UACE MATHEMATICS PAPER 1 2012 and marking guides

Section A

1. The force $\binom{0}{2}$, $\binom{0}{4}$, $\binom{0}{3}$ and $\binom{0}{a}$N at points (p, 1), (2, 3), (4, 5) and (6, 1) respectively. The resultant force is $\binom{0}{3}$ acting at (1, 1). Find the values of a and p.
2. Two events A and B are such that P(A) = $\frac{1}{5}$ and P(B) = $\frac{1}{2}$. Find P(AUB) when A and B are:
   (a) independent
   (b) mutually exclusive events
3. Use trapezium rule with four sub-interval places to estimate $\int_0^4 \frac{1}{1+sinx}$dx. Give your answer correct to 3 decimal places
4. a) Show that the final velocity v of the body which starts with initial velocity u and moves with uniform acceleration a consequently covering a distance x, is given by v = $[u2 + 2ax]^{\frac{1}{2}}$
   b) Find the value of x in (a) if v = 30m/s, u = 10m/s and a = 5m/s^2.
5. A teacher gave two tests in chemistry. Five students were graded as follows

	Grade
Test 1	A B C D E
Test II	B A C D E

   Determine the rank correlation coefficient between the two tests on your result.
6. A light inextensible string passes over a smooth pulley fixed at the top of a smooth plane inclined at 300 to the horizontal. A mass of 4kg is attached to one end of the string and hangs freely. A mass, m is attached to the other end of the string and rests on the inclined plane. if the system is in equilibrium, find m.
7. The table below shows the cost y shillings for hiring a motor cycle for a distance x kilometres.

x(km)	10	20	30	40
Cost(shs)	2800	3600	4400	5200

   Use linear interpolation or extrapolation to calculate the:
   (a) cost of hiring the motorcycle for a distance of 45km
   (b) the distance mukasa travelled if he paid shs. 4000.
8. A random variable X has the following probability distribution P(X = 0) = $\frac{1}{8}$, P(X = 1) = P(X = 2) = $\frac{3}{8}$ and P(X = 3) = $\frac{1}{8}$. Find the
   (a) Mean value of x
   (b) Variance of x.

Section B

9. The table below shows the marks obtained in an examination of 200 candidates

Marks (%)	Number of candidates
10 – 19	18
20 – 29	34
30 – 39	58
40 – 49	42
50 – 59	24
60 – 69	10
70 – 79	6
80 – 89	8

(a) Calculate the
   (i) Mean mark
   (ii) Model mark
(b) Draw a cumulative frequency curve for the data.
   Hence estimate the lowest mark for a distinction one if the top 5% of the candidates qualify for the distinction.

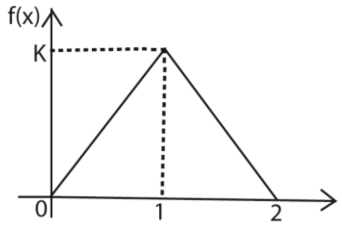
10. At 11.45 a.m., ship A has position vector $\begin{pmatrix}0\\2\end{pmatrix} km$ and moving at 8kmh^{-1} in the direction N300E.

At 12noon, another ship B has position vector $\begin{pmatrix}0\\2\end{pmatrix}$km and moving at 3kmh^{-1} in the direction South East.

(a) Find the position vector of ship A at noon
(b) If the ship after 12 noon maintains their courses, find
   (i) time when they are closest
   (ii) the least distance between them

11. (a) (i) Show that the equation e^x -2x – 1 = 0 has a root between x = 1 and x = 1.5.
   (ii) Use linear interpolation to obtain an approximation for the root.
(b) (i) Solve the equation in (a)(i), Using each formula below twice
   Take the approximation in (a)(ii) as the initial value.
   Formula I: $x_{n+1} = \frac{1}{2}(e^{x_n} + 1)$
   Formula I: $x_{n+1} = \frac{e^{x_n}(e^{x_n}-1)+1}{e^{x_n}-2}$
   (ii) Deduce with a reason which of the two formulae is appropriate for solving the given equation in (a)(i).
   Hence write down a better approximate root, correct to two decimal places.

