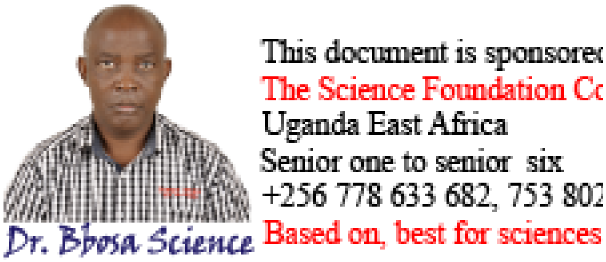


UACE MATHEMATICS PAPER 1 2012 and marking guides

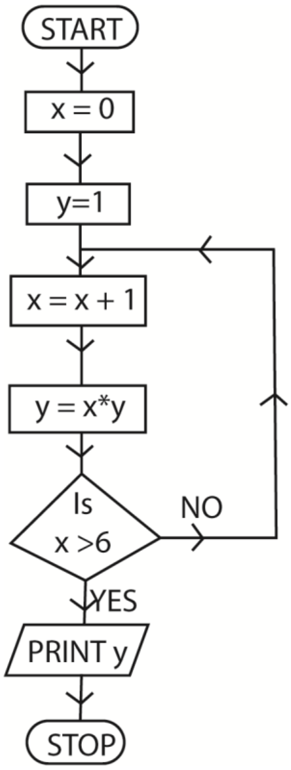
Section A

1. The data below represents the lengths of leaves in centimetres; 4.4, 6.2, 9.4, 12.6, 10.0, 8.8, 3.8 and 13.6. Find the
   (a) mean length
   (b) variance
2. A particle of mass 2kg moves under the action of three forces, F_1, F_2 and F_3 at a time, t.
   $F_1 = \left(\frac{1}{4}t - 1\right)i + (t - 3)\,jN$
   $F_2 = \left(\frac{1}{2}t + 2\right)i + \left(\frac{1}{2}t - 4\right)jN$
   $F_1 = \left(\frac{1}{4}t - 4\right)i + \left(\frac{3}{2}t + 1\right)jN$
   Find the acceleration of the particle when t = 2 seconds
3. The table below shows delivery charges by a courier company.

Mass(g)	200	400	600
Charge (shs)	700	1200	3000

   Using linear interpolation or extrapolation find the
   (a) delivery charge of a parcel weighing 352g
   (b) mass of a parcel whose delivery charge is shs. 3,300.
4. Two events A and B are such that P(A'∩B) = 3x, P(A∩B') = x and P(b) = $\frac{4}{7}$.
   Using a Venn diagram, find the values of
   (a) x
   (b) P(A∩B)
5. A can row a boat in still water at 6kmh^{-1}. He wishes to cross a river to a point directly opposite his starting point. The river flows at 4kmh^{-1} and has a width of 250m. Find the time the man would take to cross the river.

6. Study the flow chart given below



(a) Performa dry run
(b) What is the purpose of the flow chart

7. Given that X ~N(2,2.89), find P(X<0)

8. Particles of weight 12N, 8N and 4N act at points (1, -3), (0, 2) and (1, 0) respectively. find the centre of gravity of the particles.

SECTION B

9. The continuous random variable X has the probability density function (p.d.f) given by

$$f(x) = \begin{cases} k_1 x, & 1 \le x \le 3 \\ k_2(4-x), & 3 < x \le 4 \\ 0 & otherwise \end{cases}$$

where k_1 and k_2 are constants.

(a) Show that $k_2 = 3k_1$
(b) Find
   (i) The value of k_1 and k_2
   (ii) E(X), the expectation of X

10. An inelastic string of length a metres is fixed at one end P and carries a particle of mass 3kg at its other end Q. The particle is describing a horizontal circle of radius 80cm with an angular speed of 5 rad^{-1}.

Determine the
(a) (i) angle the string makes with the horizontal,
    (ii) tension in the string
(b) Value of a
(c) Linear speed of the particle

11. (a) Use the trapezium rule with five subintervals to estimate $\int_0^{\frac{\pi}{3}} tanx dx$ correct to three decimal places.

   (b) (i) Find the value $\int_0^{\frac{\pi}{3}} tanx dx$ to 3 decimal places

   (ii) Calculate the percentage error in your estimation in (a) above.

   (iii) suggest how the percentage error may be reduced.

12. The heights and masses of ten students are given in the table below

Height (cm)	Mass (kg)
156	62
151	58
152	63
146	58
160	70
157	60
149	55
142	57
158	68
141	56

   (a) (i) Plot the data on a scatter diagram.

   (ii) Draw the line of best fit. Hence estimate the mass corresponding to height 155cm.

