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Coordinate geometry 1

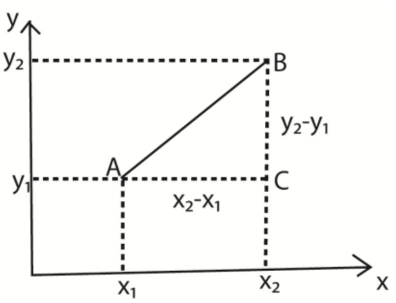
This is the area of mathematic where geometrical relationships are described algebraically by reference to the coordinates

The length of the line segment

Given two points A(x1, y1) and B(x2, y2) in x-y plane, the distance between A and B, denoted by $\overline{AB}$ is $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Proof

Geometrical approach



Using Pythagoras theorem

$\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$

$\overline{AB}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Vector method approach

AB = OB – OA

$= \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$

$= \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$

Using Pythagoras theorem

$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1

(a) Find the between the points

(i) A(1, 3) and B(7, 11)

Solution

Using $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{AB} = \sqrt{(7-1)^2 + (8-3)^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{100}$ = 10

(ii) P(-1, 2) and Q(3, 7)

Solution

$\overline{PQ} = \sqrt{(-1-3)^2 + (2-7)^2}$

$= \sqrt{(-4)^2 + (-5)^2}$

$= \sqrt{25}$ = 5

(b) The points A, B and C have coordinates A(-3, 2), B(-1, -2) and C(0, n) where n is a constant. Given that $\overline{BC} = \frac{1}{5}AC$, find the possible values of n.

Solution

Using $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\overline{BC} = \sqrt{(0-(-1))^2 + (n-(-2))^2}$

$= \sqrt{1^2 - (n+2)^2}$

$=\sqrt{n^2 + 4n + 5}$

$\overline{AC} = \sqrt{(0-(-3))^2 + (n-2)^2}$

$= \sqrt{3^2 - (n-2)^2}$

$=\sqrt{n^2 - 4n + 13}$

Bu t$\overline{BC} = \frac{1}{5}AC$

⇨ $\sqrt{n^2 + 4n + 5} = \frac{1}{5}\sqrt{n^2 - 4n + 13}$

$5\sqrt{n^2 + 4n + 5} = \sqrt{n^2 - 4n + 13}$

Squaring both sides

$25(n^2 + 4n + 5) = n^2 - 4n + 13$

$24n^2 + 104n + 112 = 0$

$3n^2 + 13n + 14 = 0$

$(3n + 7)(n + 2) = 0$

Either n = $-\frac{2}{3}$ or n = -2

Hence the values of n are $-\frac{2}{3}$ and -2

To show that given points are vertices of a right-angle triangle.

Suppose that the points A, B and C are vertices of a triangle ABC, to show that ABC is a right angled triangle, then by applying the Pythagoras theorem

Either $\overline{AB}^2 = \overline{AC}^2 + \overline{BC}^2$
or $\overline{BC}^2 = \overline{AB}^2 + \overline{AC}^2$

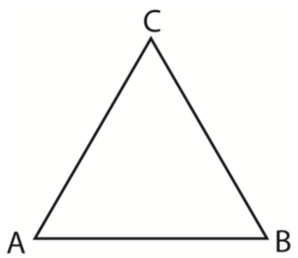
or $\overline{AC}^2 = \overline{AB}^2 + \overline{BC}^2$

Example 2

Prove that the following points are vertices of a right-angled triangle

(a) A(2, 3), B(5, 6) and C(8, 3)

Solution



$\overline{AC}^2 = (8-2)^2 + (3-3)^2 = 36$

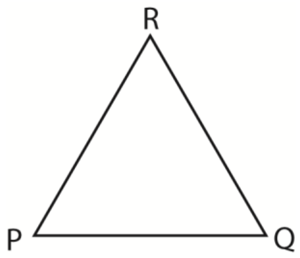
$\overline{AB}^2 = (5-2)^2 + (6-3)^2 = 18$

$\overline{BC}^2 = (8-5)^2 + (3-6)^2 = 18$

Since $\overline{AB}^2 + \overline{BC}^2 = \overline{AC}^2 = 36$, the triangle ABC is a right angled triangle

(b) P(2, 1), Q(5, -1) and R(9, 5)

Solution



$\overline{P2}^2 = (5-2)^2 + (-1-1)^2 = 13$

$\overline{PR}^2 = (9-2)^2 + (5-1)^2 = 65$

$\overline{QR}^2 = (9-5)^2 + (5-(-1))^2 = 52$

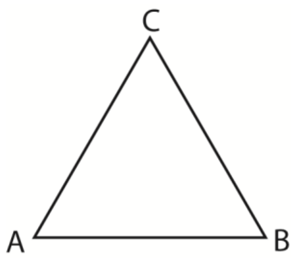
Since $\overline{PQ}^2 + \overline{QR}^2 = \overline{PR}^2 = 65$, the triangle PQR is a right angled triangle

Note: if the triangle is **isosceles**, then two of the sides must be equal and for **equilateral** triangle all the sides must be equal

Example 3

Prove that the following points A(1, 2), B(3, 7) and C(6, 14) are vertices of an isosceles triangle.

Solution



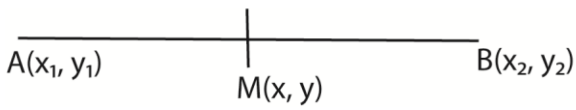
$\overline{AC}^2 = (6-1)^2 + (14-2)^2 = 169$

$\overline{AB}^2 = (13-1)^2 + (7-2)^2 = 169$

$\overline{BC}^2 = (6-13)^2 + (14-7)^2 = 98$

Since $\overline{AC}^2 = \overline{AB}^2 = 169$, hence, the triangle ABC, is isosceles triangle.

The mid-point of a line segment



The mid-point , M of a line segment AB with A(x_1, y_1) and B(x_2, y_2) is given as

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example 4

(a) Find the coordinates of the midpoint of the line joining each of the following pairs of points

(i) A(8, 4) and B(2, -4)

Solution

The midpoint of AB = $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

= $M\left(\frac{8+2}{2}, \frac{4-4}{2}\right)$

= M(5, 0)

(ii) P(-6, -2) and Q(-4, -5)

Solution

The midpoint of PQ = $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

= $M\left(\frac{-6-4}{2}, \frac{-2-5}{2}\right)$

= M(-5, -3.5)

(b) Find the coordinated of point S given that M(3, -2) is the midpoint of the straight line joining S to T(9, -2)

Solution

The midpoint of PQ = $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

⇨ $\frac{x+9}{2} = 3$

$x = -3$

Also $\frac{y-2}{2} = -2$

y = -2

Hence S(-3, -2)

