

Hence x-intercept are (0, 0) and (4, 0)

- As x→+∞, y →+∞ and x→-∞, y →-∞
- Finding turning point(s)

$y = x^2(x - 4) = x^3 - 4x^2$

$\frac{dy}{dx} = 3x^2 - 8x$

At turning point, $\frac{dy}{dx} = 0$

⇨ $3x^2 - 8x = x(3x - 8) = 0$

Either x = 0

Or x = $\frac{8}{3}$

When x = 0; y=0

When x =$\frac{8}{3}$; =$3\left(\frac{8}{3}\right)^2 - 8\left(\frac{8}{3}\right) = \frac{-256}{27}$

Hence turning points are (0,0) and$\left(\frac{8}{3}, \frac{-256}{27}\right)$

- Finding the nature of turning points

$\frac{dy}{dx} = 3x^2 - 8x$

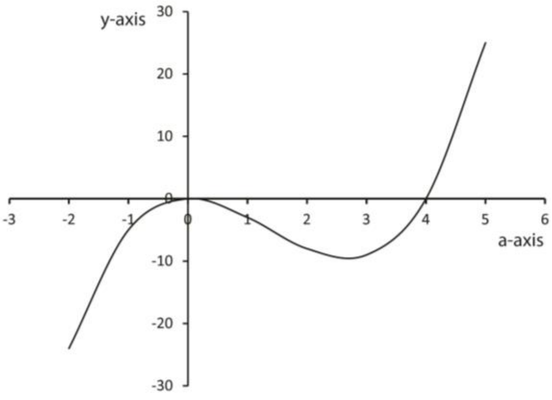
$\frac{d^2y}{dx^2} = 6x - 8$

For (0, 0); $\frac{d^2y}{dx^2} = 6(0) - 8 = -8\ (< 0)$

Hence (0, 0) is maximum

For $\left(\frac{8}{3}, \frac{-256}{27}\right); \frac{d^2y}{dx^2} = 6\left(\frac{8}{3}\right) - 8 = 8\ (> 0)$

Hence $\frac{8}{3}$ is minimum



Graphs of rational functions

Rational functions are fractions expressed in the form $y = \frac{f(x)}{g(x)}$.

The basic principles followed when sketching rational curves

(a) Determine if the curve is symmetrical about either or both axes of coordinates.

(b) Find the intercepts on both axes.

(c) Examine the behaviour of the curve as x tends to infinity.

(d) Find the turning points and their nature

(e) Determine the possible asymptotes of the curve

- Vertical asymptote is the value of x which make(s)y tend to infinity. Here we equate the denominator of the function to zero
- Horizontal asymptote is the value of x which make(s)x tend to infinity. Here we divide terms of the numerator and denominators by x with the highest power.
  Alternatively; when finding the horizontal asymptote, we re-arrange the equation and solve for x or make x the subject and then observe the limits, i.e. x→∞, see how y behaves
- Slating asymptotes; this only occurs if horizontal asymptote does not exist and the frictions is improper. Here we divide the terms of the numerator by those of the denominator an y equated to the quotient becomes the asymptote, i.e. asymptote is the y-quotient.

(f) Determine the region where the curve exists/does not exist. This is done by finding a quadratic equation in x such that for real values of x; $b^2>4ac$

Example 2

(a) Sketch the graph of $y = \frac{(x-1)(x+2)}{(x-2)(x+1)}$

Solution

Steps taken

- Finding intercepts

For y-intercepts; x= 0, $y = \frac{(-1)(2)}{(-2)(+1)} = 1$

Hence the y-intercept = (0, 1)

For x-intercept y= 0, $\frac{(x-1)(x+2)}{(x-2)(x+1)} = 0$;

x=1 or x = -2

Hence the x-intercept are (1, 0) and (-2, 0)

- Finding turning points

$y = \frac{(-1)(2)}{(-2)(+1)} = \frac{x^2+x-2}{x^2-x-2}$

$\frac{dy}{dx} = \frac{(x^2-x-2)(2x+1)-(x^2+x-2)(2x-1)}{(x^2-x-2)^2}$

At turning point, $\frac{dy}{dx} = 0$

⇨ $\frac{(x^2-x-2)(2x+1)-(x^2+x-2)(2x-1)}{(x^2-x-2)^2} = 0$

$(2x^3 - x^2 - 5x - 2)$

$-(2x^3 + x2 - 5x + 2) = 0$

$2x^2 + 4 = 0$

$x^2 + 2 = 0$

There is no real value of x, hence there is no turning points.

