UACE MATHEMATICS PAPER 2 2010 and marking guides

Section A

1. Two events M and N are such that P(M) = 0.7, P(M∩N) = 0.45 and P(M∩N') = 0.18. find
   (a) P(N')
   (b) P(M or N but not both M and N)
2. P. Q and R are points on a straight road such that PQ = 20m and QR = 55m. A cyclist moving with uniform acceleration passes P and then notices that it takes him 10s and 15s to travel between (P and Q) and (Q and R) respectively. Find the uniform acceleration.
3. Find the approximate value to one decimal place of $\int_0^1 \frac{dx}{1+x}$, using the trapezium rule with five strips.
4. The probability mass function of a discrete random variable X is given by

   $$f(x) = \begin{cases} \frac{16}{15}x2^{-x}, & x = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

   Find the
   (a) mean of X
   (b) variance of x
5. A carton of mass 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal. The coefficient of friction between the carton and the plane is 1/3. Find a a horizontal force that should be applied to make the carton just about to move up the plane.
6. The table below shows the values of continuous function f with respect to t.

t	0	0.3	0.6	1.2	1.8
f(t)	2.72	3.00	3.32	4.06	4.95

   Using linear interpolation
   (a) f(t) when t = 0.9
   (b) t when f(t) = 4.48
7. The table below shows the expenditure (in Ug. Shs) of a student during the first and second term.

Item	Expenditure		weight
	1st term	2nd term	
Clothing	46,500	49,350	5
Pocket money	55,200	57,500	3
books	80,000	97,500	8

   Using first term expenditure as the base, calculate the average weighted price index to one dType equation here.ecimal place.

8. A particle moving with simple harmonic motion (SHM) travels from a point xm from the centre O to a point on the opposite side of O and xm from O in 3s. The particle takes a further 2s to reach the extreme point of motion. Find the period of the motion.

SECTION B

9. (a) If a is the first approximation to the root of the equation $x^5 - b = 0$, show that the second approximation is given by $\frac{4a+\frac{b}{a^4}}{5}$
   (b) Show that the positive real root of the equation $x^5 - 17 = 0$ lies between 1.5and 1.8. Hence use the formulas in (a) above to determine the root to 3 decimal places.

10. The table below shows the marks obtained by students in a physics test.

Marks	frequency
25 – 29	9
30 – 34	12
35 – 39	10
40 – 44	17
45 – 49	13
50 – 54	25
55 – 59	18
60 – 64	14
65 – 69	8
70 - 74	8

   (a) Draw a histogram and use it to estimate the modal mark.
   (b) Find the
      (i) mean
      (ii) standard deviation

11. Two equal particles of mass, m are attached to the ends of an inelastic string of length a and are placed close together on horizontal plane. If one of the particles is projected vertically upwards with a velocity $\sqrt{2gh}$ where h >a
   (a) show that the other particle will rise a distance $\frac{1}{4}$(h – a) before coming to rest.
   (b) Determine the loss in kinetic energy when the string become taut if a = 20m, h = 54m and m = 4.8kg

12. (a) The probabilities that player A, B and C score in a netball game are $\frac{1}{5}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. If they play in a game, what is the probability that:
   (i) only C scores
   (ii) at least one player scores
   (iii) two and only two players score
   (b) There are 100 students taking principal mathematics in a certain school. 56 of the students are boys and the remainder are girls. The probability that a student takes principal mathematics given that a student is a boy is $\frac{1}{5}$ The probability that a student takes principal mathematics given that a student is a girl is $\frac{1}{11}$. If a student is chosen at random from the school, find the probability that the student:
      (i) is a boy given that the student takes principal mathematics.
      (ii) Does not take principal mathematics.

13. The centre of a regular hexagon ABCDEF of side 2a is O. Forces of magnitude 4N, sN, tN, 1N, 7N and 3N act along the sides AB, BC, CD, DE, EF and FA respectively. Their directions are in the order of the letters.
    (a) Given that the resultant of the six forces is of magnitude $2\sqrt{3}N$ acting in a direction perpendicular to BC, determine the values of s and t.
    (b) (i) Show that the sum of moments of the forces about O is $27a\sqrt{3}$Nm.
        (ii) If the midpoint of BC is M, find the equation of the line of action of the resultant; refer to OM as x-axis and OD as y-axis.
14. (a) The positive real numbers N_1, and N_2 are rounded off to give n_1 and n_2 respectively. Determine the maximum relative error in using n_1n_2 for N_1N_2. State any assumptions made.
    (b) If $N_1 = 2.765$, $N_2 = 0.72$, determine the range within which the exact values of
        (i) $N_1N_2(N_1 – N_2)$
        (ii) $\frac{N_2-N_1}{N_1N_2}$
15. Two aircrafts P and Q are flying at the same height. P is flying due north 500kmh^{-1} while is flying due west at 600kh^{-1}. When the aircrafts are 100km apart, the pilots realized that they are about to collide. The Pilot of P then changes course to 345^0 and maintain the speed of 500kmh^{-1}. The pilot of Q maintains his course but increases speed. Determine the
    (a) Distance each aircraft would have travelled if the pilots had not realized that they were about to collide.
    (b) New speed beyond which the aircraft Q must fly in order to avoid collision.
16. (a) The chance that a cow recovers from a certain mouth disease when treated is 0.72. If 100 cows are treated by the same vaccine, find the 95% confidence limits for the mean number of covers that recover.
    (b) The ages of taxis on a route are normally distributed with standard deviation of 1.5 years; a sample of 100 taxis inspected on a particular day gave a mean age of 5.6years. Determine
    (i) A 99% confidence interval for the mean age of all taxis that operate on the route
    (ii) The probability that the taxis were ages between 5.4 and 5.8 years.