Exercise 1

1. Find the distance between each of the following pairs of points
   (a) (-7, 3) and (-2, 5) $[\sqrt{29}]$
   (b) (2, -3) and (7, 7) $[5\sqrt{5}]$
   (c) (4, -1) and (-2, 1) $[2\sqrt{10}]$
2. Prove that a triangle with vertices (1, 2), (13, 7) and (6, 14) is isosceles.
3. Find the midpoint of the following point
   (a) (2, 1) and (4, 5) [3, 3]
   (b) (-1, 4) and (3, 1) [1, 3]
   (c) (-2, 6) and (0, 2) [-1, 4]
4. Prove that the points A(-2, 0), B(0, $2\sqrt{5}$) and C(2, 0) are vertices of an equilateral triangle.
5. The points L, M and N have coordinates (3, 1), ((2, 6) and (x, 5) respectively. Given that the distance LM is equal to the distance MN, calculate the possible values of x. [-3 or 7]
6. Given that the distance between P(r, 4) and Q(2, 3) is equal to the distance between R(3, -1) and S(-2, 4). Calculate the possible value of r. [-5 or 7]
7. A triangle has vertices A(6, 2), B(x, 6) and C(-2, 6). Given that the triangle is isosceles with AB = BC, Calculate the value of x. [3]
8. F(5, 1), G(x, 7) and H(8, 2) are vertices of a triangle. Given that the length of the side FG is twice the length of side FH, find the value of x. [3 or 7]
9. Given that the distance from A(13, 10) to B(1, y) is three times the distance from B to C(-3, -2), find the value of y. [1 or -8]
10. M(6,5) is the midpoint of a straight line joining the point A to point B, find the coordinates of B [10, 7]

Gradient of a straight line

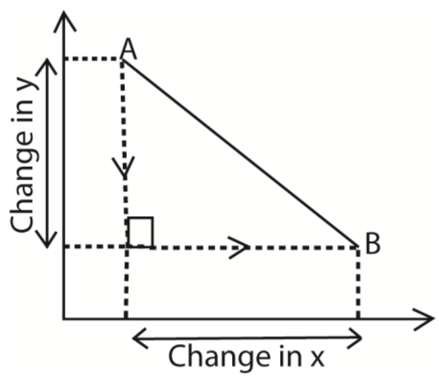
The gradient of a line joining points A(x1, y1) and B(x2, y2) is the measure of steepness of the line AB and it is a ratio of the change in y-coordinate to the change in x-coordinate.

i.e., gradient $=\frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$

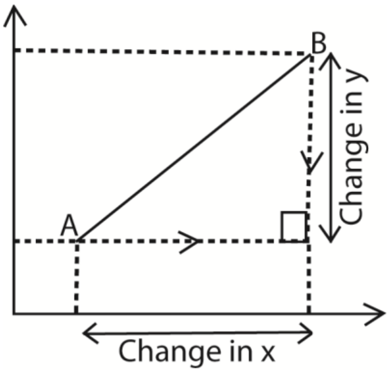
The gradient is usually denoted by m which may be positive or negative

Note:

(i) If the line slopes downwards from left to right, the gradient

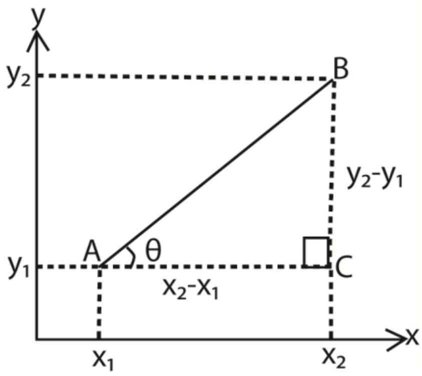


(ii) On the other hand, if the line slopes upwards from left to right, the gradient is positive.



Angle of a straight line to the horizontal

Suppose that the line in the second illustration makes θ is the horizontal as shown below



From trigonometry, $\tan\theta = \frac{BC}{AC}$

$= \frac{y_2-y_1}{x_2-x_1}$

$$\theta = \tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right)$$

This means that if m = $\frac{y_2-y_1}{x_2-x_1}$

$$\theta = \tan^{-1} m$$

Example 5

(a) Find the gradient of the straight line joining each of the following pairs of points.

(i) A(7, 4) and B(-1, -2)

Solution

m = $\frac{y_2-y_1}{x_2-x_1} = \frac{-2-4}{-1-7} = \frac{-6}{-8} = \frac{3}{4}$

(ii) A(-3, -2) and B(-4, -5)

Solution

m = $\frac{y_2-y_1}{x_2-x_1} = \frac{-5-(-2)}{-4-(-3)} = \frac{-3}{-1} = 3$

(b) Find the angle which the straight line joining each of the following pairs of points makes with the horizontal

(i) A(5, 4) and B(6, 8)

Solution

Angle $\theta = \tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right)$

$= \tan^{-1}\left(\frac{8-4}{6-5}\right)$

$= 76^0$

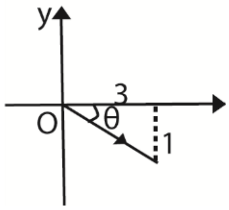
(ii) A(-3, -5) and (-4, -2)

Angle $\theta = \tan^{-1}\left(\frac{y_2-y_1}{x_2-x_1}\right)$

$= \tan^{-1}\left(\frac{-2-(-5)}{-4-(-2)}\right)$

$= -71.6^0$

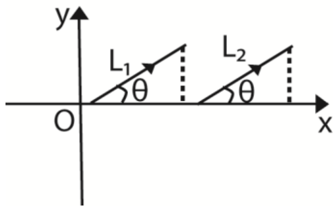
i.e.



Hence the angle which AB makes with the horizontal is 71.6^0 with positive x-axis downwards as shown in the diagram.

Gradient of parallel lines

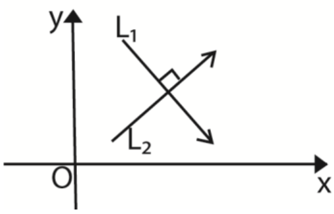
If two are parallel, their gradients are equal.



In the diagram above L_1 is parallel to L_2 and both L_1 and L_2 have the same gradient.

Gradient of perpendicular lines

If two lines are perpendicular, the product of their gradients is -1.



If m_1 and m_2 are the gradients of L_1 and L_2 respectively, then $m_1 \times m_2 = -1$

Example 6

(a) Given the points A(2, 3), B(5, 5), C(7,2) and D(4, 0)

(i) Prove that AB is parallel to DC

Gradient of AB, $m_1 = \frac{5-3}{5-2} = \frac{2}{3}$

Gradient of DC, $m_2 = \frac{2-0}{7-4} = \frac{2}{3}$

Since the gradient of AB and DC are equal, the lines AB and DC are parallel

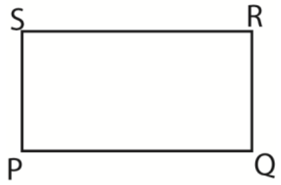
(ii) Prove that AC is perpendicular to BD

Gradient of AC, $m_1 = \frac{2-3}{7-2} = \frac{-1}{5}$

Gradient of BD, $m_2 = \frac{0-5}{4-5} = 5$

Since $m_1 \times m_2 = -1$, AC is perpendicular BD

(b) Prove that the point P(1, 3), Q(3, 4), R(5, 0) and S(3, -1) form a parallelogram.