- Finding asymptotes;

Vertical asymptote

$(x - 2)(x + 1) = 0$

Either (x – 2) = 0; x = 2

Or (x + 1) = 0; x = -1

Hence the vertical asymptotes are x = 2 and x = 1

Horizontal asymptotes

$y = \frac{x^2+x-2}{x^2-x-2}$

Dividing terms on the LHS by x^2

$y = \frac{1+\frac{1}{x}-\frac{2}{x^2}}{1-\frac{1}{x}-\frac{2}{x^2}}$

As x →∞, y→1

Hence the horizontal asymptote y = 1

- Finding the regions where the curve does not exist

$y = \frac{x^2+x-2}{x^2-x-2}$

$y(x^2 - x - 2) = x^2 + x - 2$

$y(x^2 - x - 2) - x^2 - x + 2 = 0$

$(y-1)x^2 + (-y-1)x + (2-2y) = 0$

For real value of x, $b^2 > 4ac$

⇨ $(-y-1)^2 > 8(y-1)(1-y)$

$(y+1)^2 + 8(y-1)^2 > 0$

$9y^2 - 6y + 9 > 0$

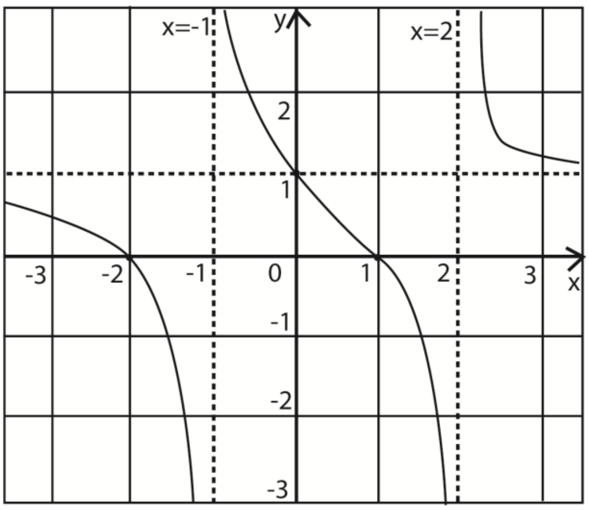
There is no real value of y which means that there is no restriction on y

- Determining the sign of the function throughout its domain. The function will only change sign where the curve cuts the x-axis and vertical asymptotes

The critical value are -, -1, 1, 2

	X<-2	-2<x<-1	-1<x<1	1<x<2	x>2
x-1	-	-	-	+	+
x+2	-	+	+	+	+
x-2	-	-	-	-	+
X+1	-	-	+	+	+
y	+	-	+	-	+

Graph of $y = \frac{(x-1)(x+2)}{(x-2)(x+1)}$



(b) Sketch the graph

Sketch the graph of $y = \frac{x(x-2)}{x+1}$

Steps taken

- Finding the intercepts

For y – intercept; x = 0, and y =0

Hence the y-intercept is (0, 0)

For x- intercept y= 0

⇨ $\frac{x(x-2)}{x+1} = 0$

$x(x-2) = 0$

Either x = 0

Or (x-2) = 0; x = 2

Hence the x-intercepts are(0, 0) and (2, 0)

- Finding turning points

$y = \frac{x^2-2x}{x+1}$

$\frac{dy}{dx} = \frac{x^2-2x(1)-(x+1)(2x-2)}{(x+1)^2} = \frac{x^2+2x-2}{(x+1)^2}$

At turning points, $\frac{dy}{dx} = 0$

⇨ $x^2 + 2x - 2 = 0$

$x = \frac{-2\pm\sqrt{2^2-(4x1x-2)}}{2x1}$

$x = \frac{-2\pm\sqrt{12}}{2} = -1 \pm 1.732$

When x =$-1 + 1.732 = 0.732$

$y = \frac{0.732(0.732-2)}{0.732+1} = -0.54$

When x = $-1 - 1.732 = -2.732$

$y = \frac{-2.732(-2.732-2)}{-2.732+1} = -7.46$

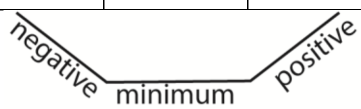
Hence the turning points are

(0.73, -0.54) and (-2.73, -7.46)

- Finding the nature of the turning points

For (0.73, -0.54)

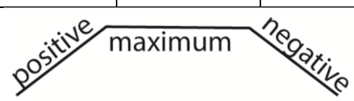
x	O	0.73	3
$\frac{dy}{dx}$	-2	0	0.25



Hence the turning point (0.73, -0.54) is minimum

For (-2.73, -7.46)

x	-3	-2.73	-2
$\frac{dy}{dx}$	0.25	0	-2



Hence the turning point (-2.73, -7.46) is maximum

- Finding asymptotes

For vertical asymptote, the denominator = 0

⇨ x+ 1 = 0; x = -1

since the function is improper fraction, there must be slating asymptote.

Dividing the numerator by denominator;

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          x-3
x+1 ) x²-2x
      - x² +x
      -------
       - 3x-3
       -------
            3
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The slanting asymptote is y = x – 3

X	0	3
y	-3	0

- Finding the region where the curve does not exist.