   (b) (i) Calculate the rank correlation coefficient for the data.

   (ii) Comment on the significance of the height on the masses of students. [Spearman's, ρ =0.79 and Kendall's, τ = 0.64 at 1% level of significance bases on 10 observations]

13. A football player projected a ball at a speed of 8ms-1 at an angle of 30^0 with the ground. The ball strikes the ground at a point which is level with the point of projection. After impact with the ground, the ball bounced and the horizontal component of velocity of the ball remained the same but the vertical component was reversed in the direction and halved in magnitude. The player running after the ball kicked it again at a point which was at a horizontal distance of 1.0m from the point where it bounced, so the ball continued in the same direction. Find the

   (a) Horizontal distance between the point of projection and the point at which the ball first stroked the ground. [Take g = $10ms^{-2}$]

   (b) (i) the time interval between the ball striking the ground and the player kicking it again

   (ii) the height of the ball above the ground when it is kicked again. [Take g = $10ms^{-2}$]

14. (a)(i) On the same axes, draw graphs $y = x^2$ and y = cosx for $0 \leq x \leq \frac{\pi}{2}$ at intervals of $\frac{\pi}{8}$

   (ii) From your graphs, obtain to one decimal place, an approximate root of the equation $x^2 - cosx = 0$

   (a) Using Newton-Raphson method, find the root of the equation $x^2 - cosx = 0$, taking the approximate root in (a) as an initial approximation. Give your answer correct to three decimal places.

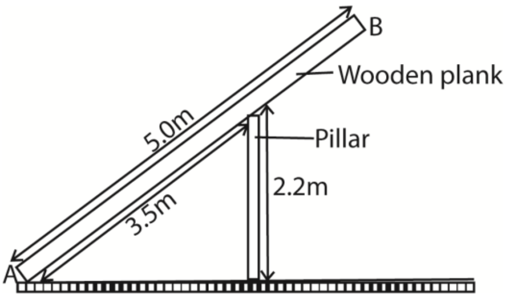
15. Box A contains 4 red sweets and 3 green sweets. Box B contains 5 red sweets and 6 green sweets. Box A is twice as likely to be picked as box B. If a box is chosen at random and two sweets are removed from it, one at a time without replacement;

   (a) find the probability that the two sweets removed are of the same colour.

   (b) (i) construct a probability distribution table for the number of red sweets removed

   (ii) find the mean number of red sweets removed

16. The diagram below shows a uniform wooden plank AB of mass 70kg and length 5m. the end A rests on a rough horizontal ground. The plank is in contact with the top of a rough pillar at C. The height of the pillar is 2.2m and AC = 3.5m



Given that the coefficient of friction at the ground is 0.6and the plank is just to slip, find the

(a) angle the plank makes with the ground at A

(b) normal reaction at

  (i) A

  (ii) C

(c) Coefficient of friction at C.

END

Marking guide

Section A

1. The data below represents the lengths of leaves in centimetres; 4.4, 6.2, 9.4, 12.6, 10.0, 8.8, 3.8 and 13.6. Find the
   (a) mean length
   (b) variance

   Solution

x	x^2
3.8	14.44
4.4	19.36
6.2	38.44
8.8	77.44
9.2	84.64
10.0	100.00
12.6	158.76
13.6	184.96
$\sum x = 68.6$	$\sum x^2 = 678.04$

(a) mean length = $\frac{\sum x}{n} = \frac{68.6}{8} = 8.575$cm

(b) Var(x) = $\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$= \frac{678.04}{8} - \left(\left(\frac{\sum x}{n}\right)^2\right)^2$

=11.224cm (3d.p)

2. A particle of mass 2kg moves under the action of three forces, F_1, F_2 and F_3 at a time, t.

$$F_1 = \left(\frac{1}{4}t - 1\right)i + (t-3)\,jN$$
$$F_2 = \left(\frac{1}{2}t + 2\right)i + \left(\frac{1}{2}t - 4\right)jN$$
$$F_1 = \left(\frac{1}{4}t - 4\right)i + \left(\frac{3}{2}t + 1\right)jN$$

