UACE MATHEMATICS PAPER 1 2011 and marking guides

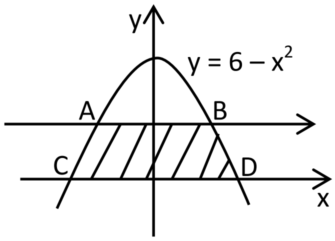
Section A

1. Solve the equation $\log_{25} 4x^2 = \log_5(3 - x^2)$
2. Find the equation of a line through the point (2, 3) and perpendicular to the line x + 2y + 5 = 0
3. Evaluate $\int_1^3 \left(\frac{3x^2+4x+1}{x^3+2x^2+x}\right) dx$
4. A committee of 4 men and 3 women is to be formed from 10 men and 8women. In how many ways can the committee be formed.
5. Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{7}{9}\right)$
6. Given the $R = q\sqrt{1000 - q^2}$; find
   (a) $\frac{dR}{dq}$
   (b) the value of q when R is maximum
7. Show that the points A, B and C with position vector 3i + 3j + k, 7i + 4k and 11i + 4j + 5k respectively are vertices of a triangle.
8. (a) form a differential equation by eliminating the constants a and b from x = accost + bsint
   (b) State the order of the differential equation formed in (a) above.

SECTION B

9. (a) The first term of an Arithmetic Progression (A.P) is ½. The sixth term of A.P is four times the fourth term. Find the common difference of the A.P.
   (b) The roots of a quadratic equation $x^2 + px + q = 0$ are α and β. Show that the quadratic equation whose roots are $\alpha^2 - q\alpha$ and $\beta^2 - q$ is given by $x^2 - (p^2+pq-2q)x + q^2(q + p + 1) = 0$.
10. (a) form a quadratic equation having -3 + 4i as one of its roots.
    (b) Given $z_1 = -1 + i\sqrt{3}$ and $z_2 = -1 - i\sqrt{3}$
       (i) Express $\frac{z_1}{z_2}$ in the form $a + i\sqrt{b}$, where a and b are real numbers
       (ii) Represent $\frac{z_1}{z_2}$ on an Argand diagram.
       (iii) Find $\left|\frac{z_1}{z_2}\right|$

11. In the diagram below, the curve $y = 6 - x^2$ meets the line y = 2 at A and B and the x-axis at A and D.



Find

(a) coordinates of A, B, C, and D

(b) area of the shaded region, correct to one decimal place.

12. (a) Find the angle between the line x = $\frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}$

(b) Find in vector form the equation of the line of intersection of two planes 2x + 3y – z = 4 and x – y + 2z = 5.

13. (a) Find the equation of the tangent to parabola $y^2 = \frac{x}{16}$ at $\left(t^2, \frac{1}{4}\right)$.

(b) If the tangent to the parabola in (a) above at points P$\left(p^2, \frac{1}{p}\right)$ and Q$\left(q^2, \frac{1}{q}\right)$ meet on the line y = 2.

(i) show that p + q = 16

(ii) deduce that the midpoint of PQ lies on the line y = 2

14. (a) Solve 3sinx + 4cox = 2 for $-180^0 \le x \le 180^0$.

(b) Show that in any triangle AB $\frac{a^2-b^2}{c^2} = \frac{\sin(A-B)}{\sin(A+B)}$

15. (a) Differentiate the following with respect to x.

(i) $(x+1)^{\frac{1}{2}}(x+2)^2$

(ii) $\frac{2x^2+3x}{(x-4)^2}$

(b) The base radius of a right circular cone increases and the volume changes by 2%. If the height of the cone remains constant, find the percentage increase in the circumference of the base.

16. (a) Solve the differential $\frac{dy}{dx} = \frac{sin^2x}{y^2}$, given that y = 1 when x = 0.

(b) It is observed that the rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. A body at 78^0Cis placed in a room at 20^0C and after 5minutes the body has cooled to 65^0C. What will be its temperature after further 5 minutes?