$y = \frac{x^2-2x}{x+1}$

$y(x+1) = x^2 - 2x$

$x^2 - (2+y)x - y = 0$

For real values of x, $b^2 \geq 4ac$

$(2+y)^2 > 4x\ 1\ x(-y)$

$y^2 + 8y + 4 > 0$

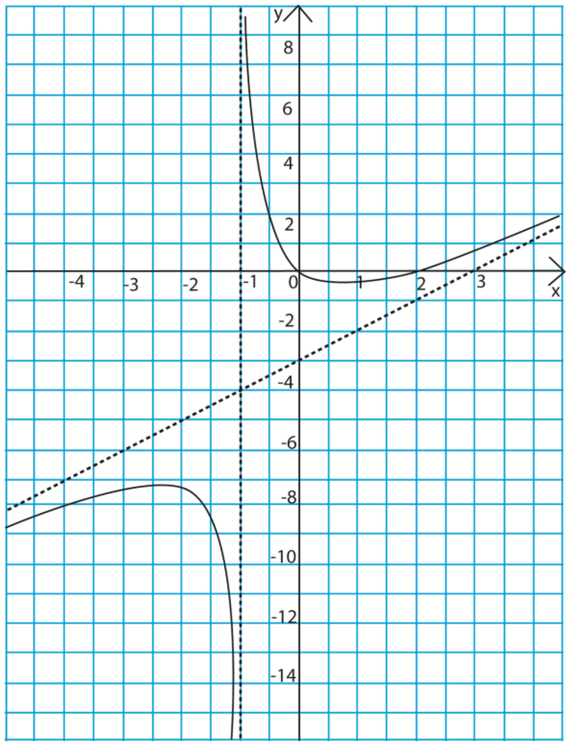
The inequality cannot be factorized therefore we may not proceed further even though there is no real value of y

- Determining the sign of the function through it domain

The critical values are -1, 0, 2

	X<-1	-1<x<0	0<x<2	x>2
x(x-1)	+	+	-	+
x+1	-	+	+	+
y	-	+	-	+

Graph of $y = \frac{x(x-2)}{x+1}$



(c) Given the curve $y = \frac{x(x-1)}{(x-2)(x+1)}$

- Finding intercept

For y-intercept x= 0; y= 0

Hence y-intercept = (0, 0)

For x-intercept y = 0

⇨ x(x-1) = 0

Either x = 0

Or x -1 =0; x = -1

Hence x-intercept as are (0, 0) and (1, 0)

- Finding turning points

$y = \frac{x^2-x}{x^2-x-2}$

$\frac{dy}{dx} = \frac{(x^2-x-2)(2x-1)-(x^2-x)(2x-1)}{(x^2-x-2)^2}$

At turning point $\frac{dy}{dx} = 0$

$(2x-1)\{(x^2-x-2)-(x^2-x)\} = 0$

(2x – 1)(-2) = 0

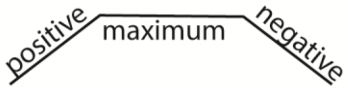
⇨ 2x – 1 = 0; x =$\frac{1}{2}$

When x = $\frac{1}{2}$, $y = \frac{\left(\frac{1}{2}\right)^2 - \frac{1}{2}}{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) - 2} = \frac{1}{9}$

Hence turning point is $\left(\frac{1}{2}, \frac{1}{9}\right)$

- Determining nature of turning point

x	0	$\frac{1}{2}$	1
$\frac{dy}{dx}$	2	0	-6



Hence the turning point $\left(\frac{1}{2}, \frac{1}{9}\right)$ is maximum.

- Finding the asymptote(s)
  For vertical asymptote
  (x – 2)(x + 1) = 0
  Either (x – 2) = 0; x = 2
  Or (x + 1) =0; x = -1
  For horizontal asymptote
  Dividing the numerator and denominator on the RHS by x^2.
  $y = \frac{1+\frac{1}{x}}{1-\frac{1}{x}-\frac{2}{x^2}}$
  As x → ∞, y→1
- Finding the region where x does not exist

  $y = \frac{x(x-1)}{(x-2)(x+1)}$

  $y(x-2)(x+1) = x(x-1)$

  $y(x^2 - x - 2) = x^2 - x$

  $yx^2 - yx - 2y - x^2 + x = 0$

  $(y-1)x^2 + (1-y)x - 2y = 0$

  For real values of x, $b^2 \geq 4ac$

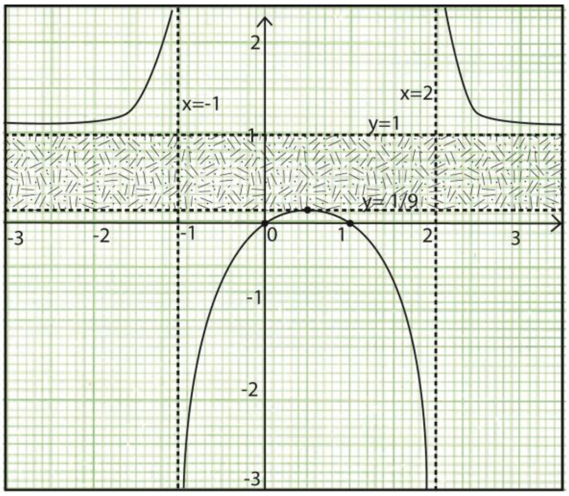
  $(1-y)^2 > -8y(y-1)$

  $1 - 2y + y^2 + 8y(y-1) > 0$

  (9y – 1)(y – 1) = 0

	y<$\frac{1}{9}$	$\frac{1}{9} < y < 1$	Y > 1
9y – 1	-	+	+
y – 1	-	-	+
(9y – 1)(y – 1)	+	-	+