Find the acceleration of the particle when t = 2 seconds

Solution

Resultant force = $F_1 + F_2 + F_3$

$$F = \begin{pmatrix} \frac{1}{4}t - 1 \\ t - 3 \end{pmatrix} + \begin{pmatrix} \frac{1}{2}t + 2 \\ \frac{1}{2}t - 4 \end{pmatrix} + \begin{pmatrix} \frac{1}{4}t - 4 \\ \frac{3}{2}t + 1 \end{pmatrix}$$

$$= \begin{pmatrix} t - 3 \\ 3t - 6 \end{pmatrix}$$

F = (t – 3)i + (3t – 6)j

But F = ma

2a = (t – 3)i + (3t – 6)j

a = $\frac{1}{2}\left((t-3)i + (3t-6)j\right)$

At t = 2 seconds

a = $\frac{1}{2}\left((2-3)i + (3(2)-6)j\right)$

$= -\frac{1}{2}i$ ms^2

3. The table below shows delivery charges by a courier company.

Mass(g)	200	400	600
Charge (shs)	700	1200	3000

Using linear interpolation or extrapolation find the

(a) delivery charge of a parcel weighing 352g

Solution

Extract

	A	B	C
Mass(g)	200	352	400
Charge (shs)	700	x	1200

Gradient AB = gradient AC

$\frac{352-200}{x-700} = \frac{400-200}{1200-700}$

x = 1080

hence the delivery charge = shs. 1080

(b) mass of a parcel whose delivery charge is shs. 3,300.

Solution

Extract

	A	B	C
Mass(g)	400	600	y
Charge (shs)	1200	3000	3300

Gradient AB = gradient AC

$\frac{600-400}{3000-1200} = \frac{y-400}{3300-1200}$

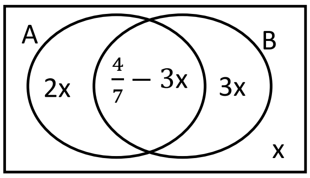
y= $633\frac{1}{3}$kg

Hence the mass of the parcel = $633\frac{1}{3}$kg

4. Two events A and B are such that P(A∩B') = 2x ,P(A'∩B) = 3x, P(A'∩B') = x and P(b) = $\frac{4}{7}$. Using a Venn diagram, find the values of
   (a) x
   (b) P(A∩B)

Solution

Note P(A'∩B) = P(B) only, P(A∩B') = P(A) only and P(A'∩B') = P(AUB)'



(a) From the Venn diagram;

Total probability in the Venn diagram = 1

⇨ $2x + \frac{4}{7} - 3x + 3x + x = 1$

$3x + \frac{4}{7} = 1$

$x = \frac{1}{7}$

(b) $P(A \cap B) = \frac{4}{7} - 3 \times \frac{1}{7} = \frac{1}{7}$

5. A can row a boat in still water at $6kmh^{-1}$. He wishes to cross a river to a point directly opposite his starting point. The river flows at $4kmh^{-1}$ and has a width of 250m. Find the time the man would take to cross the river.

Solution

Speed of the boat in still water is $6kmh^{-1}$

$= \frac{6 \times 1000}{3600}$

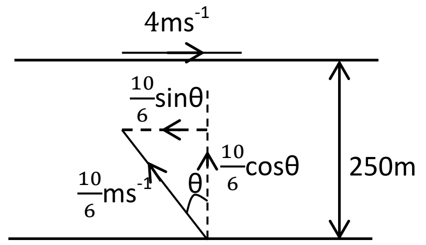
$= \frac{10}{6} ms^{-1}$

Speed of river is $4kmh^{-1}$

$= \frac{4 \times 1000}{3600}$

$= \frac{10}{9} ms^{-1}$

Width of the river = 250m



Resolving velocities

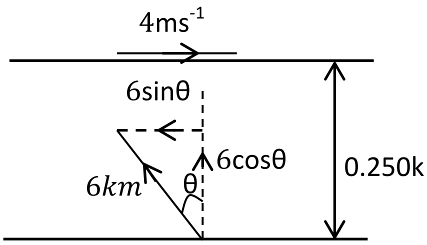
$\frac{10}{6} sin\theta = \frac{10}{9}$

$\sin\theta = \frac{2}{3}$

$\theta = \sin^{-1}\frac{2}{3} = 41.81^0$

Time taken = $\frac{250}{\frac{10}{6}cos41.81} = 201.245s$

OR



Resolving velocities

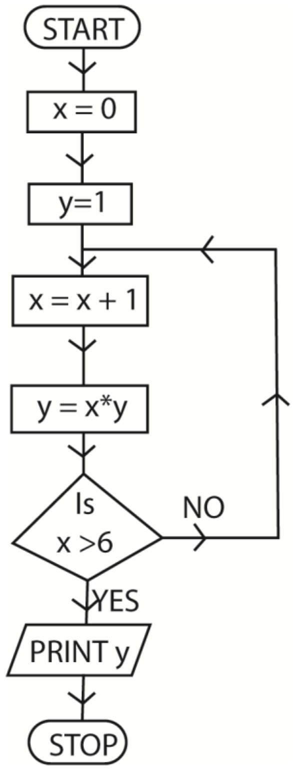
$6sin\theta = 4$

$\sin\theta = \frac{2}{3}$

$\theta = \sin^{-1}\frac{2}{3} = 41.81^0$

Time taken = $\frac{250}{\frac{10}{6}cos41.81} = 201.245s$

6. Study the flow chart given below



(a) Performa dry run

X	Y
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040

(b) What is the purpose of the flow chart

To find the factorial of n where n≤ 7 (i.e. numbers from 0 to 7)