12. A continuous random variable X has a probability density function (p.d.f) f(x) as shown in the graph below



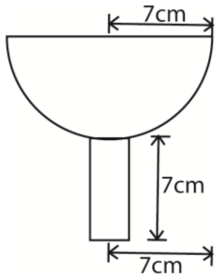
(a) Find the
   (i) value of K
   (ii) expression for the probability density function (p.d.f)of X
(b) Calculate the

(i) Mean of X

(ii) $P(X<1.5/X>0.5)$

13. (a) Show that the centre of gravity of a uniform thin hemispherical cup of radius, r is at a distance $\frac{r}{2}$ from the base

    (b) The figure below is made of a thin hemispherical cup of radius 7cm. It is welded to a stem of length 7cm. The mass of the stem is one quarter that of the cup.



    Find the distance from the base, of the centre of gravity.

14. (a) The length, width and height of a tank were all rounded off to 3.65cm, 2.14m and 2.5mrespectively.

    Determine in m^3 the least and greatest amount of water the tank can contain.

    (b) A shop offered 25% discount on all items in its store and a second discount of 5% to any customer who paid cash

    (i) Construct a flow chart which shows the amount paid for each item.

    (ii) Using the flow chart in (b)(i) , compute the amount paid for the following items

Item	price	Mode of payment
mattress	125,000	cash
TV	340,000	credit

15. (a) A box of oranges contains 20 good and 4 bad oranges. If 5 oranges are picked at random, determine the probability that 4 are good and the other is bad

    (b) An examination has 100 questions. A student has 60% chance of getting each question correct. A student fails the examination for a mark less than 55. A student gets a distinction for a mark 68 or more. Calculate the probability that a student

    (i) fails the examination

    (ii) gets a distinction

16. A gun of mass 3000kg fires horizontally a shell at initial velocity of $300ms^{-1}$. If the recoil of the gun is brought to rest by a constant opposing force 9000N in two seconds, find the

    (a) (i) initial velocity of recoil gun.

    (ii) mass of the shell

    (iii) gain in kinetic energy of the shell just after firing

    (b) (i) displacement of the gun

    (ii) Work done against the opposing force.

END

Marking guide

Section A

1. The force$\begin{pmatrix}0\\2\end{pmatrix}$, $\begin{pmatrix}0\\4\end{pmatrix}$, $\begin{pmatrix}0\\3\end{pmatrix}$ and$\begin{pmatrix}0\\a\end{pmatrix}$N at points (p, 1), (2, 3), (4, 5) and (6, 1) respectively. The resultant force is $\begin{pmatrix}0\\3\end{pmatrix}$ acting at (1, 1). Find the values of a and p.

   Solution

   By resolution of forces

   $\begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix} + \begin{pmatrix}0\\4\end{pmatrix} + \begin{pmatrix}0\\3\end{pmatrix} + \begin{pmatrix}0\\a\end{pmatrix}$

   $\begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}0\\9+a\end{pmatrix}$

   a = -6

   Taking moments about the origin

   Using analytical approach

   Sum of moments of individual forces = moment of the resultant.

   $\begin{vmatrix}p & 0\\1 & 2\end{vmatrix} + \begin{vmatrix}2 & 0\\3 & 4\end{vmatrix} + \begin{vmatrix}4 & 0\\5 & 3\end{vmatrix} + \begin{vmatrix}6 & 0\\1 & a\end{vmatrix} = \begin{vmatrix}1 & 0\\1 & 3\end{vmatrix}$

   2p – 0 + 8 – 0 + 12 – 0+6a – 0 = 3 – 0

   2p + 8 + 12 + 6(-6) = 3

   2p + 8 + 12 - 36 = 3

   2p = 19

   p = 9.2

   Hence a = -6 and p = 9.5

2. Two events A and B are such that P(A) = $\frac{1}{5}$ and P(B) = $\frac{1}{2}$. Find P(AUB) when A and B are:
   (a) independent
   Solution
   P(AUB) = P(A) + P(B) – P(A).P(B)
   $=\frac{1}{5}+\frac{1}{2}-\frac{1}{5}.\frac{1}{2}$
   $=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}$
   (b) mutually exclusive events
   Solution
   P(AUB) = P(A) + P(B)
   $=\frac{1}{5}+\frac{1}{2}$
   $=\frac{7}{10}$