Hence the curve does not lie in the range $\frac{1}{9} < y < 1$



(d) Sketch the curve $y = \frac{x^2+4x+3}{x+2}$

- Finding the range of values over which the curve does not exist
  $y = \frac{x^2+4x+3}{x+2}$
  $y(x+2) = x^2 + 4x + 3$
  $x^2 + (4-y)x + (3-2y) = 0$
  For real values of x, $b^2 \geq 4ac$
  $(4-y)^2 > 4(3-2y)$
  $16 - 8y + y^2 - 12 + 8y \geq 0$
  $y^2 + 4 \geq 0$
  Since there are no real values of y, this means that there is no restriction on y.
- Finding intercepts
  For y intercept, x= 0; $y = \frac{3}{2}$
  Hence y- intercept is $\left(0, \frac{3}{2}\right)$
  For x – intercepts y = 0
  ⇨ $\frac{x^2+4x+3}{x+2} = 0$

  $x^2 + 4x + 3 = 0$

  (x + 3)(x+ 1) = 0

  Either (x + 3) = 0; x= -3

  Or (x+1) = 0, x = -1

  Hence x-intercepts are (-1, 0) and (-3, 0)
- Finding turning points

  $y = \frac{x^2+4x+3}{x+2}$

  $\frac{dy}{dx} = \frac{(x+2)(2x+4)-(x^2+4x+3)(1)}{(x+2)^2}$

  At turning points, $\frac{dy}{dx} = 0$

$x^2 + 4x + 5x = 0$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{-4}}{2}$

Since there is no real value of x, this means that the curve has no turning point

- Finding vertical asymptote

  (x +2) = 0

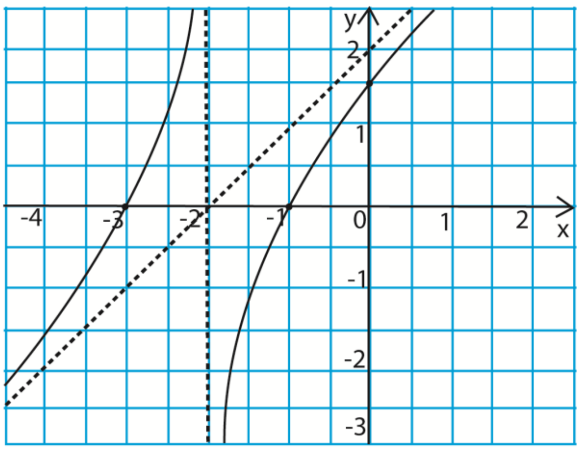
  x = -2

  Since the function is improper fraction, there could be a slating asymptote

$$\begin{array}{r|l} & x + 2 \\ (x+2) & x^2 + 4x + 3 \\ & -\ x^2 + 2x \\ \hline & 2x + 3 \\ & -\ 2x + 4 \\ \hline & -1 \end{array}$$

  Hence the slanting asymptote is y =x + 2

  A curve $y = \frac{x^2+4x+3}{x+2}$



(e) A curve is given by $y = \frac{(x-1)}{(2x-1)(x+1)}$

(i) Show that for real values of x, y cannot take on values in the interval $\left(\frac{1}{9}, 1\right)$

$y = \frac{(x-1)}{(2x-1)(x+1)}$

$y(2x-1)(x+1) = x - 1$

$y(2x^2 + x - y) = x - 1$

$2yx^2 + (y-1)x + (1-y) = 0$

For real values of x, $b^2 \geq 0$

$(y-1)^2 \geq 8y(1-y)$

$(y-1)^2 + 8y(y-1) \geq 0$

(9y – 1)(y – 1) = 0

	$y < \frac{1}{9}$	$\frac{1}{9} < y < 1$	Y > 1
9y – 1	-	+	+
y – 1	-	-	+
(9y – 1)(y – 1)	+	-	+

Hence the curve does not lie in the range $\frac{1}{9} < y < 1$

(ii) Determine the turning points of the curve

$y = \frac{(x-1)}{(2x-1)(x+1)} = \frac{(x-1)}{2x^2+x-1}$

$\frac{dy}{dx} = \frac{(2x^2+x-1)(1)-(x-1)(4x+1)}{(2x^2+x-1)^2}$

$= \frac{-2x^2+4x}{(2x^2+x-1)^2}$

At turning point $\frac{dy}{dx} = 0$

⇨ $-2x^2 + 4x = 0$

$-2x(x - 2) = 0$

Either 2x = 0; x= 0

Or (x - 2) = 0; x= 2

When x = 0; $y = \frac{-1}{-1} = 1$ => (x, y) =(0, 1)

When x = 2, $y = \frac{1}{3x3} = \frac{1}{9}$=> (x, y) =$\left(2, \frac{1}{9}\right)$