END

Marking guide

Section A

1. Solve the equation
 $\log_{25} 4x^2 = \log_5(3 - x^2)$
 Solution
 $\log_{25}(2x)^2 = \log_5(3 - x^2)$
 Expressing the L.H.S in terms of log5
 $\frac{2\log_2 2x}{\log_2 25} = \log_5(3 - x^2)$
 $\frac{2\log_2 2x}{\log_2 5^2} = \log_5(3 - x^2)$
 $\frac{2\log_2 2x}{2} = \log_5(3 - x^2)$
 $\log_2 2x = \log_5(3 - x^2)$
 ⇨ $2x = 3 - x^2$

 $x^2+2x - 3 = 0$

 $(x + 3)(x - 1) = 0$

 Either x = -3 or x = 1

 ∴x = 1

OR

$\log_{25} 4x^2 = \log_5(3 - x^2)$

Expressing the R.H.S in terms of $\log_{25}$

$\log_{25} 4x^2 = \frac{\log_{25}(3-x^2)}{\log_{25} 5}$

$\log_{25} 4x^2 = \frac{\log_{25}(3-x^2)}{\log_{25} 25^{\frac{1}{2}}}$

$= \frac{\log_{25}(3-x^2)}{\frac{1}{2}}$

$= 2\log_{25}(3 - x^2)$

$\log_{25} 4x^2 = \log_{25}(3 - x^2)^2$

$4x^2 = (3 - x^2)^2$

$2x = 3 - x^2$

$x^2+2x - 3 = 0$

$(x + 3)(x - 1) = 0$

Either x = -3 or x = 1

∴x = 1

2. Find the equation of a line through the point (2, 3) and perpendicular to the line x + 2y + 5 = 0
 Solution
 Rearranging the equation
 $y = -\frac{1}{2}x - \frac{5}{2}$
 Gradient of the line is $-\frac{1}{2}$ then the gradient of the perpendicular = 2
 ⇨ $\frac{y-3}{x-2} = 2$
 $y = 2x - 1$

3. Evaluate $\int_1^3 \left(\frac{3x^2+4x+1}{x^3+2x^2+x}\right) dx$

 Solution

 Let $u = x^3 + 2x^2 + x$

 $\frac{du}{dx} = 3x^2 + 4x + 1$

x	u
1	4
3	48

 ⇨ $\int_1^3 \left(\frac{3x^2+4x+1}{x^3+2x^2+x}\right) dx = \int_4^{48} \frac{1}{u} du$
 $= [Inu]_4^{48}$
 =In48 –In4
 $=\text{In}\left(\frac{48}{4}\right) = In12 = 2.4849$

OR

$\int \left(\frac{3x^2+4x+1}{x^3+2x^2+x}\right) dx =$
$\int \left(\frac{3x^2+4x+1}{u}\right) . \int \left(\frac{du}{3x^2+4x+1}\right)$

$=\int \frac{1}{u} du = Inu + c$

$\therefore \int_1^3 \left(\frac{3x^2+4x+1}{x^3+2x^2+x}\right) dx = [Inx^3 + 2x^2 + x]_1^3$

=In48 –In4
$=\text{In}\left(\frac{48}{4}\right) = In12 = 2.4849$

OR

$\frac{3x^2+4x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$3x^2 + 4x + 1 = A(x+1)^2 + Bx(x+1) + Cx$

When x = -1,

3 – 4 + 1 = -C => C = 0

When x = 0; A = 1

For coefficient of x2

3 = A + B => B = 2

Hence $\frac{3x^2+4x+1}{x(x+1)^2} = \frac{1}{x} + \frac{2}{x+1}$

$\int_1^3 \frac{3x^2+4x+1}{x(x+1)^2}dx = \int_1^3 \frac{1}{x}dx + 2\int_1^3 \frac{1}{x+1}dx$

$=[\text{Inx} + 2\text{In}(x+1)]_1^3$

$=[\text{Inx}(x+1)^2]_1^3$

=In48 – In4

$=\text{In}\left(\frac{48}{4}\right) = In12 = 2.4849$

4. A committee of 4 men and 3 women is to be formed from 10 men and 8women. In how many ways can the committee be formed?