3. Use trapezium rule with four sub-interval places to estimate $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}$dx. Give your answer correct to 3 decimal places
   Solution
   d = $\frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

x	y	
0 $\frac{\pi}{8}$ $\frac{\pi}{4}$ $\frac{\pi}{2}$	1 0.5	0.72323 0.58579 0.51978
sum	1.5	1.82880

   $\int_0^{\frac{\pi}{2}} \frac{1}{1+sinx}$dx= $\frac{1}{2} x \frac{\pi}{8}(1.5 + 2\ x\ 1.82880)$
   = $\frac{\pi}{16}(1.5 + 3.6576)$
   = 1.012692
   =1.013(3d.p)

4. (a) Show that the final velocity v of the body which starts with initial velocity u and moves with uniform acceleration a consequently covering a distance x, is given by v = $[u^2 + 2ax]^{\frac{1}{2}}$
   Solution
   Using $s = \frac{t}{2}(u+v)$ and v = u + at
   $$t = \frac{v-u}{a}$$
   Substituting for s = x and t
   $$x = \frac{\left(\frac{v-u}{a}\right)}{2}(u+v) = \left(\frac{v-u}{2a}\right)(u+v)$$
   $2ax = v^2 - u^2$
   v = $[u^2 + 2ax]^{\frac{1}{2}}$

   (b) Find the value of x in (a) if v = 30m/s, u = 10m/s and a = 5m/s^2.

   Solution

   Substituting for v = 30m/s, u = 10m/s and a = 5m/s^2.

   30 = $[10^2 + 10x]^{\frac{1}{2}}$
   900 = 100 + 10x
   x = 80m

5. A teacher gave two tests in chemistry. Five students were graded as follows

	Grade
Test 1	A B C D E
Test II	B A C D E

Determine the rank correlation coefficient between the two tests on your result.
Solution
Using spearman's method of rank correlation

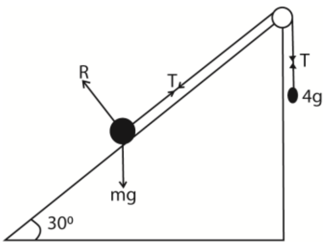
Test 1	Test II	R_1	R_2	d	d^2
A	B	1	2	-1	1
B	A	2	1	1	1
C	C	3	3	0	0
D	D	4	4	0	0
E	E	5	5	0	0
					$\sum d^2 = 2$

$p = 1 - \frac{6\sum d^2}{n(n^2-1)} = \frac{6\ x\ 2}{5(25-1)} = 1 - \frac{12}{120}$
$= 0.9$
There is a very high positive correlation between the two tests.
OR:
Using Kendall's method

	A	B	C	D	E
Tests I	1	2	3	4	5
Test II	2	1	3	4	5
Agreements	3	3	2	1	=9
Disagreement	1	0	0	0	=1

s = 9 – 1 = 8
$\tau = \frac{2s}{n(n-1)} = \frac{2x8}{5(5-1)} = \frac{16}{20} = 0.8$
There is high positive correlation between the two tests

6. A light inextensible string passes over a smooth pulley fixed at the top of a smooth plane inclined at 300 to the horizontal. A mass of 4kg is attached to one end of the string and hangs freely. A mass, m is attached to the other end of the string and rests on the inclined plane. if the system is in equilibrium, find m.



(↑): T – 4g = 0

T=4g ………………… (i)

(↗) T – mg $\sin 30^0$ = 0

T = mg$\sin 30^0$ ……..(ii)

Equating eqn. (i) and eqn. (ii)

4g = mg$\sin 30^0$

m = $\frac{4}{\sin 30^0} = 8kg$

7. The table below shows the cost y shillings for hiring a motor cycle for a distance x kilometres.

x(km)	10	20	30	40
Cost(shs)	2800	3600	4400	5200

Use linear interpolation or extrapolation to calculate the:
(a) cost of hiring the motorcycle for a distance of 45km
Solution
Extract

A	B	C
30	40	45
4400	5200	x

Gradient AB = gradient BC
$\frac{5200-4400}{40-30} = \frac{x-5200}{45-40}$
$\frac{800}{10} = \frac{x-5200}{5}$
x = shs.5600
Hence the cost of hiring a motorcycle for a distance of 45km is shs. 5600