Gradient of PQ, $m_1 = \frac{4-3}{3-1} = \frac{1}{2}$

Gradient of SR, $m_2 = \frac{0-(-1)}{5-3} = \frac{1}{2}$

∴ PQ and SR are parallel

$$\overline{PQ} = \sqrt{(3-1)^2 + (4-3)^2} = \sqrt{5}$$

$$\overline{SR} = \sqrt{(5-3)^2 + (0-(-1))^2} = \sqrt{5}$$

∴ PQ and SR are equal

Gradient of PS, $m_1 = \frac{-1-2}{3-1} = -2$

Gradient of QR, $m_2 = \frac{0-4)}{5-3} = -2$

∴ PS and QR are parallel

So the figure PQRS could either be a rectangle of parallelogram

For a rectangle

PQ and PS are perpendicular, thus the product of their gradients = -1

Since the gradient of PQ and x gradient of PS $= \frac{1}{2}x - 2 = -1$

Hence the figure PQRS is a rectangle not a parallelogram.

(c) The quadrilateral ABCD has vertices A(-2, -3), B(1, -1), C(7, -10) and D(2, -9).

(i) Prove that Ad is parallel to BC

Gradient of AD, $m_1 = \frac{-9-(-3)}{2-(-2)} = \frac{-3}{2}$

Gradient of BC, $m_2 = \frac{-10-(-1)}{7-1} = \frac{-3}{2}$

Since the gradient of AD and BC are equal, the lines AD and BC are parallel

(ii) Prove that AD is perpendicular to BC

Gradient of AD, $m_1 = \frac{-1-(-3)}{1-(-2)} = \frac{2}{3}$

Gradient of BC, $m_2 = \frac{-10-(-1)}{7-1} = \frac{-3}{2}$

Since $m_1 \times m_2 = -1$, AD is perpendicular BC

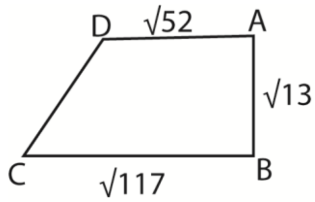
(iii) Prove that the area of the quadrilateral ABCD is 32½ sq. units

$$\overline{AB} = \sqrt{(1-(-2))^2 + (-1-(-3))^2}$$
$$= \sqrt{13}$$

$$\overline{BC} = \sqrt{(7-1)^2 + (-10-(-1))^2}$$
$$= \sqrt{117}$$

$$\overline{AD} = \sqrt{(2-(-2))^2 + (-9-(-3))^2}$$
$$= \sqrt{52}$$

Since all the sides of a quadrilateral are different, the figure is a trapezium



$$Area = \frac{1}{2}\sqrt{13}(\sqrt{52} + \sqrt{117})$$

$$= 32\frac{1}{2}\ sq.units$$

(d) A quadrilateral ABCD has vertices A(-2, 1), B(0, 4), C(3, 2) and D(1, -1)

(i) Prove that all sides of the quadrilateral have the same length.

Solution

$$\overline{AB} = \sqrt{(0-(-2))^2 + (4-1)^2}$$
$$= \sqrt{13}$$

$$\overline{BC} = \sqrt{(3-0)^2 + (2-4)^2}$$
$$= \sqrt{13}$$
$$\overline{CD} = \sqrt{(1-3)^2 + (-1-2)^2}$$
$$= \sqrt{13}$$

$$\overline{AD} = \sqrt{(-2-1)^2 + \left(1-(-2)\right)^2}$$
$$= \sqrt{13}$$

Hence all the sides of the quadrilateral are equal in length

(ii) Prove that AB is parallel to AD

Gradient of AB, $m_1 = \frac{4-1)}{0-(-2)} = \frac{3}{2}$

Gradient of DC, $m_2 = \frac{2-(-1)}{3-1} = \frac{3}{2}$

Since the gradient of AB and DC are equal, the lines AB and DC are parallel

(iii) Prove that AD is parallel to BC

Gradient of AD, $m_1 = \frac{-1-1)}{1-(-2)} = \frac{-2}{3}$

Gradient of BC, $m_2 = \frac{2-4}{3-0} = \frac{-2}{3}$

Since the gradients of AD and BC are equal, the lines AD and BCC are parallel

(iv) What is the name of the quadrilateral ABCD?

It could be a square or a rhombus

For square, gradient of AB x gradient of BC $=\frac{-2}{3} x \frac{3}{2} =$ -1

Hence the quadrilateral is a square.

Exercise 2

1. Find the gradient of the straight line of each of the following pairs of points.
   - (a) (-2, 5) and (5, -3) $\left[\frac{-8}{7}\right]$
   - (b) (3, 7) and (7, -4) $\left[\frac{-11}{4}\right]$
   - (c) (6, 3) and (7, 4) [1]
2. Find the angle between a line joining the following points with the horizontal
   - (a) (2, 5) and (-3, -2) [54.46^0]
   - (b) (3, 7) and (-6, 11) [-23.96^0]
   - (c) (5, -3) and (5, 2) [90^0]
3. A triangle has vertices A(3, -2), B(2, -14) and C(-2, -4).Find the gradients of the straight lines AB, BC and CA. Hence prove that the triangle is right-angled $\left[12, \frac{-5}{2}, \frac{2}{5}; hence\ BC\ and\ CA\ are\ perpendicular\right]$
4. The straight line joining the points P(6, 5) to Q(q, 2) is perpendicular to the straight line joining point Q to R(9, -1). Find the value of q. [3, 11]
5. Prove that the quadrilateral PQRS with vertices P(-1, 3), Q(2, 4), R(4, -2) and S(1, -3) is a rectangle and calculate the area [20 sq. units]
6. The four points A(5, 4), B(6,2), C(12, 5) and D(11, 7) are vertices of a quadrilateral. Prove that the quadrilateral is a rectangle and calculate its area. [15 sq. units]
7. Prove that the points A(2, 3), B(4, 8), C(8, 9) and D(4, -1)for a trapezium.
8. The quadrilateral ABCD has vertices S(1, 1), T(4, 5), U(12, -1) and V(1, -1) are vertices of a quadrilateral STUV.
   - (a) Prove that ST is perpendicular to TU, and that SV is perpendicular to UV.
   - (b) Calculate the length of each of the sides ST, TU, UV, and VS. [5, 10, 11, 2]
   - (c) Prove that the area of the quadrilateral STUV is 36 square units.
9. The quadrilateral CDEF has vertices, C4, 0), D(8, 4), E(2, -8) and F(0, 2). The points P, Q, R and S are midpoints of the sides CD, DE, EF and FC respectively. Prove that the Quadrilateral PQRS is a rhombus and show that its area is 15 sq. units.

Equation of a straight line

The general equation of a straight line is given by y = mx + c, where m = gradient of the line i.e. m $=\frac{y_2-y_1}{x_2-x_1}$ and c = y-intercept. For lines passing through the origin, c = 0; hence y = mx.