Solution

The committee can be formed in ${}^{10}C_4$ and ${}^{8}C_3$ ways

= ${}^{10}C_4 \times {}^{8}C_3$

= 210 x 56

= 11760 ways

5. Show that

$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{7}{9}\right)$

Solution

Let A =$\tan^{-1}\left(\frac{1}{2}\right)$; B =$\tan^{-1}\left(\frac{1}{5}\right)$, C =$\tan^{-1}\left(\frac{7}{9}\right)$

⇨ tanA = $\frac{1}{2}$; tanB =$\frac{1}{5}$; tan C = $\frac{7}{9}$

For $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{7}{9}\right)$

A + B = C

tan(A+ B) = tanC

tan C = $\frac{tanA+tanB}{1-tanAtanB} = \frac{\frac{1}{2}+\frac{1}{5}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)}$

$= \frac{\frac{7}{10}}{1-\frac{1}{10}} = \frac{7}{9}$

Hence $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{7}{9}\right)$

6. Given the $R = q\sqrt{1000-q^2}$; find

(a) $\frac{dR}{dq}$

Solution

$R = q\sqrt{1000-q^2}$

$InR = Inq + \frac{1}{2}In(1000-q^2)$

$\frac{1}{R}\frac{dR}{dq} = \frac{1}{q} + \frac{1}{2}\left(\frac{-2q}{1000-q^2}\right)$

$\frac{dR}{dq} = \left[\frac{1}{q} - \frac{q}{1000-q^2}\right].q(1000-q^2)^{\frac{1}{2}}$

$= \frac{\sqrt{1000-q^2}}{1} - \frac{q^2}{\sqrt{1000-q^2}}$

$= \frac{(1000-q^2)-q^2}{\sqrt{1000-q^2}}$

$= \frac{1000-2q^2}{\sqrt{1000-q^2}}$

OR

Let u = q => $\frac{du}{dq}$ = 1

v = $\sqrt{1000-q^2}$=> $\frac{dv}{dq} = \frac{1}{2}\left(\frac{-2q}{\sqrt{1000-q^2}}\right)$

=$\frac{-q}{\sqrt{1000-q^2}}$

$\frac{dR}{dq} = \sqrt{1000-q^2}(1) + \frac{-q}{\sqrt{1000-q^2}}.q$

$= \sqrt{1000-q^2} - \frac{q^2}{\sqrt{1000-q^2}}$

$= \frac{1000-2q^2}{\sqrt{1000-q^2}}$

(b) the value of q when R is maximum

For R_{max}; $\frac{dR}{dq} = 0$

⇨ $\frac{1000-2q^2}{\sqrt{1000-q^2}} = 0$

$2q^2 = 1000$

q =$\sqrt{500}$

7. Show that the points A, B and C with position vector 3i + 3j + k, 7i + 4k and 11i + 4j + 5k respectively are vertices of a triangle.

Solution

For a triangle to be

AB + BC + AC = 0

(OB – OA)+(OC – OB) + (OA – OC) = 0

$=\left[\begin{pmatrix}8\\7\\4\end{pmatrix} - \begin{pmatrix}3\\3\\1\end{pmatrix}\right] + \left[\begin{pmatrix}11\\4\\5\end{pmatrix} - \begin{pmatrix}8\\8\\4\end{pmatrix}\right] + \left[\begin{pmatrix}3\\3\\1\end{pmatrix} - \begin{pmatrix}11\\4\\5\end{pmatrix}\right]$

$=\begin{pmatrix}5\\4\\0\end{pmatrix} + \begin{pmatrix}3\\-3\\1\end{pmatrix} + \begin{pmatrix}-8\\-1\\-4\end{pmatrix}$

$=\begin{pmatrix}0\\0\\0\end{pmatrix}$

Hence A, B and C are vertices of a triangle

8. (a) form a differential equation by eliminating the constants a and b from x = accost + bsint

Solution

x = accost + bsint

$\frac{dx}{dt} = -asint + bcost$

$\frac{d^2x}{dt^2} = -acost - bsint$

$= -$(accost + bsint)

$= -$x

(b) State the order of the differential equation formed in (a) above.

Solution

It is a second order differential equation because the highest power of the differential coefficient is 2.