(b) the distance mukasa travelled if he paid shs. 4000.
Extract

A	B	C
30	x	40
3600	4000	5200

Gradient AB = gradient BC
$\frac{4000-3600}{x-30} = \frac{5200-4000}{40-x}$
$\frac{400}{x-30} = \frac{1200}{40-x}$
400(40-x) = 1200(x – 30)
4 (40-x) = 12 (x – 30)
16x = 520
x = 32.5km

Hence the distance that costs shs. 4000 is 32.5km

8. A random variable X has the following probability distribution $P(X = 0) = \frac{1}{8}$, $P(X = 1) = P(X = 2) = \frac{3}{8}$ and $P(X = 3) = \frac{1}{8}$. Find the

Solution

x	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
xP(X = x)	0	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{3}{8}$
$X^2P(X = x)$	0	$\frac{3}{8}$	$\frac{12}{8}$	$\frac{9}{8}$

(a) Mean value of x

Mean = E(X) $=\frac{3}{8} + \frac{6}{8}+\frac{3}{8}$ =1.5

(b) Variance of x.

$Var(X) = E(x^2) - (E(x))^2$

$E(x^2) = \frac{3}{8} + \frac{12}{8} + \frac{9}{8} = 3$

$Var(X) = 3 - (1.5)^2 = 0.75$

Section B

9. The table below shows the marks obtained in an examination of 200 candidates

Marks (%)	Number of candidates
10 – 19	18
20 – 29	34
30 – 39	58
40 – 49	42
50 – 59	24
60 – 69	10
70 – 79	6
80 – 89	8

(a) Calculate the
   (i) Mean mark
   (ii) Model mark

(b) Draw a cumulative frequency curve for the data.
Hence estimate the lowest mark for a distinction one if the top 5% of the candidates qualify for the distinction.

Solution

Marks (%)	X	f	fx	cf
10 – 19	14.5	18	261	18
20 – 29	24.5	34	833	52
30 – 39	34.5	58	2001	110
40 – 49	44.5	42	1869	152
50 – 59	54.5	24	1308	176
60 – 69	64.5	10	645	186
70 – 79	74.5	6	447	192
80 – 89	84.3	8	676	200
		∑f=200	∑fx=8040	

(a) (i) mean = $\frac{\sum fx}{\sum f} = \frac{8040}{200} = 40.2\%$

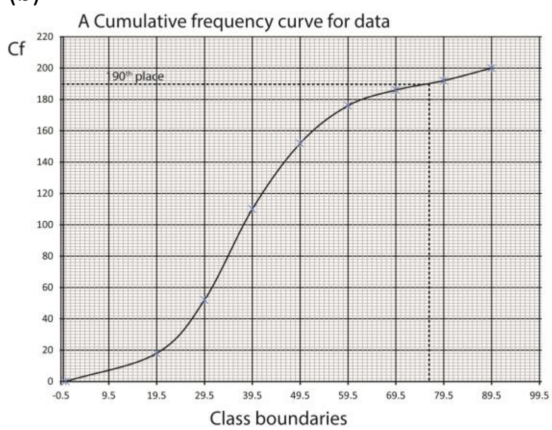
(ii) mode = Li+$\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) c$

$=29.5 +10 \left(\frac{24}{24+16}\right)$

$= 29.5 + \frac{240}{40}$

=35.5%

(b)



5% of 200= $\frac{5}{100} x200 = 10$

From the graph the 190th value is 76.5

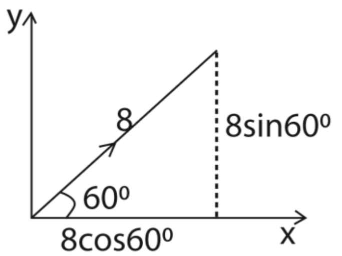
Hence the lowest mark is 76.5%

10. At 11.45 a.m, ship A has position vector $\begin{pmatrix}5\\1\end{pmatrix} km$ and moving at 8kmh^{-1} in the direction N30^{0}E. At 12noon, another ship B has position vector $\begin{pmatrix}8\\7\end{pmatrix}$km and moving at 1.5kmh^{-1} in the direction South East.