Example 7

(a) Find the equation of with gradient = 5 and passes through the point

(i) A(2, 5)

Solution

Method 1

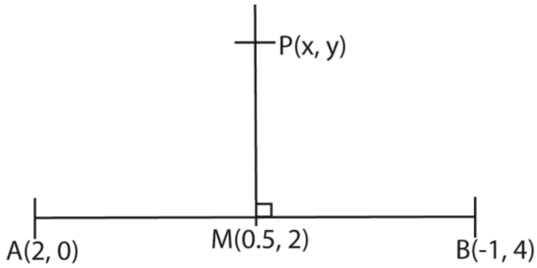
The general equation of a line is y = mx + c

Substituting for m = 5 and points of A
5 = 5(2) + c
c = -5
Hence the line is y = 5x – 5
Method 2
Let B(x, y) lie on the line
From m =$\frac{y_2-y_1}{x_2-x_1}$
5 =$\frac{y-5}{x-2}$
y – 5 = 5(x – 2)
y = 5x – 5

(ii) P(2, 7)
Using y = mx + c
Substituting for m = 5 and points of P
7 = 5(2) + c
c = 17
Hence the line is y = 5x +17

(b) Find the equation of a straight line joining the following points
(i) A(0, 5) and B(3, 4)
Solution
Gradient = $\frac{4-5}{3-0} = \frac{-1}{3}$
Substituting coordinates for A in general equation
5 =$\frac{-1}{3}(0) + c$ => C = 5
Hence equation of the line is y = $\frac{-1}{3}x + 5$
(ii) P(-2, -5) and Q(3, -7)
Solution
Gradient = $\frac{-7-(-5)}{3-(-2)} = \frac{-2}{5}$
Substituting coordinates for P in general equation
-5 =$\frac{-2}{5}(-2) + c$ => C = -5 - $\frac{4}{5}$ = -$\frac{29}{5}$
Hence equation of the line is y = $\frac{-2}{5}x - \frac{29}{5}$
(c) Find the equation of a perpendicular bisector of the line joining the following points
(i) (2, 0) and (-1, 4)
Solution
Let M be the midpoint of AB
⇨ M($\frac{2-1}{2}$, $\frac{0+4}{2}$ =M(0.5, 2)

Let P(x, y) lie on the perpendicular bisector



Gradient of AB, $m_1 = \frac{4-0}{-1-2} = \frac{4}{-3}$

Gradient of MP, $m_2 = \frac{y-2}{x-0.5}$

But $m_1 \times m_2 = -1$

$\frac{4}{-3}\left(\frac{y-2}{x-0.5}\right) = -1$

4(y – 2) = 3(x – $\frac{1}{2}$)

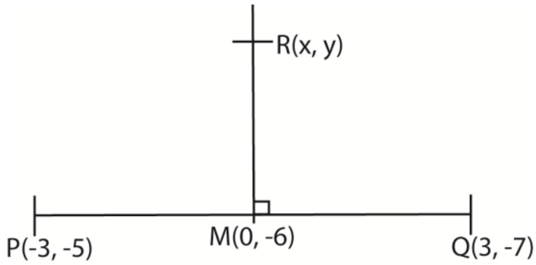
8y -16 = 6x – 3

8y – 6x = 13

(ii) P(-3, -5) and Q(3, -7)

Solution
Let M be the midpoint of AB
⇨ M($\frac{-3+3}{2}$, $\frac{-5-7}{2}$ =M$(0, -6)$

Let R(x, y) lie on the perpendicular bisector



Gradient of PQ, $m_1 = \frac{8-(-7)}{-3-3} = \frac{15}{-6} = \frac{5}{-2}$

Gradient of MR, $m_2 = \frac{y-(-6)}{x-0} = \frac{y+6}{x-0}$

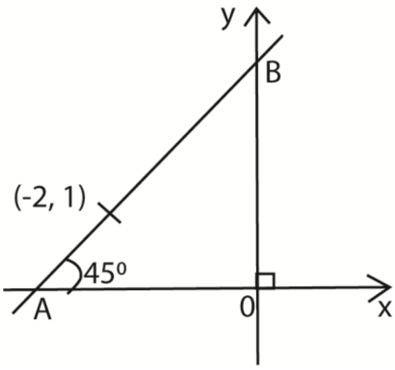
But $m_1 \times m_2 = -1$

$\frac{5}{-2}\left(\frac{y+6}{x-0}\right) = -1$

5(y+6) =2x
5y – 2x + 30 = 0

(d) A straight line, L passes through point (-2, 1) and makes an angle of 45^0 with the horizontal.

(i) Find the equation of the line L.
Solution



Gradient of the line L = tan 45^0 = 1

⇨ m = 2

Substituting the coordinates of the point in the general equation, y = mx + c

1 = 1(-2) + c => that c = 3

Hence the equation of the line L is y = x + 3

(ii) Given that L intersects the x-axis at A and the y-axis at B find the distance AB
Solution
At A, y = 0; x = -3
Hence coordinates of A(-3, 0)
At B x = 0 y = 3
Hence coordinates of B(0, 3)
$\overline{AB} = \sqrt{(0-(-3))^2 + (3-0)^2}$
$= 3\sqrt{2}$ units

Perpendicular distance of a point to a line
Just like in vectors, the shortest distance of a point from a given line is the perpendicular distance of a point from the line
Suppose that the equation of the line is in the form ax + by + c = 0
The perpendicular distance of a point $P(x_1, y_1)$=d $=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Example 8

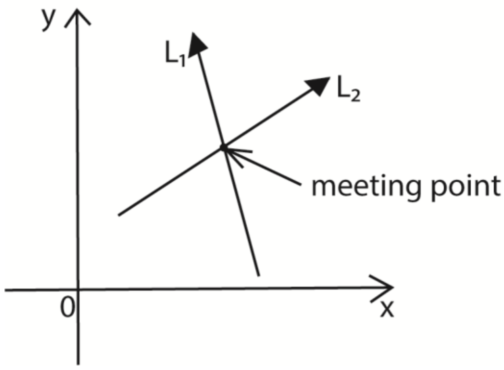
Find the perpendicular distance of the following points from the given lines

(a) (-2, 6) from x + y + 4 = 0
Solution
a =1, b = 1 and c = 4
Substituting for (x, y) = (-2, 6)
d $=\left|\frac{1(-2)+1(6)+4}{\sqrt{1^2+1^2}}\right| = \frac{8}{\sqrt{2}}=4\sqrt{2}$units

(b) (7, -4) from 3x – 5y = 7
Solution
Rearranging the equation
3x – 5y – 7 = 0
a =3, b = -5 and c = -7
Substituting for (x, y) = (7, -4)
d $=\left|\frac{3(7)-5(-4)-7}{\sqrt{3^2+(-5)^2}}\right| = \frac{34}{\sqrt{34}}=\sqrt{34}$units

(c) (8, -5) from y = 3x -1
Solution
Rearranging the equation
3x – y – 1 = 0
a =3, b = **-1** and c = -1
Substituting for (x, y) = (8, -5)
d $=\left|\frac{3(8)-1(-5)-1}{\sqrt{3^2+(-1)^2}}\right| = \frac{28}{\sqrt{10}}=8.85$units

Intersection of two lines
The point of intersection of two or more lines is obtained by solving the equations of the lines simultaneously.