Determining the nature of turning points

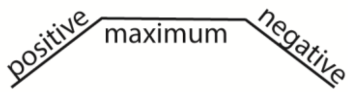
For (0,1)

x	-0.5	0	0.5
$\frac{dy}{dx}$	-2.5	0	1.5

Hence (0, 1) is minimum

For$\left(2, \frac{1}{9}\right)$

x	1	2	3
$\frac{dy}{dx}$	+0.025	0	-0.074



Hence $\left(2, \frac{1}{9}\right)$is maximum

(iii) State with reasons the asymptotes of the curve

For vertical asymptote

(2x – 1)(x + 1) = 0

Either 2x – 1 =0; x=$\frac{1}{2}$

Or (x + 1) = 0; x = -1

For horizontal asymptotes

Dividing the numerator and denominator on the RHS by x

$$y = \frac{1-\frac{1}{x}}{2x-1-\frac{1}{x}}$$

As x→ ∞, y → 0

Hence horizontal asymptote is y = 0

(iv) Sketch the curve

Finding intercepts

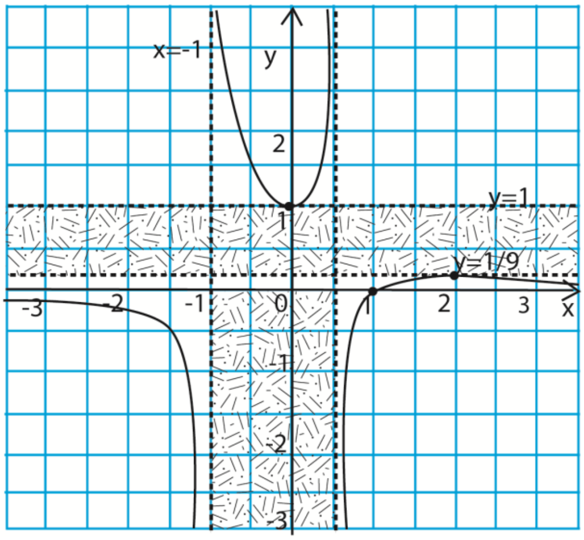
For y-intercept x = 0; y = 1

Hence y-intercept (0,1)

For x-intercept y = 0; x= 1

Hence x – intercept is(1, 0)

A graph of $y = \frac{(x-1)}{(2x-1)(x+1)}$



Sketching graphs of parametric equations

It requires eliminating parameters in the equations given and then following similar steps in above examples.

Example 3

(a) A curve is given by parametric equations

x = t + 2 and y = $\frac{t^2-t}{t+1}$

(a) Find the Cartesian equation of the curve

Solution

x = t – 2; t = x + 2

Substituting t into the equation

$$y = \frac{(x-2)^2-(x-2)}{(x-2)+1} = \frac{(x-2)(x-3)}{(x-1)}$$

Hence Cartesian equation is $y = \frac{(x-2)(x-3)}{(x-1)}$

(b) Sketch the curve

$$y = \frac{(x-2)(x-3)}{(x-1)}$$

- Finding intercepts

For y-intercept, x = 0, y = -6

For x-intercept y = 0

⇨ $(x-2)(x-3) = 0$

Either x – 2 = 0; x = 2

Or (x – 3) = 0; x = 3

Hence x- intercepts are (2, 0) and (3, 0)

- Finding the turning points

$$y = \frac{x^2-5x+6}{(x-1)}$$

$$\frac{dy}{dx} = \frac{(x-1)(2x-5)-(x^2-5x+6)(1)}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$$

At turning point $\frac{dy}{dx} = 0$

⇨ $x^2 - 2x - 1 = 0$

$$x = \frac{2 \pm \sqrt{(-2)^2-4(1)(-1)}}{2(1)}$$

$$x = \frac{2\pm\sqrt{8}}{2} = \frac{2\pm2\sqrt{2}}{2} = 1 \pm \sqrt{2}$$

Either x = 1 = $\sqrt{2}$= 2.4

Or x = 1 – $\sqrt{2}$= -0.4

When x = 2.4

$$y = \frac{(2.4-2)(2.4-3)}{(2.4-1)} = -0.17$$

When x = -0.4

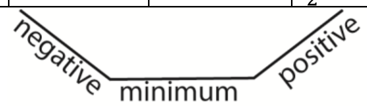
$$y = \frac{(-0.4-2)(-0.4-3)}{(-0.4-1)} = -5.83$$

Hence the turning points are (2.4, -0.17) and (-0.4, - 5.83)

- Finding the nature of turning points

For (2.4, -0.17)

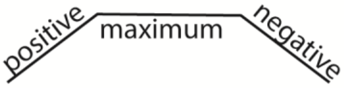
X	2	2.4	3
$\frac{dy}{dx}$	-1	0	$\frac{1}{2}$



Hence the turning point (2.4, -0.17) is minimum.

For (-0.4, -5.83)

X	-1	-0.4	0
$\frac{dy}{dx}$	$\frac{1}{2}$	0	-1



Hence the turning point (-0.4, -5.3) is maximum

- Finding asymptotes

For vertical asymptotes

x – 1 = 0, x= 1

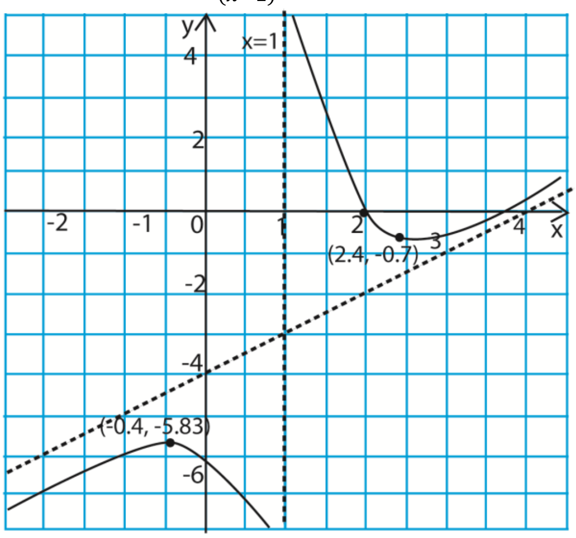
For slanting asymptote

$$\begin{array}{r l} & x - 4 \\ (x - 1) \big) & x^2 - 5x + 6 \\ & -\ x^2 - x \\ & \quad -4x + 6 \\ & -\ -4x + 4 \\ & \qquad 2 \end{array}$$