END

Marking guide

Section A

1. Two events M and N are such that P(M) = 0.7, P(M∩N) = 0.45 ans P(M∩N') = 0.18. find
   (a) P(N')
   (b) P(M or N but not both M and N)

   Solution

   (a) Using a contingency table:

I	N	N'	I
M	M∩N	M∩N'	M
M'	M'∩N	M'∩N'	M'
I	N	N'	I

From the table above

P(M) = P(M∩N) + P(M∩N')

0.7 = 0.45 + P(M∩N')

P(M∩N') = 0.25

P(N') = P(M∩N') + P(N'∩M')

= 0.25 + 0.18

= 0.43

Alternatively; from Demorgan's rule

P(M'∩N') = P(MUN)'

= 1 – P(MUN)

P(NUM) = 1- P(M'∩N')

= 1 – 0.18

=0.82

But P(MUN) = P(M) + P(N) – P(M∩N)

0.82 = 0.7 + P(N) – 0.45

P(N)= 0.82 + 0.45 – 0.7

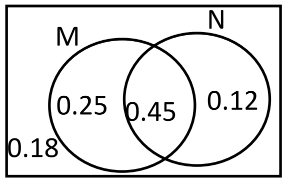
= 0.57

P(N') = 1 – P(N)

= 1 – 0.57

= 0.43

(b) Solution
P(M or N but not both M and N)
= P(M or N) only
Now P(M or N) only = P(MUN) only
P(MUN) only
= P(M) + P(N) – P(M∩N) - P(M∩N)
= 0.7 + 0.57 - 0.45 - 0.45
= 0.37
OR: Using Venn diagram



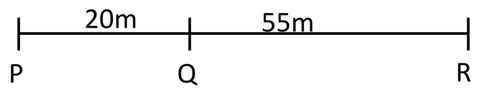
P(MUN) only = P(M) only + P(N) only
= 0.25 + 0.12
= 0.37

2. P. Q and R are points on a straight road such that PQ = 20m and QR = 55m. A cyclist moving with uniform acceleration passes P and then notices that it takes him 10s and 15s to travel between (P and Q) and (Q and R) respectively. Find the uniform acceleration.

Solution

Method 1

Using equations of motion



Let u = to velocity at P

Using $s = ut + \frac{1}{2}at^2$

20= 10u + ½ a x 100
20 = 10u + 50a
2 = u + 5a ……………………….(i)

Considering PR

75= 25u + ½ a x 625
73 = 25u + 312.5a
3 = u + 12.5a ……………………….(ii)
Eqn.(ii) – eqn. (i)
7.5a = 1

a = $\frac{2}{15}ms^2$

Hence uniform acceleration = $\frac{2}{15}ms^2$

3. Find the approximate value to one decimal place of $\int_0^1 \frac{dx}{1+x}$, using the trapezium rule with five strips.

Solution

$\int_0^1 \frac{dx}{1+x} = \frac{1}{2}d[y_0 + y_n + 2(y_1, y_2 + \ldots y_{n-1})]$

$d = \frac{1-0}{5} = 0.20$

x	0	0.20	0.40	0.60	0.80	1.00
1+ x	1	1.20	1.40	1.60	1.80	2.00
$\frac{1}{1+x}$	1	0.833	0.714	0.625	0.556	0.5

$\int_0^1 \frac{dx}{1+x}$

$= \frac{1}{2}$x 0.2[1 +0.5+2(0.833+0.714+0.625+0.556)]

= 0.1 x 6.956

= 0.7 (1d.p)

$\therefore \int_0^1 \frac{dx}{1+x}$ = 0.7 (1d.p)

4. The probability mass function of a discrete random variable X is given by

$$f(x) = \begin{cases} \frac{16}{15}x2^{-x}, & x = 1,2,3,4 \\ 0, & otherwise \end{cases}$$

Find the
(a) mean of X
(b) variance of x

Solution

x	1	2	3	4
F(x)=$\frac{16}{15}x2^{-x}$	0.5333	0.2667	0.1333	0.0667
xf(x)	0.5333	0.5334	0.3999	0.2668
x^2f(x)	0.5333	1.0668	1.1997	1.0672