Example 9

(a) Find the coordinates of the point of intersection of each of the following pairs of straight lines
(i) y = x + 3 and y = 4x + 6
Solution
Let y = x + 3 ……………………(1)
And y = 4x + 6 …………………(2)
Equating (1) and (2)
x + 3 = 4x + 6; x = -1
substituting x I (1)
y = 4(-1) + 6; y = 2
Hence the point of intersection is (-1, 2)
(ii) 2x - 3y = 7 and 3x – 7y = 13

Let 2x – 3y = 7 ……………………(1)
And 3x – 7y = 13 …………………(2)
Equating (1) and (2)
x + 3 = 4x + 6; x = -1
3eqn,(1) – 2eqn. (2)
5y = -5 => y = -1
Substituting for y into eqn. (1)
2x – 3(-1) = 7; x = 2
Hence the point of intersection (2, -1)

(b) (i) Find the equation of the straight line L, which passes through the point P(2, 4) and perpendicular to the line 5y + x = 7
Solution
Let Q(x, Y) lie on the same line
Gradient of PQ, $m_1 = \frac{y-4}{x-2}$
From5y + x = 7; $y = -\frac{1}{5}x + \frac{7}{5}$
Gradient, $m_2 = -\frac{1}{5}$
But for perpendicular lines, $m_1 \times m_2 = -1$
⇨ $-\frac{1}{5}\left(\frac{y-4}{x-2}\right) = -1$
$y - 4 = 5(x - 2)$
$y = 5x - 6$
(ii) Given that the line L meets line y = x + 6 at point S. find the coordinates of S.
Solution
At S the two equations are equal
⇨ $5x - 6 = x + 6$; x = 3
Substituting x in y = x + 6
y = 3 + 6 = 9
Hence coordinates of S are (3, 9)

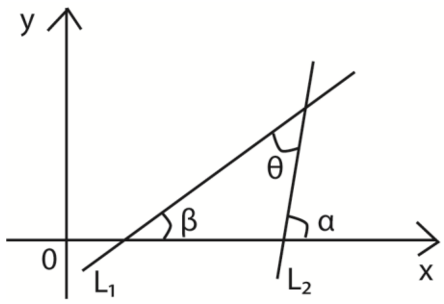
(c) The line L has equation 2x – y – 1 = 0. The line M passes through point A(0, 4) and is perpendicular to the line L. The line N passes through point B(3, 0) and is parallel to M.
(i) Find an equation of M and show that he line L and M intersect at the point A(0, 4).
Solution
Let K(x, y) lie on line M
Gradient of AK, $m_1 = \frac{y-4}{x-0} = \frac{y-4}{x}$
From 2x – y = 1; y = 2x -1
Gradient, $m_2 = 2$
For perpendicular line $m_1 \times m_2 = -1$
⇨ $2\left(\frac{y-4}{x}\right) = -1$
2(y – 4) = -x
2y + x = 8
At the point, P of intersection, the two lines are equal
2x – 1 = y ……………………….(1)
2y + x = 8 ………………………. (2)
Substituting for y in eqn. (1) into equation (2)
2(2x-1) + x = 8
x = 2
Substituting for x in eqn. (1)
y = 2(2) – 1 = 3
Hence point of intersection P (2, 3)

(ii) Find an equation of N and hence find the coordinates of point Q where the Line L and Line N intersect
Let D (x, y) lie on N
Gradient BD, $m1 = \frac{y-0}{x-3} = \frac{y}{x-3}$
From line 2y + x = 8
$y = -\frac{1}{2} + 4$
Gradient $m_2 = -\frac{1}{2}$
For parallel lines $m_1 = m_2$.
⇨ $-\frac{1}{2} = \frac{y}{x-3}$
$2y = 3 - x$
2y + 3x = 3
At point Q, Line L = line N
2x – 1 = y …………………..(1)
2y + x = 3 ……………………(2)
Substituting for y in eqn. (2)
2(2x -1) + x = 3
5x – 2 = 3
x = 1
Substituting for x in eqn. (1)
y = 2(1) -1 = 1
hence Q(1, 1)

(iii) Prove that $\overline{AP} = \overline{BQ} = \overline{PQ}$

$$\overline{AP} = \sqrt{(2-0)^2 + (3-4)^2} = \sqrt{5}$$
$$\overline{BQ} = \sqrt{(1-3)^2 + (1-0)^2} = \sqrt{5}$$
$$\overline{PQ} = \sqrt{(1-2)^2 + (1-3)^2} = \sqrt{5}$$
Hence $\overline{AP} = \overline{BQ} = \overline{PQ}$

Angle between two lines
Suppose that two lines y = m1x + c1 and y = m2x + c2 are inclined at angle α and β respectively as shown below



The sum of two interior angles = opposite exterior angle

⇨ θ + β = α

θ = α – β

tan θ = tan (α – β)

$= \frac{tan\alpha - tan\beta}{1 + tan\alpha tan\beta}$

$= \frac{m_1 - m_2}{1 - m_1 m_2}$

$\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

Hence the acute angle between two intersecting lines is $\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

Note that the angle between parallel lines is zero while that between perpendicular line is 90^0.

Example 10

(a) Find the acute angle between the following pairs of lines

(i) 4y – 3x = 6 and 2y + x = 3

Solution

For 4y – 3x = 6

$y = \frac{3}{4}x + \frac{6}{4}$ => $m_1 = \frac{3}{4}$

For 2y + x = 3

$y = -\frac{1}{2}x + \frac{3}{2}$ => $m_2 = -\frac{1}{2}$

From $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

$\theta = \tan^{-1}\left(\frac{\frac{3}{4} - \left(-\frac{1}{2}\right)}{1 + \frac{3}{4}.\left(-\frac{1}{2}\right)}\right) = \tan^{-1}\left(\frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{8}}\right)$

$= \tan^{-1} 2 = 63.4^0$

(ii) 2x – 5y = 15 and 3y + 2x = 6

Solution

For 2x – 5y = 15

$y = \frac{2}{2}x + 3$ => $m_1 = \frac{2}{5}$

For 3y + 2x = 6

$y = -\frac{2}{3}x + 2$ => $m_2 = -\frac{2}{3}$

From $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

$\theta = \tan^{-1}\left(\frac{\frac{2}{5} - \left(-\frac{2}{3}\right)}{1 + \frac{2}{5}.\left(-\frac{2}{3}\right)}\right) = \tan^{-1}\left(\frac{\frac{2}{5} + \frac{2}{3}}{1 - \frac{4}{15}}\right)$

$= \tan^{-1}\left(\frac{16}{11}\right) = 46.8^0$

(iii) y = 4 and 3y + 2x – 6 = 0

Solution

For y = 4, => $m_1 = 0$

For 3y + 2x – 6 = 0

$y = -\frac{2}{3}x - 2$ => $m_2 = -\frac{2}{3}$

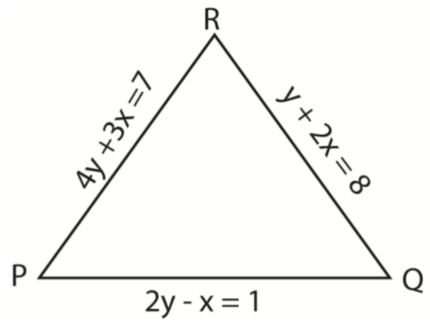
From $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

$\theta = \tan^{-1}\left(\frac{0 - \left(-\frac{2}{3}\right)}{1 + 0.\left(-\frac{2}{3}\right)}\right) = \tan^{-1}\left(\frac{\frac{2}{3}}{1}\right)$