Hence the slanting asymptote is y = x – 4

x	0	4
y	-4	0

Graph $y = \frac{(x-2)(x-3)}{(x-1)}$



(c) A curve is given by parametric equations

x = cos2θ and y =2sinθ.

(i) Find the equation of the normal to the curve at θ=$\frac{5\pi}{6}$

x = cos2θ

$\frac{dx}{d\theta} = -2sin2\theta$

y =2sinθ.

$\frac{dy}{d\theta} = 2cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta}.\frac{d\theta}{dx} = \frac{-2cos\theta}{2sin2\theta} = -\frac{cos\theta}{sin2\theta}$

At θ=$\frac{5\pi}{6}$

$\frac{dy}{dx} = \frac{cos\left(\frac{5\pi}{6}\right)}{sin\left(\frac{5\pi}{3}\right)} = -1$

Gradient of the normal =$\frac{-1}{-1} = 1$

x = cos2θ

$x = cos\frac{5\pi}{3} = \frac{1}{2}$

$y = 2sin\frac{5\pi}{6} = 1$

Let a point (x, y) lie on the normal

⇨ $\frac{y-1}{x-\frac{1}{2}} = 1$

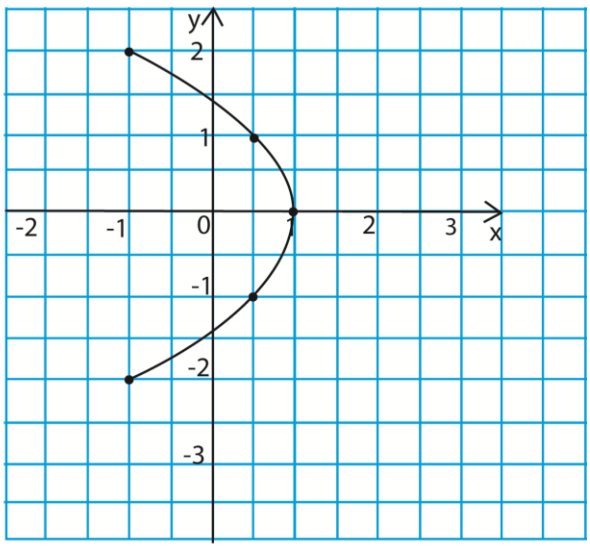
$y = x + \frac{1}{2}$

Hence the equation of the normal to the curve at $\theta = \frac{5\pi}{6}$ is $y = x + \frac{1}{2}$

(ii) Sketch the curve for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
x=cos2θ	-1	-0.5	0.5	1	0.5	-0.5	-1
y=2sinθ	-2	-1.73	-1	0	1.73	1.73	2