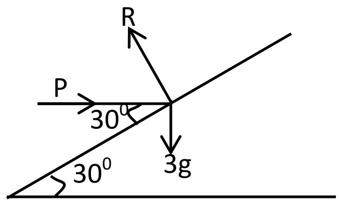
(a) E(x) = $\sum_{all} xf(x)$
= 0.5333+ 0.5334 + 0.3999 + 0.2668
= 1.7334

(b) Ver(x) = E(x^2) - (E(x))^2
E(x^2) = $\sum_{all} x^2 f(x)$
= 0.5333+ 1.0668 + 1.1997 + 1.0672
= 3.867
ver(x) = 3.867 – (1.7334)^2
= 0.8623 (4d.p)

5. A carton of mass 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal. The coefficient of friction between the carton and the plane is 1/3. Find a a horizontal force that should be applied to make the carton just about to move up the plane.

Solution

Let the horizontal force be P



By resolving forces;
Along the plane
pcos30^0 =3gsin30 + $\frac{1}{3}R$……………………..(i)
Perpendicular to the plane
R – psin30^0 = 3g cos30^0
R =3gcos30^0 + psin30^0 ……………………(ii)
Substituting eqn. (ii) into eqn. (i)
pcos30^0 =3gsin30 + $\frac{1}{3}$(3gcos30^0 + psin30^0)
p(cos30^0 - $\frac{1}{3}$sin30^0) = 3gsin30^0 +gcos30^0

p = $\frac{3gsin30^0+gcos30^0}{cos30^0-\frac{1}{6}}$

=$\frac{23.18704896}{0.6993587371}$

= 33.1547N (4d.p)

6. The table below shows the values of continuous function f with respect to t.

t	0	0.3	0.6	1.2	1.8
f(t)	2.72	3.00	3.32	4.06	4.95

Using linear interpolation
(a) f(t) when t = 0.9

Solution

Extract

t	0.6	0.9	1.2
f(t)	3.32	y	4.06
	A	B	C

Gradient of AB = gradient of AC

$\frac{y-3.32}{0.9-0.6} = \frac{4.06-3.32}{1.2-0.6}$

y = 3.69

Hence t = 0.9, f(t) = 3.39

(b) t when f(t) = 4.48

Solution

Extract

t	1.2	x	1.8
f(t)	4.06	4.48	4.95
	A	B	C

Gradient of AB = gradient of AC

$\frac{4.48-4.06}{x-1.2} = \frac{4.95-4.06}{1.8-1.2}$

x = 1.48

Hence when f(t) = 4.48, t = 1.48

7. The table below shows the expenditure (in Ug. Shs) of a student during the first and second term.

Item	Expenditure		weight
	1st term	2nd term	
Clothing	46,500	49,350	5
Pocket money	55,200	57,500	3
books	80,000	97,500	8

Using first term expenditure as the base, calculate the average weighted price index to one decimal place.

Solution

Average weighted price index

$=\frac{\Sigma\left(\frac{P_1}{P_2}w_i\right)}{\Sigma w_1}x100$

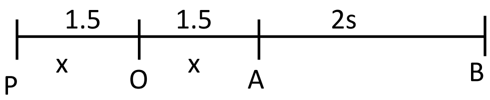
$=\left(\frac{\frac{49350}{46,500}x5+\frac{57,500}{55,200}x3+\frac{97,500}{80,00}x8}{5+3+8}\right)$

$=\frac{1818.145}{16} = 113.634$

=113.6 (d.p)

8. A particle moving with simple harmonic motion (SHM) travels from a point xm from the centre O to a point on the opposite side of O and xm from O in 3s. The particle takes a further 2s to reach the extreme point of motion. Find the period of the motion.

Solution



By symmetry, time from P to O = time from O to A = 1.5s

The time from O to B = $\frac{1}{4}T = (1.5 + 2)s$

Period = 3.5 x 4 = 14s

SECTION B

9. (a) If a is the first approximation to the root of the equation $x^5 - b = 0$, show that the second approximation is given by $\frac{4a+\frac{b}{a^4}}{5}$

Solution

Let $f(x) = x^5 - b$

$f'(x) = 5x_n^4$

Using Newton-Raphson method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{x_n^5-b}{5x_n^4}$

$= \frac{5x_n^5-x_n^5+b}{5x_n^4}$

$= \frac{4x_n^5+b}{5x_n^4}$

Dividing numerator and denominator on RHS by x_n^4

$x_{n+1} = \frac{4x_n+\frac{b}{x_n^4}}{5}$

Substituting for $x_0 = a$

$x_1 = \frac{4a+\frac{b}{a^4}}{5}$

(b) Show that the positive real root of the equation $x^5 - 17 = 0$ lies between 1.5and 1.8. Hence use the formulas in (a) above to determine the root to 3 decimal places.