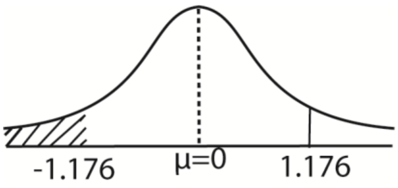
7. Given that X ~N(2,2.89), find P(X<0)

Solution

μ =2 and δ^2 = 2.89 => δ = 1.7

P(X < 0) = $P\left(z < \frac{0-2}{1.7}\right)$

= P(z < -1.176)



By symmetry, P(z< -1.176) = P(z > 1.176)

= 0.5 – P(0< z < 1.176)

= 0.5 – 0.3802

= 0.1198

8. Particles of weight 12N, 8N and 4N act at points (1, -3), (0, 2) and (1, 0) respectively. Find the centre of gravity of the particles.

Solution

Sun of moments = sum of moments

$$12\begin{pmatrix}1\\-3\end{pmatrix} + 8\begin{pmatrix}0\\2\end{pmatrix} + 4\begin{pmatrix}1\\0\end{pmatrix} = 24\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}12\\-36\end{pmatrix} + \begin{pmatrix}0\\16\end{pmatrix} + \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}24\bar{x}\\24\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}16\\-20\end{pmatrix} = \begin{pmatrix}24\bar{x}\\24\bar{y}\end{pmatrix}$$

$\bar{x} = \frac{16}{24} = \frac{2}{3}$

$\bar{y} = \frac{-20}{24} = \frac{-5}{6}$

Hence the centre of gravity of the particles is $\left(\frac{2}{3}, \frac{-5}{6}\right)$

SECTION B

9. The continuous random variable X has the probability density function (p.d.f) given by$f(x) = \begin{cases} k_1x, & 1 \le x \le 3 \\ k_2(4-x), & 3 < x \le 4 \\ 0 & otherwise \end{cases}$
where k_1 and k_2 are constants.
(a) Show that $k_2 = 3k_1$

Solution

For 1 ≤ x ≤ 3, f(x) = k_1x

f(3) = $3k_1$ ……………………..(i)

For 3 < x ≤ 4, f(x) = k1(4 – x)

f(3) = $3k_1(4 - 3) = k_2$ …….(ii)

Eqn. (i) and eqn. (ii)

$k_2 = 3k_1$

(b) Find
(i) The value of k_1 and k_2

Solution

$\int_{-\infty}^{\infty} f(x) = 1$

$k_1 \int_1^3 xdx + k_2 \int_3^4 (4-x)dx = 1$

$k_1\left[\frac{x^2}{2}\right]_1^3 + 3k_1\left[4x - \frac{x^2}{2}\right]_3^4 = 1$

$k_1\left(\frac{9}{2} - \frac{1}{2}\right) + 3k_1\left[(16-8) - \left(12 - \frac{9}{2}\right)\right] = 1$

$4k_1 + 1.5\ k_1 = 1$

$11k_1 = 2$

$k_1 = \frac{2}{11}$

$k_2 = 3 \text{ x} \frac{2}{11} = \frac{6}{11}$

(ii) E(X), the expectation of X

E(X) = $k_1 \int_1^3 x.xdx + k_2 \int_3^4 x(4-x)dx$

= $\frac{2}{11}\int_1^3 x^2dx + \frac{6}{11}\int_3^4 (4x - x^2)dx$

= $\frac{2}{11}\left[\frac{x^3}{3}\right]_1^3 + \frac{6}{11}\left[2x^2 - \frac{x^3}{3}\right]_3^4$

= $\frac{2}{11}\left(\frac{27}{3} - \frac{1}{3}\right) + \frac{6}{11}\left[\left(32 - \frac{64}{3}\right) - \left(18 - \frac{27}{3}\right)\right]$

= $\frac{52}{33} + \frac{10}{11}$

= $\frac{82}{33}$ =2.485(2d.p)

Hence E(X) = 2.485

10. An inelastic string of length a metres is fixed at one end P and carries a particle of mass 3kg at its other end Q. The particle is

describing a horizontal circle of radius 80cm with an angular speed of 5 rad^{-1}. Determine the