$= \tan^{-1}\left(\frac{2}{3}\right) = 33.7^0$

(b) Calculate the area of the triangle which has sides given by the equations 2y – x = 1, y + 2x = 8 and 4y + 3x = 7

Solution



At point P

4y + 3x = 7 ……………………………….(1)

2y – x = 1 …………………………………..(2)

Eqn. (1) – 2eqn.(2)

10y = 10; y = 1

Substituting for y in eqn. (2)

x = 2(1) – 1 = 1

Hence P(1, 1)

At point Q

y + 2x = 8 ……………………………….(1)

2y – x = 1 …………………………………..(2)

Eqn. (1) + 2eqn.(2)

5y = 10; y = 2

Substituting for y in eqn. (2)

x = 2(2) – 1 = 3

Hence Q(3, 2)

At point R

y + 2x = 8 ………………………………(1)

4y + 3x = 7 ……………………………..(2)

3Eqn. (1) - 2eqn.(2)

-5y = 10; y = -2

Substituting for y in eqn. (1)

2x = 8 + 2 =10 => x = 5

Hence r(-2, 5)

Finding dimensions

$\overline{PR} = \sqrt{(5-1)^2 + (-2-1)^2} = 5$

$\overline{PQ} = \sqrt{(3-1)^2 + (2-1)^2} = \sqrt{5}$

$\overline{QR} = \sqrt{(5-3)^2 + (-2-2)^2} = 2\sqrt{5}$

Finding <QPR

For 4y + 3x = 7

y = $-\frac{3}{4}x + \frac{7}{4}$ => $m_1 = -\frac{3}{4}$

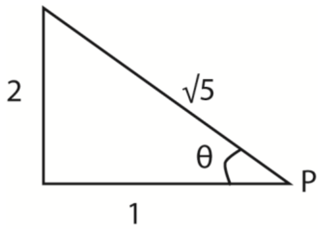
For 2y – x = 1

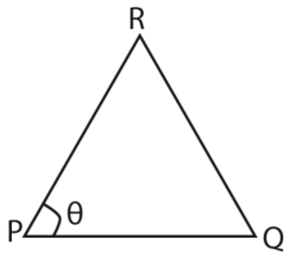
y = $\frac{1}{2}x + \frac{1}{2}$ => $m_2 = \frac{1}{2}$

From θ = $\tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

<QPR = $\tan^{-1}\left(\frac{\frac{1}{2} - \left(-\frac{3}{4}\right)}{1 + \frac{1}{2}\cdot\left(-\frac{3}{4}\right)}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{3}{8}}\right)$

$= \tan^{-1}(2)$





Method 1 using sine rule

Area of PQR = $\frac{1}{2} x PQ\ x\ PC\ sin\theta$

$= \frac{1}{2} x\sqrt{5}\ x\ 5x\ \frac{2}{\sqrt{5}} = 5$sq. units

Method 2 using Heron's formula

Area of PQR = $\sqrt{s(s-a)(s-b)(s-c)}$

Where a, b, and c are sides of a triangle and

s = $\frac{1}{2}(a + b + c)$

s = $\frac{1}{2}(5 + \sqrt{5} + 2\sqrt{5} = \frac{5+3\sqrt{5}}{2}$

Area = $\sqrt{\left(\frac{5+3\sqrt{5}}{2}\right)\left(\frac{3\sqrt{5}-5}{2}\right)\left(\frac{5+\sqrt{5}}{2}\right)\left(\frac{5-\sqrt{5}}{2}\right)}$

= 5sq. units

Exercise 3

1. Find the gradient of each of the following straight lines
   (a) y = 4x -2 [4]
   (b) y = 2x + 3 [3]
   (c) y = 2 – 5x [-5]
   (d) $\frac{y}{2} - \frac{x}{5} = 4$ $\left[\frac{2}{5}\right]$
2. Find the equation of the straight line that has the following properties
   (a) Gradient 1 and passes through (2, 4) [y = x + 2]
   (b) gradient$\frac{1}{4}$ and passes through (2, 5) $\left[y = \frac{1}{4}x + \frac{9}{2}\right]$
3. Find the equation of a straight line that has the following properties
   (a) Passes through (-2, 3) and parallel to y = 5x + 4 $[y = 5x + 13]$
   (b) Passes through (6, -2 and is perpendicular to y = -3x + 4 $\left[y = \frac{1}{3}x - 4\right]$
   (c) Passes through $\left(-\frac{1}{3}, -\frac{2}{3}\right)$ and is perpendicular to 3y + 10x – 8 = 0 $[10y = 3x - 3]$
4. Find the equation of a straight line joining the following pairs of points
   (a) (2, 4) and (-1, 0) [3y = 4x +4]
   (b) (-4, 1) and(6, 2) [10y = x + 14]
   (c) (3, 4) and (-1, 4) [y =4]
5. Find the equation of the perpendicular bisector of the straight lines joining each of the following pairs of point
   (a) (5, 5) and (2, -2) [4y+ 2x = 11]
   (b) (-1, 4)and (3, 3) [2y – 8x + 1 = 0]
   (c) (3, 2) and (-4, 1) [y = -7x – 2]
6. Find the equation of a straight line:
   (a) L_1 which is perpendicular bisector of points A(-2, 3) and B(1, -5) [16y – 6x + 13 = 0]
   (b) L_2 which is a perpendicular bisector of the points B(1, -5) and (17, 1) [3y + 8x – 60 = 0]
   (c) Show that L_1 is perpendicular to L_2.

7. The perpendicular bisector of a straight line joining the points (3, 2) and (5, 6) meet the x-axis at A and they-axis at B. Prove that the distance AB is equal to $\sqrt{5}$.
8. A is a point (1, 2) and B is a point (7, 4). The straight line L_1 passes through B and is perpendicular to AB; the straight line L_2 passes through A and is also perpendicular to AB. The line L_1 meets the x-axis at P and the y-axis at Q. Line L_2 meets the x-axis at R and the y-axis at S.
   (a) Find the equations of each of L_1 and L_2. [y = -3x + 25, y = -3x + 5]
   (b) Calculate the area of the triangle OPQ. $\left[104\frac{1}{6}\right]$
   (c) Calculate the area of the triangle ORS. $\left[4\frac{1}{6}\right]$
   (d) Find the area of the trapezium PQSR. [100 sq. units]
9. P is the point with coordinates (2, 1) and L is the straight line which is perpendicular to OP and which passes through P.
   (a) Find the equation of L. [y = -2x + 5]
   (b) Given that line L meets the x-axis at A and y-axis at B. calculate
      (i) the area of the triangle OAP. [1.25]
      (ii) The area of the triangle OBP [5]
      (iii) Find the ratio of the area OAP to that of OBP [1:4]
10. Find the shortest distance between each of the following
   (a) The point (2, 4) and the line 3x – 4y + 8 = 0 $\left[\frac{2}{5}\right]$
   (b) The point (5, -1) and the line 12x + 5y – 3 = 0 [4]
   (c) The point (9, -3) and the line y = x. $[6\sqrt{2}]$
11. Find the coordinates of the point of intersection of each of the following pairs of straight lines
   (a) y = 2x + 3 and y = 4x + 1 [1, 5]
   (b) y = x + 3 and y = 4x + 6 [-1, 2]
   (c) 2x – 3y = 7 and 3x – 7y = 13 [2, -1]
   (d) x + 3y – 2 = 0 and 3x + 5y – 8 = 0 $\left[\frac{7}{2}, -\frac{1}{2}\right]$
12. Find the equation of the straight line L, which passes through the point (2, 4) and perpendicular to the line 5y + x = 7 [y = 5x – 6]
   (b) Given that the line L meets the line y = x + 6 at point S, find the coordinates of point S.[3, 9]
13. Calculate the area of the triangle which has sides given by the equations 2y – x = 1, y + 2x = 8 and 4y + 3x = 7 [5sq. units]
14. The point A has coordinates (2, -5). The straight line 3x + 4y – 36 = 0 cuts the x-axis at B and the y – axis at C. Find
   (a) The equation of the line through A which is perpendicular to the line BC. [4x – 3y = 23]
   (b) The perpendicular distance from the line BC. [10]
   (c) The area of the triangle ABC. [75 sq. units]