Solution

Using the sign change method

Let $f(x) = x^5 - 17$

$f(1.5) = (1.5)^5 - 17 = -9.4025$

$f(1.8) = (1.8)^5 - 17 = 1889567$

since f(1.5) x f(1.8)<0; the root lies between 1.5 and 1.8

Since f(1.5) is nearer to zero than f(1.8), we take $x_0 = 1.5$

$x_1 = \frac{4(1.5)+\frac{17}{(1.5)^4}}{5}$= 1.8716

Error = |1.8716 − 1.5| = 0.3716>0.0005

$x_2 = \frac{4(1.8716)+\frac{17}{(1.8716)^4}}{5}$= 1.7744

Error = |1.7744 − 1.8716| = 0.0.0972>0.0005

$x_3 = \frac{4(1.7744)+\frac{17}{(1.7744)^4}}{5}$= 1.7625

Error = |1.7625 − 1.7744| = 0.0119>0.0005

$x_3 = \frac{4(1.7625)+\frac{17}{(1.7625)^4}}{5}$= 1.7623

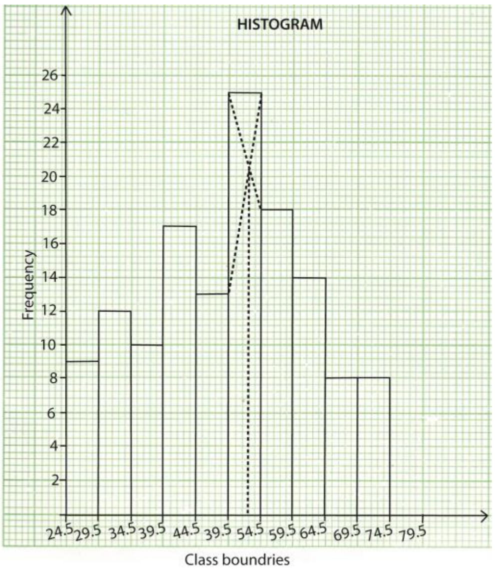
Error = |1.7623 − 1.7625| = 0.0002<0.0005

Hence the root = 1.762(3d.p)

10. The table below shows the marks obtained by students in a physics test.

Marks	frequency
25 – 29	9
30 – 34	12
35 – 39	10
40 – 44	17
45 – 49	13
50 – 54	25
55 – 59	18
60 – 64	14
65 – 69	8
70 - 74	8

(a) Draw a histogram and use it to estimate the modal mark.



Modal mark = 49.5 + 3 = 52.5 ± 0.5

(b) Find the

(iii) mean

(iv) standard deviation

Solution

Marks	x	f	fx	fx^2
25 – 29	27	9	243	6561
30 – 34	32	12	384	12288
35 – 39	37	10	370	13690
40 – 44	42	17	714	29988
45 – 49	47	13	611	28717
50 – 54	52	25	1300	67600
55 – 59	57	18	1026	58482
60 – 64	62	14	868	53816
65 – 69	67	8	536	35912
70 - 74	72	8	576	41472
sum		134	6628	348526

(a) $\bar{x} = \frac{\sum fx}{\sum f} = \frac{6628}{134} = 49.4627$

(b) S.D =$\sqrt{var(x)}$

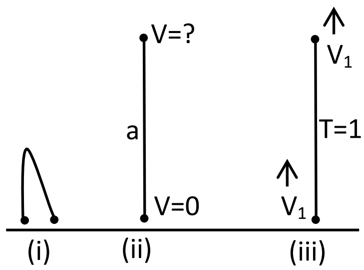
=$\sqrt{\frac{348526}{134} - \left(\frac{6628}{134}\right)^2}$

= 12.424

11. Two equal particles of mass, m are attached to the ends of an inelastic string of length a and are placed close together on horizontal plane. If one of the particles is projected vertically upwards with a velocity $\sqrt{2gh}$ where h >a

(a) Show that the other particle will rise a distance $\frac{1}{4}$(h – a) before coming to rest.

Solution



(i) Shows particles resting on horizontal plane close to each other.

(ii) In diagram (ii) one of the particles is lurched into motion within the distance =a above the ground and is experiencing acceleration due to gravity.

From $v^2 = u^2 + 2as$

Velocity (v) attained by the first moving first moving particle at a distance a above ground is

$v^2 = (\sqrt{2gh})^2 - 2ga$

$v^2 = 2gh - 2ga = 2g(h - a)$

$v = \sqrt{2g(h-a)}$

Let v_1 be the velocity common velocity of the two particles immediately above a distance a above the ground. By the principle of conservation of momentum

$m\sqrt{2g(h-a)} = 2mv_1$

$v_1 = \frac{1}{2}\sqrt{2g(h-a)}$

When the 2nd particle comes to rest $v_1 = 0$,

let the distance travelled by the 2nd particle be x

From $v^2 = u^2 + 2as$

$0 = \left(\frac{1}{2}\sqrt{2g(h-a)}\right)^2 - 2gx$

$2gx = \frac{1}{4}(2g(h-a)$

$x = \frac{1}{4}(h - a)$

Hence the second particle will rise a distance $\frac{1}{4}(h - a)$

(b) Determine the loss in kinetic energy when the string become taut if a = 20m, h = 54m and m = 4.8kg