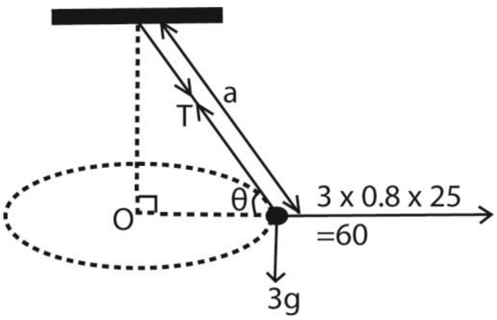
(a) (i) angle the string makes with the horizontal,

(ii) tension in the string

(b) Value of a

(c) Linear speed of the particle

Solution



(a) (i) Let θ be the angle

(↑): Tsinθ = 3g ……………… (i)

(→): Tcosθ = 60 ………………(ii)

Eqn. (i) ÷ eqn. (ii)

tanθ= $\frac{3 \, x \, 9.81}{60}$ = 0.4905

θ = $\tan^{-1} 0.4905$

= 26.13^0

(ii) From equation (i) in(a)(i)

T = $\frac{3 \, x \, 9.81}{sin26.13}$ = 66.82N

(b) acos26.13 = 0.8

a = $\frac{0.8}{\cos 26.13}$ = 0.891m

(c) From v = rω

v = o.8 (5)

= 4ms^{-1}

11. (a) Use the trapezium rule with five subintervals to estimate $\int_0^{\frac{\pi}{3}} tanxdx$ correct to three decimal places.

Solution

Let y = tan x

d = $\frac{\frac{\pi}{3}-0}{5} = \frac{\pi}{15}$

x	y=tanx	
0	0	
$\frac{\pi}{15}$		0.21256
$\frac{2\pi}{15}$		0.44523
$\frac{\pi}{5}$		0.72654
$\frac{4\pi}{15}$		1.11061
$\frac{\pi}{3}$	1.73205	
Sum	1.73205	2.49494

$\int_0^{\frac{\pi}{3}} tanxdx = \frac{1}{2} x \frac{\pi}{15} [1.73205 = 2(2.49494)]$

= $\frac{\pi}{30}$(6.72193)

$= 0.704 \ (3d.p)$

(b) (i) Find the value $\int_0^{\frac{\pi}{3}} tanxdx$ to 3 decimal places

Solution

$\int_0^{\frac{\pi}{3}} tanxdx = \int_0^{\frac{\pi}{3}} \frac{sinx}{cosx} dx$

$= -[Incosx]_0^{\frac{\pi}{3}}$

= -$\left[Incos\frac{\pi}{3} - Incos(0)\right]$

= -(-0.6931471806 – 0)

=0.6931471806

= 0.693(3d.p)

(ii) Calculate the percentage error in your estimation in (a) above.

Solution

Percentage error

= $\left(\frac{0.704-0.693}{0.693}\right) x100\%$

= 1.587%

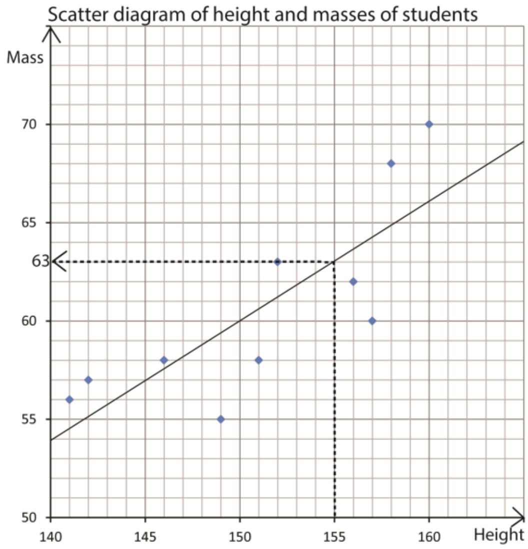
(iii) Suggest how the percentage error may be reduced.

The percentage error may be reduced by increasing the number of subintervals.

12. The heights and masses of ten students are given in the table below

Height (cm)	Mass (kg)
156	62
151	58
152	63
146	58
160	70
157	60
149	55
142	57
158	68
141	56

(a) (i) Plot the data on a scatter diagram.



(ii) Draw the line of best fit. Hence estimate the mass corresponding to height 155cm.

[63 ± 2]

(b) (i) Calculate the rank correlation coefficient for the data.