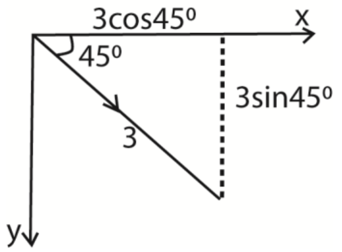
(a) Find the position vector of ship A at noon

Solution

Let V_a = velocity of A
r_a = position vector of A
Vb = velocity of B
r_b = position vector of B



$V_a = \begin{pmatrix} 8cos60^0 \\ 8sin60^0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4\sqrt{3} \end{pmatrix}$



$V_b = \begin{pmatrix} 3cos45^0 \\ 3sin45^0 \end{pmatrix} = \begin{pmatrix} 1.5\sqrt{2} \\ -1.5\sqrt{2} \end{pmatrix}$

Let ra = position vector of A at time t

At noon t = $\frac{15}{60} = \frac{1}{4}$

$r_a = \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 4\sqrt{3} \end{pmatrix} t$

$= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 4\sqrt{3} \end{pmatrix} \cdot \frac{1}{4}$

$= \begin{pmatrix} 6 \\ 1+\sqrt{3} \end{pmatrix} = \begin{pmatrix} 6 \\ 2.73205 \end{pmatrix}$km

(b) If the ship after 12 noon maintains their courses, find

(i) time when they are closest

(ii) the least distance between them

Solution

a = $\begin{pmatrix} 6 \\ 1+\sqrt{3} \end{pmatrix}$

b = $\begin{pmatrix} 8 \\ 7 \end{pmatrix}$

Let t = time taken for ships to be closest to each other.

r = a + Vt

$r_a = \begin{pmatrix} 6 \\ 1+\sqrt{3} \end{pmatrix} + \begin{pmatrix} 4 \\ 4\sqrt{3} \end{pmatrix} t$

$r_b = \begin{pmatrix} 8 \\ 7 \end{pmatrix} + \begin{pmatrix} 1.5\sqrt{2} \\ -1.5\sqrt{2} \end{pmatrix} t$

${}_Ar_B = r_a - r_b$

$= \begin{pmatrix} -2 \\ -6+\sqrt{3} \end{pmatrix} + \begin{pmatrix} 4-1.5\sqrt{2} \\ 4\sqrt{3}+1.5\sqrt{2} \end{pmatrix} t$

${}_Ar_B = r_a - r_b = \begin{pmatrix} -2 \\ -4.268 \end{pmatrix} + \begin{pmatrix} 1.87868 \\ 9.04952 \end{pmatrix} t$

$= \begin{pmatrix} -2 + 1.87868t \\ -4.268 + 9.04952t \end{pmatrix}$

$({}_Ar_B)^2 = (-2 + 1.87868t)^2 + (-4.268 + 9.04952t)^2$

$= 85.42325t^2 - 84.7617t + 22.2158$

The minimum distance, ds is attained when

$\frac{d}{dx}({}_Ar_B) = \frac{d}{dx}({}_Ar_B)^2 = 0$

⇨ 170.8465t – 84.7617 = 0

t = $\frac{84.7617}{170.8465} x60 = 30min$ 44mins

Time = 12.00+ 30min = 12:30pm

Minimum distance, ds = $|{}_Ar_B|$

$d_s{}^2 = (-2 + 1.87868x0.5)^2 + (-4.268 + 9.04952x0.5)^2$

=1.191

d_s = 1.091km

Or

For minimum distance to be reached

${}_AV_B \cdot {}_Ar_B = 0$

Naw ${}_AV_B = V_A - V_B$

$= \begin{pmatrix} 1.87868 \\ 9.04952 \end{pmatrix}$

⇨ $\begin{pmatrix} -2 + 1.87868t \\ -4.26795 + 9.04952t \end{pmatrix} \cdot \begin{pmatrix} 1.87868 \\ 9.04952 \end{pmatrix} = 0$

85.42325t = 42.3803

$t = 0.5$h

Hence ds = 1.091km as before

11. (a) (i) Show that the equation $e^x - 2x - 1 = 0$ has a root between x = 1 and x = 1.5.

Solution

Using sign change method

Let $f(x) = e^x - 2x - 1$

f(1) = e – 2 – 1 = -2.817

f(1.5) = $e^{1.5}$ – 3-1 = 0.4817

Since f(1).f(1.5) < 0, the root lies between x = 1 and x = 1.5.