Solution

The loss in kinetic energy is due to the first particle

The loss in kinetic energy

= K.E before – K.E after

$= \frac{1}{2}mv^2 - \frac{1}{2}m{v_1}^2$

$= \frac{1}{2}x4.8(2gh - 2g(h-a))$

$= \frac{1}{2}x4.8x\ 2\ x\ 9.8\ x\ 20$

=940.8J

12. (a) The probabilities that player A, B and C score in a netball game are $\frac{1}{5}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. If they play in a game, what is the probability that:

(i) only C scores

P(ONLY C scores) = P(A'∩B'∩C)

$= \frac{4}{5}x\frac{3}{4}x\frac{1}{3} = 0.2$

(ii) at least one player scores

P(at least one player scores)

1 - P(A'∩B'∩C)

$1 - \frac{4}{5}x\frac{3}{4}x\frac{2}{3} = 1 - \frac{24}{60} = \frac{36}{60} = 0.6$

(iii) two and only two players score

P(two and only two players score)

P(A'∩B∩C) + P(A∩B'∩C) + P(A∩B∩C')

$= \frac{4}{5}x\frac{1}{4}x\frac{1}{3} + \frac{1}{5}x\frac{3}{4}x\frac{1}{3} + \frac{1}{5}x\frac{1}{4}x\frac{2}{3}$

$= \frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20} = 0.15$

(c) There are 100 students taking principal mathematics in a certain school. 56 of the students are boys and the remainder are girls. The probability that a student takes principal mathematics given that a student is a boy is $\frac{1}{5}$ The probability that a student takes principal mathematics given that a student is a girl is $\frac{1}{11}$. If a student is chosen at random from the school, find the probability that the student:

(i) is a boy given that the student takes principal mathematics.

Solution

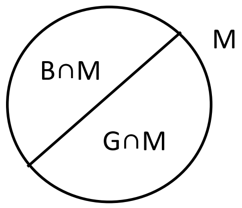
Let B = boys. G= girls, and M = principal math

$P(B) = \frac{56}{100} = 0.56$

P(G) = 0.44

$P(M/B) = \frac{1}{5}$

$P(M/G) = \frac{1}{11}$



P(M) = P(B∩M) + P(G∩M)

= P(B).P(M/B) + P(G).P(M/G)

$= 0.56 \text{ x} \frac{1}{5} + 0.44\ x\frac{1}{11}$

= 0.112 + 0.04

= 0.152

$P(B/M) = \frac{P(B\cap M)}{P(M)} = \frac{0.112}{0.152} = 0.7368$

Alternatively

$P(B/M) = \frac{P(B \cap M)}{P(M)}$

$= \frac{P(B).P(\frac{M}{B})}{P(B).P(\frac{M}{B})+P(G).P(\frac{M}{G})}$

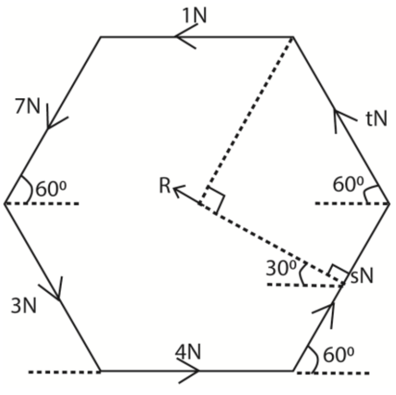
$= \frac{0.56x\frac{1}{5}}{0.56x\frac{1}{5}+0.44x\frac{1}{11}}$

= 0.7368

(ii) Does not take principal mathematics.
P(M') = 1- P(M')
= 1 – 0.152
= 0.848

13. The centre of a regular hexagon ABCDEF of side 2a is O. Forces of magnitude 4N, sN, tN, 1N, 7N and 3N act along the sides AB, BC, CD, DE, EF and FA respectively. Their directions are in the order of the letters.
    (a) Given that the resultant of the six forces is of magnitude $2\sqrt{3}N$ acting in a direction perpendicular to BC, determine the values of s and t.
    (b) (i) Show that the sum of moments of the forces about O is 27a$\sqrt{3}$Nm.
        (iii) If the midpoint of BC is M, find the equation of the line of action of the resultant; refer to OM as x-axis and OD as y-axis.