(ii) Comment on the significance of the height on the masses of students. [Spearman's ρ =0.79 and Kendall's, τ = 0.64 at 1% level of significance bases on 10 observations]

Solution

Either: using Spearman's rank correlation coefficient

Height (x)	Mass (y)	Rx	Ry	d	d^2
156	62	4	4	0	0
151	58	6	6.5	-0.5	0.25
152	63	5	3	2	4
146	58	8	6.5	1.5	2.25
160	70	1	1	0	0
157	60	3	5	-2	4
149	55	7	10	-3	9
142	57	9	8	1	1
158	68	2	2	0	0
141	56	10	9	1	1
					$\sum d^2$ =21.5

Spearman's correlation coefficient ,

ρ = $1 - \frac{6\sum d^2}{n(n^2-1)}$

$=1 - \frac{6x21.5}{10(100-1)}$

=0. 87 (2d.p)

Since 0.87 > 0.79, there is significant correlation or relationship between the height of students and their masses at 1% level.

OR: using Kendall's method

By naming pairs we have

A(156, 62), B(151, 58), C(152, 63), D(146, 58), E(160, 70), F(157, 60), G(149, 53), H(142, 67), I(158, 68) and J(141, 56)

	E	I	F	A	C	B	G	D	H	J
x	1	2	3	4	5	6	7	8	9	10
y	1	2	5	4	3	6.5	10	6.5	8	9
A	9	8	5	5	5	3	0	2	1	**38**
D	0	0	2	1	0	0	3	0	0	**6**

S = 38 – 6 = 32

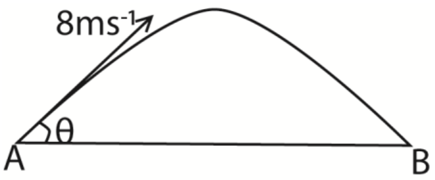
$$\tau = \frac{2S}{n(n-1)} = \frac{2x32}{10x9} = 0.71$$

Since 0.71 > 0.64, there is significant correlation or relationship between the height of students and their masses at 1% level.

13. A football player projected a ball at a speed of 8ms-1 at an angle of 30^0 with the ground. The ball strikes the ground at a point which is level with the point of projection. After impact with the ground, the ball bounced and the horizontal component of velocity of the ball remained the same but the vertical component was reversed in the direction and halved in magnitude. The player running after the ball kicked it again at a point which was at a horizontal distance of 1.0m from the point where it bounced, so the ball continued in the same direction. Find the

(b) Horizontal distance between the point of projection and the point at which the ball first stroked the ground. [Take g = $10ms^{-2}$]

Solution



Let A and B be the point of projection and striking on the ground respectively.

Vertical distance at any time, t is given by y = 8tsin30 - $\frac{1}{2}gt^2$

At point A and B, y = 0

⇨ 0 =8tsin30 - $\frac{1}{2}gt^2$
t(8sin30 - $\frac{1}{2}$gt) = 0

At point B

8sin30 = $\frac{1}{2}$gt

t =$\frac{16sin30}{10} = 0.8s$

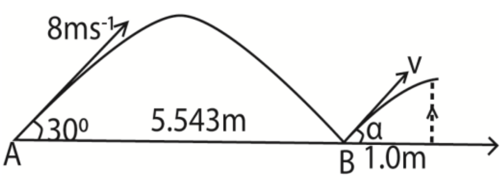
Distance AB = 8tcos30 = 8 x 0.8cos30

= 5.543m

Hence horizontal distance between the point projection of the ball and striking the ground = 5.543m

(c) (i) the time interval between the ball striking the ground and the player kicking it again
(ii) the height of the ball above the ground when it is kicked again.
[Take g = 10ms^{-2}]

Solution



After impact

Horizontal component of velocity,

$v_x = 8\cos30^0$

Vertical component of velocity,

$V_y = 8\sin30^0$

Horizontally, 1.0 = 8cos30^{0}t

t = $\frac{1.0}{8cos30^0} = 0.1443$s (4d.p)

Hence the time taken is 0.1443s

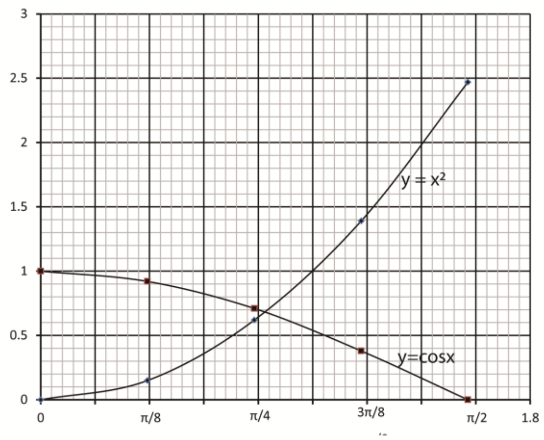
(ii) Verticall, h = 4sin30^{0}t - $\frac{1}{2}$gt^2

= 4sin30^0(0.1443) - $\frac{1}{2}$x10(0.1443)^2

= 0.1845m (4d.p)

Hence vertical distance is 0.1845m

14. (a)(i) On the same axes, draw graphs y = x^2 and y = cosx for 0≤ x ≤$\frac{\pi}{2}$ at intervals of $\frac{\pi}{8}$



(ii) From your graphs, obtain to one decimal place, an approximate root of the equation x^2 – cosx = 0
[0.8(1d.p)]

(b) Using Newton-Raphson method, find the root of the equation x^2 – cosx = 0, taking the approximate root in (a) as an initial approximation. Give your answer correct to three decimal places.