Locus

A locus is the set of all points in a plane that satisfy some condition. For example the locus of points equidistant from two given points say A and B is a perpendicular bisector of AB.

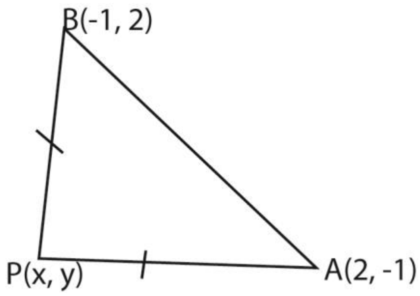
Locus can be expressed in terms of the Cartesian coordinates (x, y) of the form (r, θ)

Example 11

(a) Find the locus of point P(x, y) that is equidistant from the point A(2, -1) and B(-1, 2)

Solution

Method 1

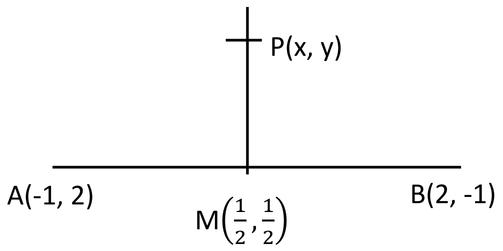


$\overline{AP} = \overline{PB}$ i.e. $\overline{AP}^2 = \overline{PB}^2$

$(x-2)^2 + (y+1)^2 = (x+1)^2 + (y-2)^2$

y = x

Method 2



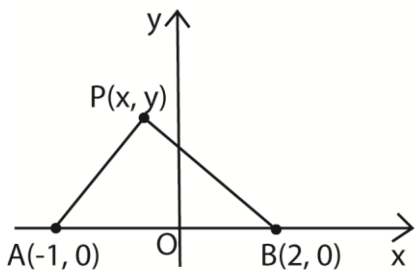
Gradient of AB = $\frac{2+1}{-1-2} = -1$

Since AB and MP are perpendicular, the product of their gradients = -1

Gradient of MP=$\frac{x-\frac{1}{2}}{y-\frac{1}{2}} = 1$

y = x

(b) Find the locus of a point which moves so that of the squares of its distance from points A(-2, 0) and B(2, 0) is 25 units.



$\overline{AP}^2 + \overline{BP}^2$ = 25

$(x+1)^2 + y^2 + (x-2)^2 + y^2$ = 25

$x^2 + y^2 = 25$

(c) The locus of P(x, y) is such that the distance OP is half the distance PR, where O is the origin and R is the point (-3, 6)

(i) Show that the locus of P describes a circle in the x-y plane

Solution

Given OP = $\frac{1}{2}PR$ i.e. $4OP^2 = PR^2$

$4(x^2+ y^2) = (x+ 3)^2 + (y - 6)^2$

$4x^2 + 4y^2 = x^2 + 6x + 9 + y^2 - 12y + 36$

$x^2 + y^2 - 2x + 4y - 15 = 0$ (a circle in x – y plane)

(ii) Determine the radius of the circle and the centre of the circle.

Solution

$x^2 + y^2 - 2x + 4y = 15$

After completing squares we have

$(x - 1)^2 + (y + 2)^2 = 20$

The centre of the circle is (1, -2) and the radius= $\sqrt{20} = 2\sqrt{5}$ units

(iii) Where does P cut the line x = 3?

Solution

Substituting x = 3 in the equation

$(x - 1)^2 + (y + 2)^2 = 20$

$((3 - 1)^2 + (y + 2)^2 = 20$

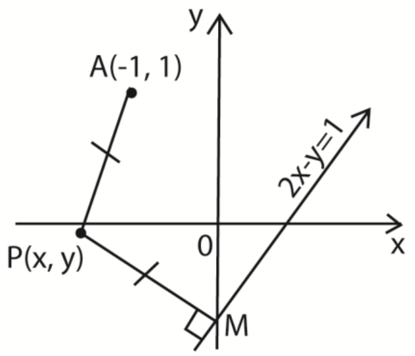
$(y + 2)^2 = 16$ i.e. $y + 2 = \pm 4$

$y = -2 \pm 4$ i.e. y = -6 or y = 2

∴P cuts the line x = 3 at the point (3, 2) and (3, -3)

(d) Find the locus of P(x, y) if its distance from A(-1, 1) is equal to the distance form the line 2x – y = 1

Solution



$\sqrt{(x+1)^2 + (y-1)^2} = \frac{|2x-y-1|}{\sqrt{(2^2+(-1)^2)}}$

⇨ $5[(x+1)^2 + (y-1)^2] = (2x-y-1)^2$

$5x^2+10x + 5 + 5y^2 - 10y + 5 = 4x(y+1)+ (y+1)^2$

$x^2 + 4y^2 + 4xy + 14x - 12y + 9 = 0$ is the locus

(e) A point R moves so that its distance from point (2, 0) is twice its distance from (0, -1). Show that the locus of R is a circle and determine its radius and its centre.

Solution

Let P (2, 0), Q(0, 1) and R (x, y)

Given PR = 2QR => $PR^2 = 4QR^2$

$(x - 2)^2 + y^2 = 4[x^2 + (y+ 1)^2]$

$x^2 - 4x + 4 + y^2 = 4x2 + 4y^2 + 8y + 4$

$3x^2 + 3y^2 + 4x + 8y = 0$ hence a circle

⇨ $x^2 + \frac{4}{3}x + y^2 + \frac{8}{3}y = 0$

$\left(x+\frac{2}{3}\right)^2 + \left(y+\frac{4}{3}\right)^2 = \frac{20}{9}$

The centre is at $\left(-\frac{2}{3},\frac{4}{3}\right)$ and the radius

$= \sqrt{\frac{20}{9}} = \frac{2\sqrt{5}}{3}$

(f) Find the locus of a point P(x, y) whose distance from the point A(3, -2) is always 5units.

Solution

AP = 5 => AP2 = 25

i.e. $(x - 3)^2 + (y + 5)^2 = 25$

$x^2 + y^2 - 6x + 4y - 12 = 0$

The locus is a circle with centre (3, -2) and radius 5 units.

(g) A is a point on the x-axis and C is a point (2, 3). The perpendicular to AC through C meets the y-axis at B. Find the locus of the midpoint of AB.