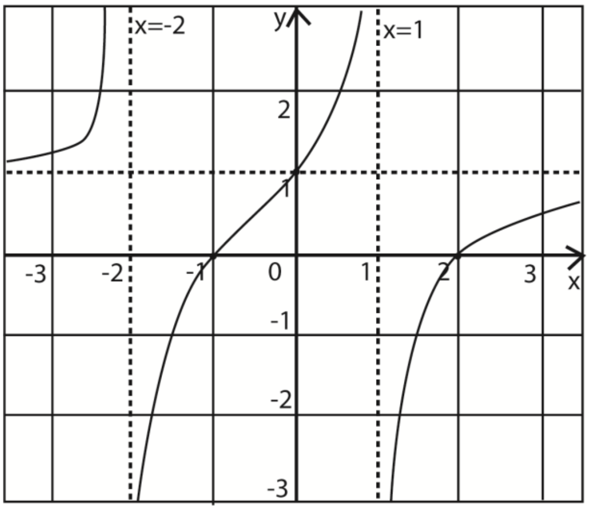
A graph of x = 1 $-\frac{y^2}{2}$



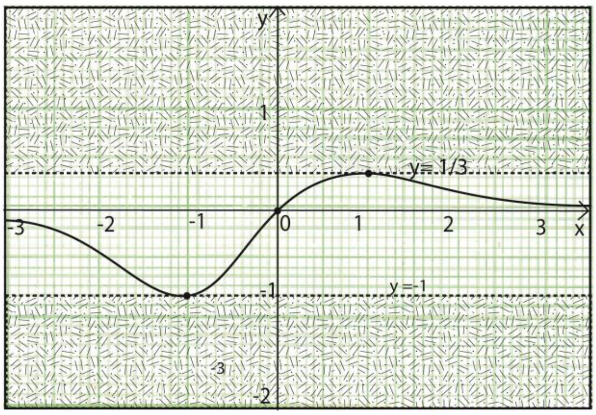
Revision question 1

1. Sketch the graph $y = \frac{(x-2)(x+1)}{(x-1)(x+2)}$

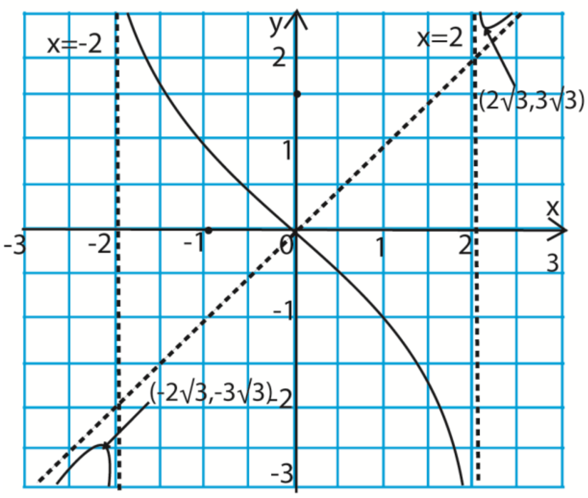
   A graph $y = \frac{(x-2)(x+1)}{(x-1)(x+2)}$



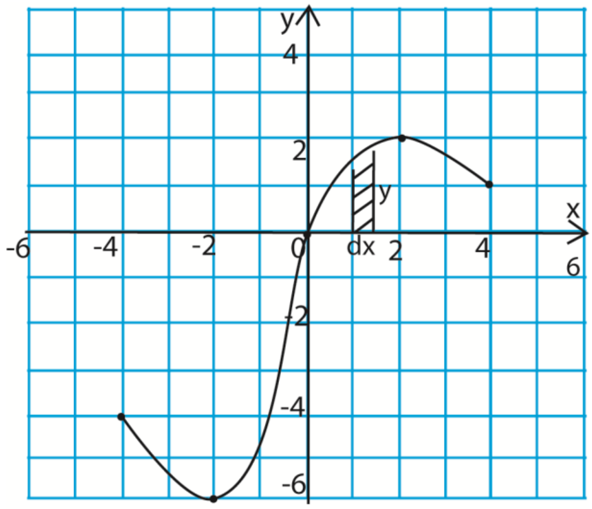
2. Sketch the graph $y = \frac{x}{x^2+x+1}$



3. Sketch the curve$y = \frac{x^2}{x^2-4}$



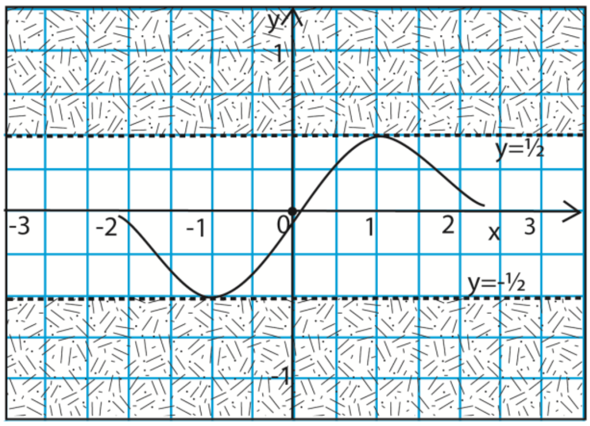
4. (a) Sketch the curve $y = \frac{12}{x^2+2x+4}$



   (b) Find the area enclosed by the curve, x-axis and 0≤ x ≤4 [0.259 to3dp)

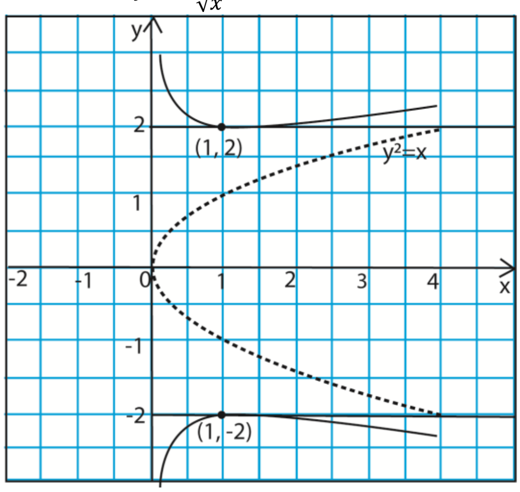
5. Determine the stationary points (including points of inflexion) of the curve $y = \frac{x}{x^2+1}$.