Solution

Let f(x)= x^2 – cosx

$f(x_n) = xn^2 - cox_n$

$f'(x_n) = 2x_n - \sin x_n$

Newton-Raphson method for finding the root is

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{{x_n}^2 - cosx_n}{2x_n - sinx_n}$

$= \frac{2{x_n}^2 + x_n sinx_n - {x_n}^2 + cosx}{2x_n - sinx_n}$

Taking $x_0 = 0.8$

$x_1 = \frac{(0.8)^2 + 0.8sin\left(\frac{180}{\pi}x0.8\right)^0 + cos\left(\frac{180}{\pi}x0.8\right)^0}{2(0.8)^2 + 0.8sin\left(\frac{180}{\pi}x0.8\right)^0}$

$= \frac{1.910591582}{2.317356091} = 0.84470434$

$|error| = 0.84470434 - 0.8$

$= 0.024470434 > 0.0005$

$\therefore x_1 = 0.8447$ (4d.p)

$x_2 = \frac{(0.8447)^2 + 0.8sin\left(\frac{180}{\pi}x0.8447\right)^0 + cos\left(\frac{180}{\pi}x0.8447\right)^0}{2(0.8447)^2 + 0.8sin\left(\frac{180}{\pi}x0.8447\right)^0}$

$= \frac{2.009116708}{2.437171959} = 0.8243639521$

$|error| = 0.8243639521 - 0.8447$

$= 0.0203360479 > 0.0005$

$\therefore x_2 = 0.8244$ (4d.p)

$x_3 = \frac{(0.8244)^2 + 0.8sin\left(\frac{180}{\pi}x0.8244\right)^0 + cos\left(\frac{180}{\pi}x0.8244\right)^0}{2(0.8244)^2 + 0.8sin\left(\frac{180}{\pi}x0.8244\right)^0}$

$= \frac{1.963858373}{2.382940516} = 0.8241323524$

$|error| = 0.8241323524 - 0.8244$

$= 0.000267647634 < 0.0005$

Hence root is 0.824(3d.p)

15. Box A contains 4 red sweets and 3 green sweets. Box B contains 5 red sweets and 6 green sweets. Box A is twice as likely to be picked as box B. If a box is chosen at random and two sweets are removed from it, one at a time without replacement;
    (a) find the probability that the two sweets removed are of the same colour.

    Solution

A
Red = 4
Green = 3
Total = 7

B
Red = 5
Green = 6
Total = 11

Let P(B) = x; then P(A) = 2x

But P(A) + P(B) = 1

2x + x = 1

$x = \frac{1}{3}$

Hence P(A) = $\frac{2}{3}$ and P(B) = $\frac{1}{3}$

P(sweets are of the same colour)

$= P(A \cap R_1 \cap R_2) + P(A \cap G_1 \cap G_2) + P(B \cap R_1 \cap R_2) + P(B \cap G_1 \cap G_2)$

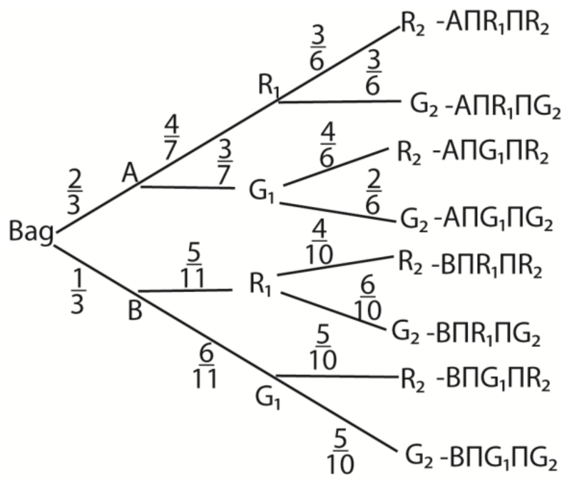
$= \frac{2}{3}x\frac{4}{7}x\frac{3}{6} + \frac{2}{3}x\frac{3}{7}x\frac{2}{6} + \frac{1}{3}x\frac{5}{11}x\frac{4}{10} + \frac{1}{3}x\frac{6}{11}x\frac{5}{10}$