SECTION B

9. (a) The first term of an Arithmetic Progression (A.P) is ½. The sixth term of A.P is four times the fourth term. Find the common difference of the A.P.

Solution

U_6 = a + 5d ....(i)

U_4 = a + 3d ....(ii)

But $U_6 = 2U_4$

⇨ a + 5d = 4(a + 3d)

-3a = 7d

d =$-\frac{3}{7}a$

Given a = ½

d =$-\frac{3}{7}x\frac{1}{2} = -\frac{3}{14}$

(b) The roots of a quadratic equation $x^2 + px + q = 0$ are α and β. Show that the quadratic equation whose roots are $\alpha^2 - q\alpha$ and $\beta^2 - q$ is given by $x^2 - (p^2+pq -2q)x + q^2(q + p + 1) = 0$

Solution

α+β = -p

αβ = q

sum of roots = $\alpha^2 - q\alpha + \beta^2 - q$

$= (\alpha^2 + \beta^2) - q(\alpha+\beta)$

$=(\alpha+\beta)^2 - 2\alpha\beta - q(\alpha+\beta)$

$= p^2 = 2q + pq$

Product of roots

$(\alpha^2 - q\alpha)(\beta^2 - q)= \alpha^2\beta^2 - q\beta\alpha^2 - qa\beta^2 + q^2\alpha\beta$

$=(\alpha\beta)^2 - q\alpha\beta(\alpha+\beta) + q^2\alpha\beta$

$=q^2 + q^2p + q^3)$

$= q^2(q+ p + 1)$

Hence the equation

$x^2 - (p^2+pq -2q)x + q^2(q + p + 1) = 0$

10. (a) form a quadratic equation having -3 + 4i as one of its roots.

Solution

Since -3 + 4i is a root, then its conjugate -3 - 4i is a root

Sum of roots = -3 + 4i -3 - 4i= -6

Product of roots = (-3 + 4i)(-3 - 4i)= 25

Hence equation is $z^2 + 6z + 25 = 0$

OR

Let z = -3 + 4i => z + 3 – 4i = 0

Also z = -3 + 4i => z + 3 + 4i = 0

Multiplying the two equations together

(z + 3 – 4i)(z + 3 + 4i) = 0

$z^2 + 6z + 25 = 0$

(a) Given $z_1 = -1 + i\sqrt{3}$ and $z_2 = -1 - i\sqrt{3}$

(i) Express $\frac{z_1}{z_2}$ in the form $a + i\sqrt{b}$, where a and b are real numbers

Solution

$\frac{z_1}{z_2} = \frac{-1+i\sqrt{3}}{-1-i\sqrt{3}} = \frac{(-1+i\sqrt{3})(-1+i\sqrt{3})}{(-1-i\sqrt{3})(-1+i\sqrt{3})}$

$= \frac{1-i\sqrt{3}-i\sqrt{3}-3}{1+3}$

$=\frac{-2-2i\sqrt{3}}{4}$

$=\frac{-1}{2} - i\frac{\sqrt{3}}{2}$

Or

$|z_1| = 2$ and $|z_2| = 2$

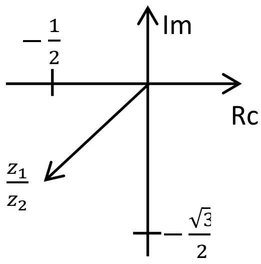
Arnz_1 = 120^0 and Arg z_2 = -120^0

$\frac{z_1}{z_2} = \frac{2}{2}$[cos($120^0$- $^-$ 120^0)+i sin(120^0- $^-$ 120^0)

= cos24^0 + isin240^0

=$\frac{-1}{2} - i\frac{\sqrt{3}}{2}$

(ii) Represent $\frac{z_1}{z_2}$ on an Argand diagram.