   Sketch the curveSketch

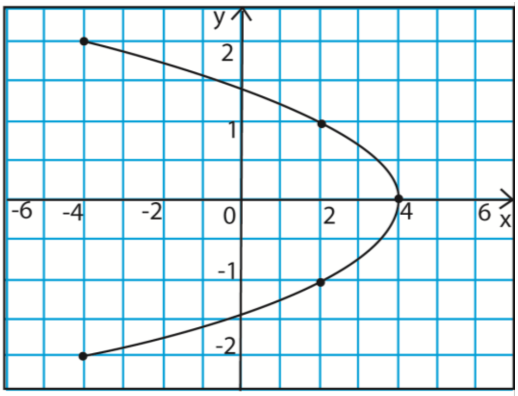


6. Sketch the curve give by the following parametric equations x= t^2 and $y = t + \frac{1}{t}$.

   A graph of $y = \frac{x+1}{\sqrt{x}}$

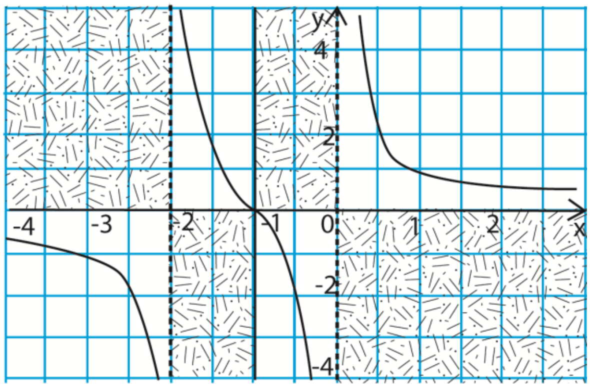


7. A curve is given by the parametric equations x= 4cos2t and y = 2sint
   (i) Find the equation of the normal to the curve at t = $\frac{5\pi}{6}$ [y = 4x – 7]
   (ii) Sketch the curve for $-\frac{\pi}{2} < t < \frac{\pi}{2}$

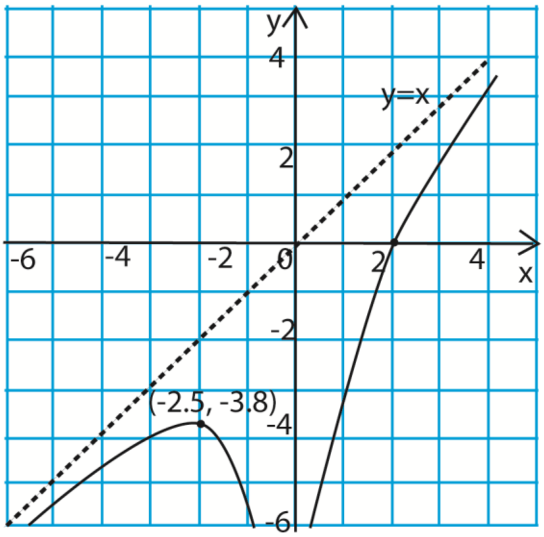


   (iii) Find the area enclosed by the curve and the y-axis [7.543 units (3d,p)]

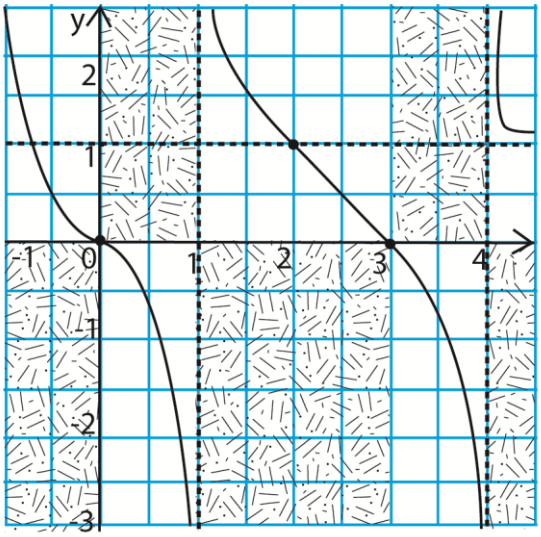
8. Sketch the curve $y = \frac{x+1}{x^2+2x}$



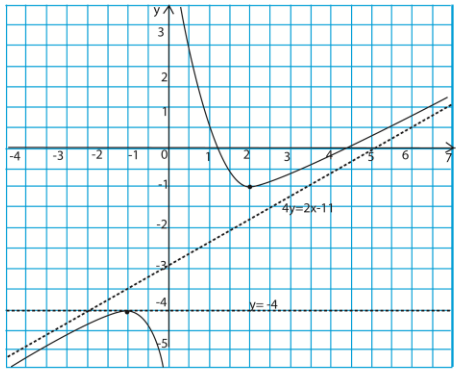
9. Sketch the curve =$y = x - \frac{8}{x^2}$



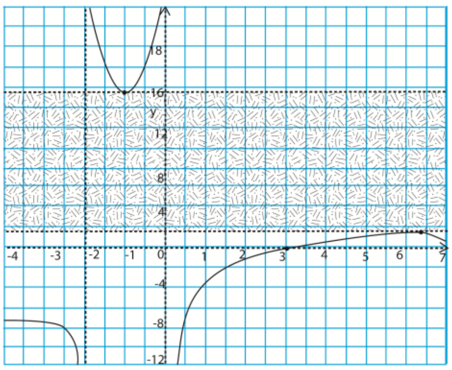
10. Given the curve $y = \frac{x(x-3)}{(x-1)(x-4)}$
   (i) Show that the curve does not have turning points $\left[\frac{dy}{dx} = 0; has\ no\ roots)\right]$
   (ii) Find the equations of asymptotes. Hence sketch the graph



11. Determining the nature of the turning points of the curve $y = \frac{x^2-6x+5}{2x+1}$, sketch the graph of the curve for x= -2 to x=7. Show any asymptotes.



12. Sketch the curve $y = \frac{4(x-3)}{x(x+2)}$



Thank you

Dr. Bbosa Science