Solution



$R=\binom{4}{0} + \binom{scos60}{ssin60} + \binom{-tcos60}{tsin60} + \binom{-1}{0} + \binom{-7cos60}{-7sin60} + \binom{3cos60}{-3sin60} = \binom{-2\sqrt{3}cos30}{2\sqrt{3}sin30}$

$\binom{-2\sqrt{3}cos30}{2\sqrt{3}sin30} = \binom{4+scos60-tcos60-1-4cos60}{0+ssin60+tsin60-10sin60}$

$\begin{pmatrix}-2\sqrt{3}.\frac{\sqrt{3}}{2}\\ 2\sqrt{3}.\frac{1}{2}\end{pmatrix} = \begin{pmatrix}4+\frac{s}{2}-\frac{1}{2}-3\\ \frac{s\sqrt{3}}{2}+\frac{t\sqrt{3}}{2}-\frac{10\sqrt{3}}{2}\end{pmatrix}$

For i : $-3 = \frac{s}{2} - \frac{1}{2} + 1$

$-4 = \frac{s}{2} - \frac{1}{2}$

-8 = s – t

s = t – 8 ………………………………..(i)

For j: $2\sqrt{3}.\frac{1}{2} = \frac{s\sqrt{3}}{2} + \frac{t\sqrt{3}}{2} - \frac{10\sqrt{3}}{2}$

2 = s + t – 10 ……………………… (ii)

Substituting (i) into (ii)

2 = t – 8 + t – 10

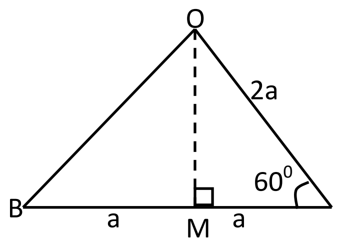
2t = 20

t = 10

From (i) s = 10 -8 = 2

Hence s = 2 and t = 10

(b)(i)



$OM^2 = (2a)^2 - a^2$

$= 3a^2$

OM = a$\sqrt{3}$

$\overset{\curvearrowright}{O}$ =4x a$\sqrt{3}$ +2x a$\sqrt{3}$ + 10x a$\sqrt{3}$+1x a$\sqrt{3}$+7x a$\sqrt{3}$ + 3x a$\sqrt{3}$

=4a$\sqrt{3}$ +2a$\sqrt{3}$ + 10a$\sqrt{3}$+1a$\sqrt{3}$+7$\sqrt{3}$ + 3a$\sqrt{3}$

=27a$\sqrt{3}$

(ii) Since the magnitude of the resultant acts in the direction of OM, this implies that X = 2$\sqrt{3}$ and Y = 0
Equation of the line of action of the resultant is given by
G – xY + yX = 0

Substituting for G = $27a\sqrt{3}$, Y = 0 and X = $2\sqrt{3}$

$27a\sqrt{3} - x(0) + y(2\sqrt{3}) = 0$

$y(2\sqrt{3}) = 27a\sqrt{3}$

$y = \frac{27a\sqrt{3}}{2\sqrt{3})} = 13.5a$

14. (a) The positive real numbers N_1, and N_2 are rounded off to give n_1 and n_2 respectively. Determine the maximum relative error in using n_1n_2 for N_1N_2. State any assumptions made.

Solution

Let $z = n_1n_2$

$Z = N_1N_2$

$= (n_1+\Delta n_1)(n_2 + \Delta n_2)$

$\Delta z = Z - z$

$=(n_1+\Delta n_1)(n_2 + \Delta n_2) - n_1n_2$

$= n_1n_2 + \Delta n_1 n_2 + \Delta n_1 n_2 + \Delta n_1 \Delta n_2 - n_1n_2$

$=\Delta n_1 n_2 + \Delta n_1 n_2 + \Delta n_1 \Delta n_2$

As $\Delta n_1 \to 0$ and $\Delta n_2 \to 0$, $\Delta n_1 \Delta n_2 \to 0$

⇨ $\Delta z = \Delta n_1 n_2 + \Delta n_1 n_2$

Relative error = $\frac{\Delta z}{z}$

$= \frac{\Delta n_1 n_2 + \Delta n_2 n_1}{n_1 n_2}$

$= \frac{\Delta n_1}{n_2} + \frac{+\Delta n_2}{n_1}$

$\left|\frac{\Delta z}{z}\right| = \left|\frac{\Delta n_1}{n_2} + \frac{+\Delta n_2}{n_1}\right|$

$\leq \left|\frac{\Delta n_1}{n_2}\right| + \left|\frac{\Delta n_2}{n_1}\right|$

(b) If $N_1 = 2.765$, $N_2 = 0.72$, determine the range within which the exact values of

(i) $N_1N_2(N_1 - N_2)$

Solution

$N_1N_2(N_1 - N_2) = (2.765)(0.72)(2.765 - 0.72)$

Maximum value

$=(2.7655)(0.725)(2.7655 - 0.715)$

$=4.111$(3dp)

Minimum value

$=(2.7645)(0.715)(2.7645 - 0.725)$

$=4.031$(3dp)