(ii) Use linear interpolation to obtain an approximation for the root.

Solution

Extract

A	B	C
1	x	1.5
-0.2817	0	0.4817

Gradient AB = gradient AC

$$\frac{0+0.2817}{x-1} = \frac{0.4817+0.2817}{1.5-1}$$

$$\frac{0.2817}{x-1} = \frac{0.7634}{0.5}$$

x = 1.1845

(c) (i) Solve the equation in (a)(i), Using each formula below twice
Take the approximation in (a)(ii) as the initial value.

Formula I: $x_{n+1} = \frac{1}{2}(e^{x_n} + 1)$

Solution

x_0 =1.18

$x_1 = \frac{1}{2}(e^{1.18} + 1) = 2.127187$

$x_2 = \frac{1}{2}(e^{2.127187} + 1) = 4.6956145$

Formula I: $x_{n+1} = \frac{e^{x_n}(x_n-1)+1}{e^{x_n}-2}$

Solution

x_0 =1.18

$$x_1 = \frac{e^{1.18}(1.18-1)+1}{e^{1.18}-2} = 1.2642$$

$$x_2 = \frac{e^{1.2642}(1.2642-1)+1}{e^{1.2642}-2} = 1.2565$$

(ii) Deduce with a reason which of the two formulae is appropriate for solving the given equation in (a)(i). Hence write down a better approximate root, correct to two decimal places.

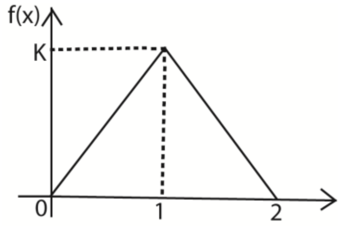
Solution

In formula I, the sequence 1.18, 2.12187, 4.6926 diverges, hence the formula is not suitable

In formula II. The sequence 1.18, 1.2642, 1.2565 converges, hence the formula is suitable for solving equation.

A better approximation = 1.26 (2d.p)

12. A continuous random variable X has a probability density function (p.d.f) f(x) as shown in the graph below



(a) Find the

(i) value of K

Solution

Area of the graph = 1

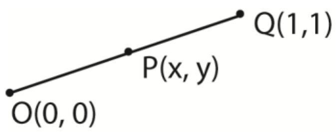
⇨ $\frac{1}{2}x2\ x\ K = 1$

$K = 1$

(ii) expression for the probability density function (p.d.f)of X

Solution

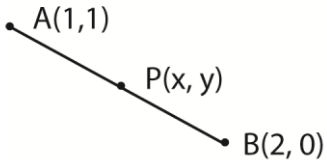
Let f(x) = y



Gradient OP = gradient OQ

$\frac{y-0}{x-0} = \frac{1-0}{1-0}$

y = x



Gradient AP = gradient AB

$\frac{y-1}{x-1} = \frac{0-1}{2-1}$

y = 2-x

$$f(x) \begin{cases} x & 0 \le x \le 1 \\ 2-2 & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

(b) Calculate the

(i) Mean of X

Solution

E(x) = $\int_{all} xf(x)dx$

$=\int_0^1 x.xdx + \int_1^2 x(2-x)dx$

$=\left[\frac{x^3}{3}\right]_0^1 + \left[x^2 - \frac{x^3}{3}\right]_1^2$

$=\left(\frac{1}{3}-0\right) + \left[\left(4-\frac{8}{3}\right) - \left(1-\frac{1}{3}\right)\right]$

$=\frac{1}{3}+\frac{4}{3}-\frac{2}{3} = 1$

(ii) P(X<1.5/X>0.5)

Solution

P(X<1.5/X>0.5) = $\frac{p(x<1\cap x>0.5)}{P(x>0.5)}$

$= \frac{P(0.5<x<1.5)}{1-P(x<0.5)}$

$= \frac{\int_{0.5}^1 xdx + \int_1^{1.5}(2-x)dx}{1-\int_0^{0.5} xdx}$

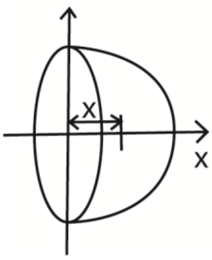
$= \frac{\left[\frac{x^2}{2}\right]_{0.5}^1 + \left[2x-\frac{x^2}{2}\right]_1^{1.5}}{1-\left[\frac{x^2}{2}\right]_0^{0.5}}$