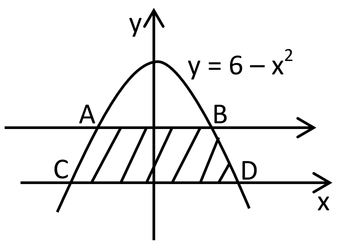
(iii) Find $\left|\frac{z_1}{z_2}\right|$

$\left|\frac{z_1}{z_2}\right| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = 1$

OR

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{2}{2} = 1$

11. In the diagram below, the curve y = 6 – x^2 meets the line y = 2 at A and B and the x-axis at A and D.



Find

(a) coordinates of A, B, C, and D

Solution

At point A and B

6 – x^2 = 2

x^2 = 4

x = ±2

A(-2, 2) and B(2,2)

At point C and D

6 – x^2 = 0

x^2 = 6

x = $\pm\sqrt{6}$

C($-\sqrt{6}, 0$) and D($\sqrt{6}, 0$)

(b) area of the shaded region, correct to one decimal place.

Solution

Area of CABD

$= \int_{-\sqrt{6}}^{\sqrt{6}}(6 - x^2)dx - \int_{-2}^{2}(6 - x^2)dx$

$=\left[6x - \frac{x^2}{3}\right]_{-\sqrt{6}}^{\sqrt{6}} - \left[6x - \frac{x^2}{3}\right]_{-2}^{2}$

=8.9sq. units (1d.p)

12. (a) Find the angle between the line x = $\frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x}{2} = \frac{y+1}{3} = \frac{z+2}{4}$

Let v_1 and v_2 be the vectors parallel to the lines respectively

⇨ $v_1 = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $v_1 = \begin{pmatrix}2\\3\\4\end{pmatrix}$

$|v_1| = \sqrt{1+4+9} = \sqrt{14}$

$|v_2| = \sqrt{4+9+16} = \sqrt{29}$

$v_1.v_2 = \begin{pmatrix}1\\2\\3\end{pmatrix}.\begin{pmatrix}2\\3\\4\end{pmatrix} = 20$

Cosθ=$\frac{v_1.v_2}{|v_1||v_2|} = \frac{20}{\sqrt{14}.\sqrt{29}} = 0.9889$

θ = $\cos^{-1} 0.9889$

= 8.53^0

(b) Find in vector form the equation of the line of intersection of two planes 2x + 3y – z = 4 and x – y + 2z = 5.

Solution

2x + 3y – z = 4 …………………(i)

x – y + 2z = 5 …………………(ii)

Eqn. (i) – 2Eqn. (ii)

5y – 5z = -6

y = z - $\frac{6}{5}$

Substituting y into equation 2

x = 5 + z - $\frac{6}{5}$ – 2z

= $\frac{19}{5} - z$

$$r = \begin{pmatrix} \frac{19}{5} \\ \frac{-6}{5} \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

OR: Eliminating z and making x the subject

$$r = \begin{pmatrix} \frac{13}{5} \\ 0 \\ \frac{6}{5} \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

OR: Eliminating z and making y the subject

$$r = \begin{pmatrix} 0 \\ \frac{13}{5} \\ \frac{10}{5} \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

13. (a) Find the equation of the tangent to parabola $y^2 = \frac{x}{16}$ at $\left(t^2, \frac{t}{4}\right)$.

Solution

For point$\left(t^2, \frac{1}{t}\right)$, x = t^2 and y = $\frac{t}{4}$

$\frac{dx}{dt} = 2t$ and $\frac{dy}{dx} = \frac{1}{4}$

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{1}{4} \cdot \frac{1}{2t} = \frac{1}{8t}$

$\frac{y - \frac{t}{4}}{x - t^2} = \frac{1}{8t}$

8ty – 4t^2 = x – t^2

8ty = x + t^2

x – 8ty + t^2 = 0

(b) If the tangent to the parabola in (a) above at points P$\left(p^2, \frac{1}{p}\right)$ and Q$\left(q^2, \frac{1}{q}\right)$ meet on the line y = 2.

(i) show that p + q = 16

Solution

Substituting for t = p

8py = x + p^2

Also 8qy = x + q^2

When y = 2

16p = x + p^2 ………………..(i)

And

16q = x + q^2 …………….. (ii)

Subtracting (ii)from (i)

16(p – q) = (p – q)(p + q)

Dividing by (p –q)

(p + q) = 16

(ii) deduce that the midpoint of PQ lies on the line y = 2

Solution

The midpoint of PQ = M$\left(\frac{p^2+q^2}{2}; \frac{p+q}{8}\right)$

Hence y = $\frac{p+q}{8}$

But p + q = 16

=> y = $\frac{16}{8} = 2$

14. (a) Solve 3sinx + 4cox = 2 for -180^0 ≤ x ≤ 180^0.