Range of values = $4.031 \leq N_1N_2(N_1 - N_2) \leq 4.111$

(ii) $\frac{N_2 - N_1}{N_1 N_2}$

Maximum value $= \frac{0.725 - 2.7645}{(2.7655)(0.725)}$

$= -1.0172$

Maximum value $= \frac{0.715 - 2.7655}{(2.7645)(0.715)}$

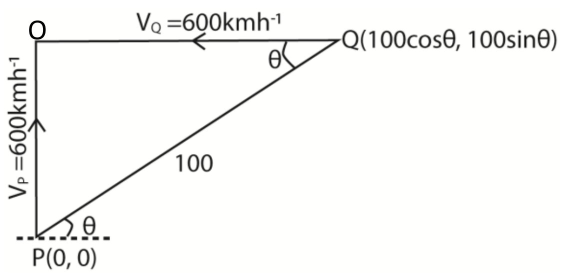
$= -1.037$

Range of values

$-1.037 \leq N_1N_2(N_1 - N_2) \leq -1.017$

15. Two aircrafts P and Q are flying at the same height. P is flying due north 500kmh^{-1} while is flying due west at 600kh^{-1}. When the aircrafts are 100km apart, the pilots realized that they are about to collide. The Pilot of P then changes course to 345^0 and maintain the speed of 500kmh^{-1}. The pilot of Q maintains his course but increases speed. Determine the

(a) distance each aircraft would have travelled if the pilots had not realized that they were about to collide.



$sin\theta = \frac{500}{100\sqrt{61}} = \frac{5}{\sqrt{61}}$

At point of collision, O, $r_P = r_Q$

⇨ $\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}0\\500\end{pmatrix} t = \begin{pmatrix}\frac{600}{\sqrt{61}}\\ \frac{500}{\sqrt{61}}\end{pmatrix} + \begin{pmatrix}-600\\0\end{pmatrix} t$

Equating corresponding unit vectors; for i

$0 = \frac{600}{\sqrt{61}} - 600t$

$t = \frac{1}{\sqrt{61}}$

Distance moved by P = $V_P.t$

$= 500 \times \frac{1}{\sqrt{61}}$

= 64km

Distance moved by Q= $V_Q.t$

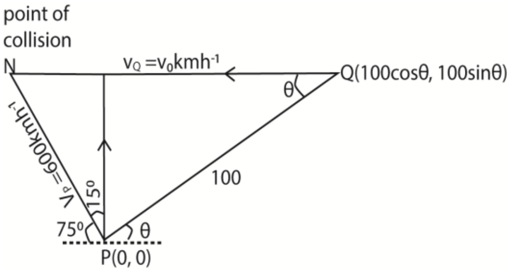
$= 600 \times \frac{1}{\sqrt{61}}$

$= 76.82$km

(b) New speed beyond which the aircraft Q must fly in order to avoid collision.

Solution

Let v_o = critical value of the speed of Q for collision to take place



At point of collision, N; $r_P = r_Q$

$$\binom{0}{0} + \binom{-500cos75^0}{500sin75^0} t = \begin{pmatrix} \frac{600}{\sqrt{61}} \\ \frac{500}{\sqrt{61}} \end{pmatrix} + \binom{-v_0}{0} t$$

Equating corresponding unit vectors

For i; $-500cos75^0 t = \frac{600}{\sqrt{61}} - v_0 t$ ....(i)

For j; $500cos75^0 t = \frac{500}{\sqrt{61}}$

$t = \frac{1}{\sqrt{61}.sin75^0}$

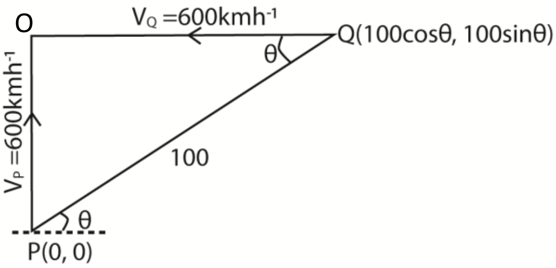
Substituting t in equation (i)

$-500cos75^0 \left(\frac{1}{\sqrt{61}.sin75^0}\right) = \frac{600}{\sqrt{61}} - v_0 \left(\frac{1}{\sqrt{61}.sin75^0}\right)$

$v_0 = 708.965$kmh^{-1}

Hence the speed of Q must be 708.965kmh^{-1}

Method II: Geometric approach



$$V_{PQ} = \sqrt{500^0 + 600^0}$$

$$= \sqrt{610000} = 100\sqrt{61}$$

$$cos\theta = \frac{600}{100\sqrt{61}} = \frac{6}{\sqrt{61}}$$

$$sin\theta = \frac{500}{100\sqrt{61}} = \frac{5}{\sqrt{61}}$$

For collision to occur at point O, when OQ and OP are perpendicular.