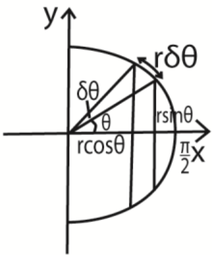
$= \frac{(0.5-0.125)+(1.875-1.5)}{0.875}$

$= \frac{0.75}{0.875} = 0.8751$

13. (a) Show that the centre of gravity of a uniform thin hemispherical cup of radius, r is at a distance $\frac{r}{2}$ from the base

Solution





If we divide the hemisphere into slices parallel to its plane face each slice is approximately a circular ring with its centre of gravity at its centre.

Considering a ring shown in the diagram above, its is approximately a cylinder of radius rsinθ and width rδθ

Surface area of the ring = (2πrsinθ)rδθ

Let w = weight per unit area.

Body	weight	Distance of C.O.G from y-axis
Element (ring)	$(2\pi r^2\sin\theta)w$	rcosθ
Hemisphere	$2\pi r^2 w$	$\overline{x}$

Equating moments along y-axis

$\int_0^{\frac{\pi}{2}} 2\pi r^2 \sin\theta w)r\cos\theta d\theta = 2\pi r^2 w\overline{x}$

$\pi r^3 w \int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta d\theta = 2\pi r^2 w\overline{x}$

$r\int_0^{\frac{\pi}{2}} \sin 2\theta\ d\theta = 2\overline{x}$

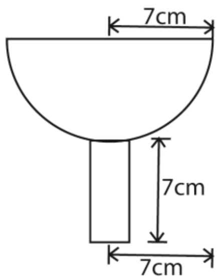
$-\frac{1}{2}r[cos 2\theta]_0^{\frac{\pi}{2}} = 2\overline{x}$

$-\frac{1}{2}r(-1-1) = 2\overline{x}$

$r = 2\overline{x}$

$\overline{x} = \frac{r}{2}$

(b) The figure below is made of a thin hemispherical cup of radius 7cm. It is welded to a stem of length 7cm. The mass of the stem is one quarter that of the cup.



Find the distance from the base, of the centre of gravity.

Solution

Area of hemispherical cup = $2\pi r^2$

= 2π(49)

= 98π

Area of a circle = πr^2

= π(49)

= 49π

Let w be weight per unit area

Body	weight	Distance of C.O.G from y-axis
Circular base	49πw	0
stem	$\frac{1}{4}(98\pi w)$	3.5cm
Hemispherical cup	98πw	10.5cm
Whole figure	171.5 πw	$\bar{x}$

Sum of moments = sum of moments

49πw x 0 +$\frac{1}{4}(98\pi w)$ x 3.5 + 98πw x 10.5

=171.5 πw$\bar{x}$

0 + 85.75 πw + 1029 πw = 171.5 πw$\bar{x}$

1114.75 πw= 171.5 πw$\bar{x}$

$\bar{x} = \frac{1114.75}{171.5} = 6.5$

Hence the distance from the base of the centre of gravity of the figure is 6.5cm

14. (a) The length, width and height of a tank were all rounded off to 3.65cm, 2.14m and 2.5m respectively. Determine in m^3 the least and greatest amount of water the tank can contain.

Solution

Let V = volume of water in the tank

L= length of the tank

W = width of the tank

H = height of the tank

V = LWH

Maximum volume (Vmax)

Vmax = (L + Δl)(W + Δw)(H + Δh)

= (3.65+0.005)(2.14+0.005)(2.5+0.05)

=(3.655)(2.145)(2.55)

= 19.992m^3 (3d.p)

Minimum volume(Vmin)

Vmin = (L - Δl)(W - Δw)(H - Δh)