Solution

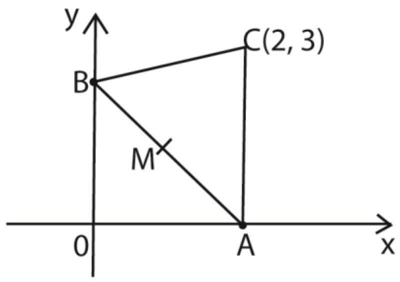
Let A(a, 0), B(0, b) and M(x, y) is the midpoint of AB

⇨ $M\left(\frac{a+0}{2}, \frac{0+b}{2}\right)$ i.e. $M\left(\frac{a}{2}, \frac{b}{2}\right)$

Since AC is perpendicular BC

⇨ $\frac{0-3}{a-2} \cdot \frac{b-3}{0-2} = -1$

$3(b - 3) = 2(2 - a)$ ……………………(i)



Now at M , $x = \frac{a}{2}$ and $y = \frac{b}{2}$

⇨ a = 2x and b = 2y

Substituting into eqn. (i)

3(2y - 3) =2(2 – 2x)

i.e. 4x + 6y= 13

(h) P is a variable point given by parametric equations

$x = \frac{1+t}{1-t}$ and $y = \frac{2t}{1-t}$

Show that the locus of P is a straight line

Solution

By eliminating t

From $x = \frac{1+t}{1-t}$; $t = \frac{x-1}{x+1}$

From $y = \frac{2t}{1-t}$; $t = \frac{y}{2+y}$.

Equating t

$\frac{x-1}{x+1} = \frac{y}{2+y}$

(x -1)(2 +y) =y(x – 1)

y – x = -1 (which is a line)

(i) A point P moves such that its distance from two points A(-2, 0) and B (8,6) are in ratio AP: PB = 3:2. Show that the locus of P s a circle. (05marks)

$\frac{\overline{AP}}{\overline{PB}} = \frac{3}{2}$ => $2\overline{AP} = 3\overline{PB}$

$2\sqrt{(x+2)^2 + y^2} =$

$3\sqrt{(x-8)^2 + (y-6)^2}$

Squaring both sides

$4(x^2 + 4x + 4 + y^2)$

$= 9(x^2 - 16x + 64 + y^2 - 12y + 36)$

$4x^2 + 16x + 16 + 4y^2$

$= 9x^2 - 144x + 900 + 9y^2 - 108y$

$5x^2 + 5y^2 - 160x - 108y + 884 = 0$

Exercise 4

1. Find the equation to the locus of a point which moves so that its distance from the point (3, 4) is always 5 units.
   $[x^2 + y^2 - 6x - 8y = 0]$
2. A and B are points (1, 0) and (7, 8) respectively. A point P moves so that the angle APB is a right angle. Find the equation of locus of P.

   $[x^2 + y^2 - xx - 8y + 7 = 0]$

3. A variable point P is given by parametric equation x = ct, $y = \frac{c}{t}$. Show that the locus of P is $xy = c^2$.
4. Find the locus of point P which is equidistant from the line x = 2and the circle $x^2 + y^2 = 1$.
   $[x^2 + 6x - 9 = 0]$
5. A Point P is twice as far from the line x + y = 5 and from the point (3, 0). Find the locus of P.

   $[7x^2 + 7y^2 - 38x + 10y - 2xy + 47 = 0]$

6. Find the Cartesian equation of a curve given parametrically by $x = \frac{1+t}{1-t}$ and $y = \frac{2t}{1-t}$
   [y – x + 1= 0]
7. P(3, 2) and Q(5, 6) are two fixed points and R moves so that angle PRQ is right angle. Show that the locus of R is given by the equation $x^2 + y^2 - 9x - 6y + 26 = 0$

8. Show that the locus of P is $\frac{1}{4}x^2 + \frac{1}{3}y^2 = 1$, given that PA + PB = 4 where A and B are the points (1, 0) and (-1, 0) respectively

Exercise 5 (Topical question)

1. ABCD is a quadrilateral with A(2, -2), B(5, -1), C(6, 2) and D(3, 1). Show that the quadrilateral is a rhombus.
2. PQRS is a quadrilateral with vertices P(2, -1), Q(4, -1) and S(2, 1). Show that the quadrilateral is a rhombus
3. The Locus of P is such that the distance OP is half the distance PR, where O is the origin and R id the point (-3, 6).
   (a) Show that the locus of P describes a circle in x – y plane
   [$x^2 + y^2 - 2x + 4y - 15 = 0$ (a circle in x – y plane)]
   (b) Determine the centre and radius of the circle.
   [The centre of the circle is (1, -2) and the radius= $\sqrt{20} = 2\sqrt{5}$ units]
   (c) Where does P cuts the line x = 3
   [(3, 2) and (3, -3)]
4. A Point P is twice as far from the line x + y = 5 as from the point (3, 0). Find the locus of P.
   [$7x^2 + 7y^2 - 38x + 10y + 47 = 0$]
5. Find the locus of point P which is equidistant from the line x = 2and the circle $x^2 + y^2 = 1$.
   [$x^2 + 6x - 9 = 0$]
6. The points R(2, 0) and P (3. 0)lie on the x-axis and Q(0, -y) lie on the y-axis. The perpendicular from the origin to RQ meets PQ at S(X, -Y). Determine the locus Of S in terms of X and Y. [$2X^2 + 3Y^2 - 6X = 0$]
7. The point A(2, 1), P(α, β) and point B(1, 2) lie in the same plane. PA meets the x-axis at point (h, 0) and PB meets the y-axis at point (0, k). Find h and k in terms of α and β.
   $\left[h = \frac{2\beta-\alpha}{\beta-1}; k = \frac{2\alpha-\beta}{\alpha-1}\right]$
8. A is a point (1, 3) and B is a point (4, 6). P is a variable point which moves in such a way that $\overline{AP}^2 + \overline{PB}^2 = 34$. Show that the locus of P describes a circle. Find the centre and radius of the circle
   $\left[centre\ is\left(\frac{5}{2},\frac{9}{2}\right), radius = \frac{2\sqrt{5}}{2}units\right]$
9. Find the locus of the point P(x, y) which moves such that its distance from the point S(-3, 0) is equal to its distance from a fixed line x = 3 [$y^2 + 12x = 0$]
10. Given the vector a = i – 3j + 3k and b = -i – 3j + 2k. find
    (j) acute angle between vectors a and b
    [30.86^0]
    (ii) equation of the plane containing a and b
    [-3x+ 5y +6z = 0]
11. The points A and B lie on the positive sides of the x-axis and y-axis respectively. If the length of AB is 5 units and angle OAB is θ, where O is the origin, find the equation of the line AB (leave θ in your answer)
    [y = -xtanθ + 5sinθ]
12. (a) Find the equation of the locus of a point which moves such that its distance from D(4, 5) is twice its distance from the origin.
    [$8x^2 + 8y^2 + 8x + 10y - 41 = 0$]
    (b) the line y = mx intersects the curve 2x2 – x at point A and B. Find the equation of the locus of point P which divides AB in the ratio 2:5. [$y = 7x^2 - x$]

Thank you

Dr. Bbosa Science