$= \frac{24}{126} + \frac{12}{126} + \frac{20}{330} + \frac{30}{330}$

=0.4372 (4d,p)

(b) (i) construct a probability distribution table for the number of red sweets removed
    (ii) find the mean number of red sweets removed

Solution



(i) Let x = number of red sweets removed

$P(x = 0) = P(A \cap G_1 \cap G_2) + P(B \cap G_1 \cap G_2)$

$= \frac{2}{3}x\frac{3}{7}x\frac{2}{6} + \frac{1}{3}x\frac{6}{11}x\frac{5}{11} = 0.1861$

$P(x = 1) = P(A \cap R_1 \cap G_2) + P(A \cap G_1 \cap R_2) + P(B \cap R_1 \cap G_2) + P(B \cap G_1 \cap R_2)$

$= \frac{2}{3}x\frac{4}{7}x\frac{3}{6} + \frac{2}{3}x\frac{3}{7}x\frac{4}{6} + \frac{1}{3}x\frac{5}{11}x\frac{6}{10} + \frac{1}{3}x\frac{6}{11}x\frac{5}{10}$

$=\frac{24}{126}+\frac{24}{126}+\frac{30}{330}+\frac{30}{330}=0.5628$

$P(x = 2) = P(A \cap R_1 \cap R_2) + P(B \cap R_1 \cap R_2)$

$=\frac{2}{3}x\frac{4}{7}x\frac{3}{6}+\frac{1}{3}x\frac{5}{11}x\frac{4}{10}$ =0.2511

x	0	1	2
P(X = x)	0.1861	0.5628	0.2511
xP(X=x)	0	0.5628	0.5022

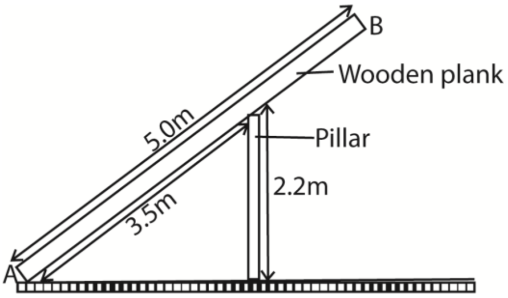
E(x) $=\sum_{x=0}^{x=2} xP(X=x)$

= 0 + 0.5628 + 0.5022

= 1.065

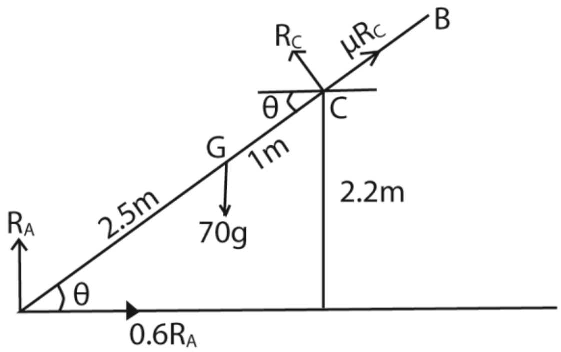
Hence the mean of red sweets removed is 1

16. The diagram below shows a uniform wooden plank AB of mass 70kg and length 5m. the end A rests on a rough horizontal ground. The plank is in contact with the top of a rough pillar at C. The height of the pillar is 2.2m and AC = 3.5m



Given that the coefficient of friction at the ground is 0.6and the plank is just to slip, find the

(a) angle the plank makes with the ground at A



sinθ = $\frac{2.2}{3.5}$

θ = $\sin^{-1}\frac{2.2}{3.5}$ = 38.945^0

(b) normal reaction at

(i) A

C: R_A x $3.5\cos(38.945^0)$

= $0.6R_A$x 2.2 + 70g x 1 $\cos(38.945^0)$

R_A [$3.5\cos(38.945^0)$-1.32]

= 70x9.8$\cos(38.945^0)$

$R_A=\frac{686\cos(38.945)}{1.402123976}$ =380.52N (2d.p)

A: R_C x 3 = 70x9.8$\cos(38.945^0)$ x 2.5

$R_C=\frac{2.5x686\cos(38.945)}{3.5}$

= 381.1N (1d.p)

(c) Coefficient of friction at C.

Solution

(→); 0.6 x 380.52 + 381.1$\cos(38.945^0)\mu$

=381.1$\cos(51.055^0)$

381.1$\cos(38.945^0)\mu$

= 381.1$\cos(51.055^0)$-228.312

$\mu=\frac{11.23570175}{381.1\cos(38.945^0)}$

= 0.0379 (4d.p)

Thank you

Dr. Bbosa Science