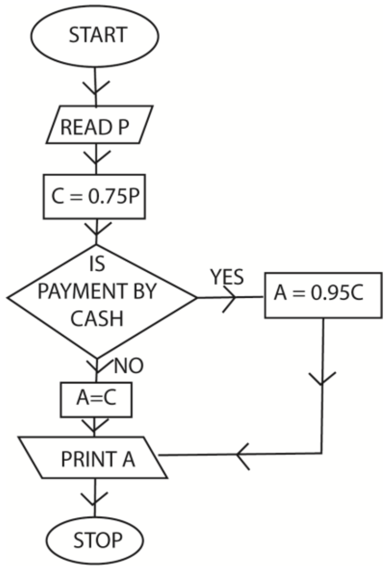
= (3.65- 0.005)(2.14-0.005)(2.5-0.05)

=(3.645)(2.135)(2.45)

= 19.066m^3 (3d.p)

(b) A shop offered 25% discount on all items in its store and a second discount of 5% to any customer who paid cash

(i) Construct a flow chart which shows the amount paid for each item.



(ii) Using the flow chart in (b)(i) , compute the amount paid for the following items

Item	price	Mode of payment
mattress	125,000	cash
TV	340,000	credit

Solution

Amount paid for mattress is

A = 0.95C

= 0.95 x 0.75 x 125,000

= 89062.5

Amount paid for TV is

A = 0.75P

= 0.75 x 340,000

= 225000

15. (a) A box of oranges contains 20 good and 4 bad oranges. If 5 oranges are picked at random, determine the probability that 4 are good and the other is bad

Solution

P(X = 4) = $\frac{{}^{20}C_4 . {}^{4}C_1}{{}^{24}C_5} = 0.456$

(b) An examination has 100 questions. A student has 60% chance of getting each question correct. A student fails the examination for a mark less than 55. A student gets a distinction for a mark 68 or more. Calculate the probability that a student

(i) fails the examination

(ii) gets a distinction

Solution

n = 100

p = 60% = 0.6

q = 40%= 0.4

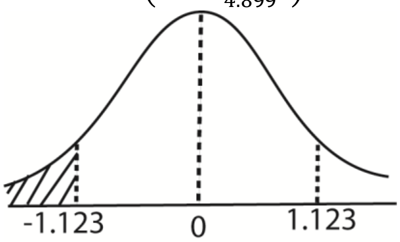
μ = np = 100 x 0.6 = 60

σ = $\sqrt{npq}$= $\sqrt{100\ x\ 0.6\ x\ 0.4}$ = 4.8990

let x = mark scored

(i) P(x<55) = P(x≤ 54)

$= P\left(z \le \frac{54.5-60}{4.899}\right)$



P(z< -1.123) = P(z>1.123)

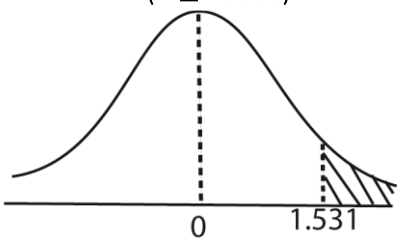
= 0.5 –(P(0 < z< 1.123)

= 0.5 – 0.3692

= 0.1308

(ii) P(x ≥ 68) = $P\left(z \le \frac{67.5-60}{4.899}\right)$

=P(z ≥1.5309)



P(z ≥ 1.531) = 0.5 – P(0 ≤ z ≤ 1.531)

= 0.5 – 0.4371

= 0.0629

16. A gun of mass 3000kg fires horizontally a shell at initial velocity of 300ms^{-1}. If the recoil of the gun is brought to rest by a constant opposing force 9000N in two seconds, find the

(a) (i) initial velocity of recoil gun.

Solution

F = ma

9000 = -3000a

a = -3ms^{-1}

using v = u + at

0 = u – 3x2

u = 6ms^{-1}

(ii) mass of the shell

Solution

Back momentum

= forward momentum

3000 x 6 =m x 300

m = $\frac{3000\ x\ 6}{300} = 60kg$

(iii) gain in kinetic energy of the shell just after firing

Solution

Gain in K.E = $\frac{1}{2}mv^2$

=$\frac{1}{2}x60\ x\ 300^2$

=$2700{,}000J$

=$2700kJ$

(b) (i) displacement of the gun

Solution

$v^2 = u^2 + 2as$

$0 = 6^2 - 2x\ 3\ x\ s$

6s = 36

s = 6m

(ii) Work done against the opposing force.

Work = force x distance

= 9000 x 6

= 54,000J = 54kJ

Thank you

Dr. Bbosa Science