Distance covered by P = OP

=100sinθ

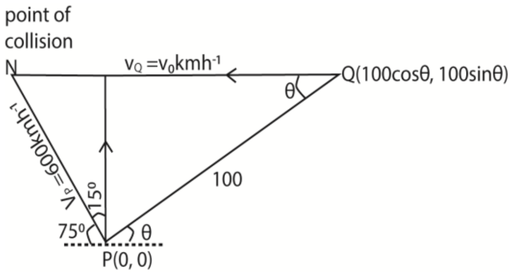
$=100 \times \frac{5}{\sqrt{61}}$ = 64km

Distance covered by Q = OQ

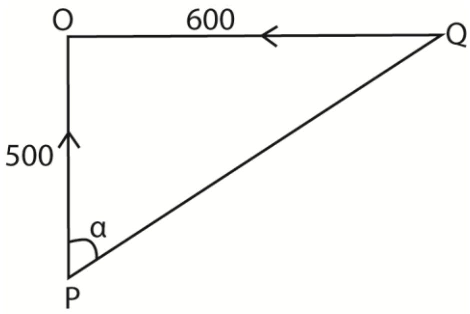
=100cosθ

$=100 \times \frac{6}{\sqrt{61}}$ = 76.822km

(b) Let v_0 = critical value of speed of Q

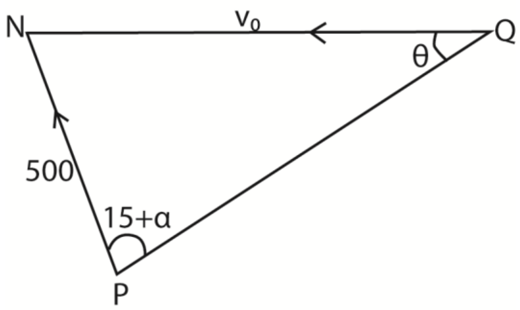


Considering triangle OPQ



$\tan\alpha = \frac{600}{500} = \frac{6}{5} \Rightarrow \alpha = \tan^{-1}\frac{6}{5} = 50.1944^0$

Considering triangle NPQ



Using sine rule; $\frac{v_0}{\sin(15+\alpha)} = \frac{500}{sin\theta}$

$v_0 = \frac{500\sin(15+\alpha)}{sin\left(\tan^{-1}\frac{5}{6}\right)}$

$= \frac{500\sin(65.1944)}{0.64018} = 708.956$

Hence the speed of Q must be 708.965kmh^{-1}

16. (a) The chance that a cow recovers from a certain mouth disease when treated is 0.72.If 100 cows are treated by the same vaccine, find the 95% confidence limits for the mean number of covers that recover.

Solution

$\overline{X}$=np = 100 x 0.72 = 72

$\sigma = \sqrt{npq} = \sqrt{100x\ 0.72x0.28}$ = 4.49

The(1 – α)% confidence interval is given by $\overline{X} \pm z_{\frac{\alpha}{2}}.\frac{\sigma}{\sqrt{n}}$

1 – α = 0.95

α = 0.05

of

v

P(0< z < $z_{\frac{\alpha}{2}}$) = 0.475

$z_{\frac{\alpha}{2}}$ =1.96

Since the sample mean is not given, we use z =$\frac{x-\mu}{\sigma}$

⇨ -1.96 = $\frac{x_1-72}{4.49}$

x_1 = 63.2

1.96 = $\frac{x_2-72}{4.49}$

x_2 = 80.8

Hence the confidence limits are (63.2, 80.8)

(b) The ages of taxis on a route are normally distributed with standard deviation of 1.5 years; a sample of 100 taxis inspected on a particular day gave a mean age of 5.6years. Determine

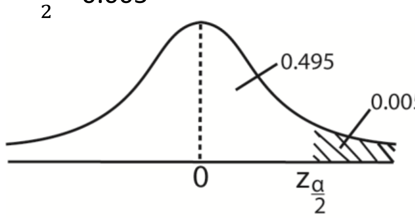
(i) A 99% confidence interval for the mean age of all taxis that operate on the route

Solution

1 – α = 0.99

α = 0.01

$\frac{\alpha}{2}$ = 0.005



P(0< z < $z_{\frac{\alpha}{2}}$) = 0.495

$z_{\frac{\alpha}{2}}$ =2.576

Confidence interval =$\overline{X} \pm z_{\frac{\alpha}{2}}.\frac{\sigma}{\sqrt{n}}$

Lower limit = 5.6 -2.576x$\frac{1.5}{\sqrt{100}}$ =5.2136

Upper limit = 5.6 + 2.576x$\frac{1.5}{\sqrt{100}}$

=5.9864

Hence 99% confidence interval is

5.2136≤μ≤ 5.9864

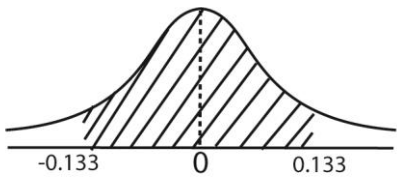
(ii) The probability that the taxis were ages between 5.4 and 5.8 years.

Solution

Let x = ages of taxis

P5.4<x< 5.8)= P$\left(\frac{5.4-5.6}{1.5} < z < \frac{5.8-5.6}{1.5}\right)$

=P(-0.133<z<0.133)



P(-0.133<z<0.133)=2P(0<z<0.133)

= 3 x 0.0529

= 0.1058

Hence the probability that the taxis were of ages between 5.4 and 5.8years is 0.1058

Thank you

Dr. Bbosa Science