Solution

Rsin(x +α) = 3sinx + 4cosx

Rcosαsinx + Rsinαcosx = 3sinx + 4cosx

Rcosα = 3

Rsinα = 4

R^2 =3^2 + 4^2

R = 5

tanα =$\left(\frac{4}{3}\right)$ => α = 53.13^0

5sin(x + 53.13^0) = 2

x + 53.13^0 = $\sin^{-1}$(0.4) = 23. 58^0, 156.42^0

x = -29.55^0, 103.29^0

OR

Rcos(x - β) = 3sinx + 4cosx

Rcosβcosx + Rsinβsinx =3sinx + 4cosx

Rcosβ = 4

Rsinβ = 3

R^2 =3^2 + 4^2

R = 5

tanβ =$\left(\frac{3}{4}\right)$ => β = 36.87^0

5cos(x + 36.87^0) =2

x + 36.87^0 = $\cos^{-1}$(0.4) = ±66.24^0

x = -29.55^0, 103.29^0

OR

$3\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right) = 2$

6t + 4 – 4t^2 = 2 + 2t^2

6t^2 – 6t – 2 = 0

3t^2 – 3t – 1 = 0

$t = \frac{3 \pm \sqrt{9+12}}{6}$

= 1.2638 or -0.2638

tan$\left(\frac{x}{2}\right)$= 1.2638

x =2$\tan^{-1}$ 1.2638 = 103.290

tan$\left(\frac{x}{2}\right)$= -0.2638

x =2$\tan^{-1}$ -0.2638 = -29.55^0

(b) Show that in any triangle ABC

$\frac{a^2-b^2}{c^2} = \frac{\sin(A-B)}{\sin(A+B)}$

Solution

$\frac{a^2-b^2}{c^2} = \frac{(2RsinA)^2-(2RsinB)^2}{(2RsinC)^2}$

$= \frac{sin^2A-sin^2B}{sin^2C}$

$= \frac{(sinA+sinB)(sinA-sinB)}{(sinc)(sinC)}$

$= \frac{\left(2sin\frac{A+B}{2}cos\frac{A-B}{2}\right)\left(2cos\frac{A+B}{2}sin\frac{A-B}{2}\right)}{\sin(A+B)\sin(A+B)}$

$= \frac{\left(2sin\frac{A+B}{2}cos\frac{A+B}{2}\right)\left(2sin\frac{A-B}{2}cos\frac{A-B}{2}\right)}{\sin(A+B)\sin(A+B)}$

$= \frac{[\sin(A+B)][Sin(A-B)]}{[\sin(A+B)][\sin(A+B)]}$

$= \frac{\sin(A-B)}{\sin(A+B)}$

15. (a) Differentiate the following with respect to x.

(i) $(x+1)^{\frac{1}{2}}(x+2)^2$

Solution

Let y = $(x+1)^{\frac{1}{2}}(x+2)^2$

Iny = $\frac{1}{2}$In(x + 1) + 2In(x + 2)

$\frac{dy}{dx}\frac{1}{y} = \frac{1}{2(x+1)} + \frac{2}{(x+2)}$

$\frac{dy}{dx} = \frac{(x+2)+4(x+1)}{2(x+1)(x+2)}.(x+1)^{\frac{1}{2}}(x+2)^2$

$\frac{dy}{dx} = \frac{(5x+6)(x+2)}{2(x+1)^{\frac{1}{2}}}$

OR

$\frac{dy}{dx} = \frac{1}{2}(x+1)^{-½}(x+2)^2 + 2(x+1)^{½}(x+2)$

$= \frac{(x+2)^2+4(x+1)(x+2)}{2(x+1)^{\frac{1}{2}}}$

$= \frac{(x+2)(x+2+4+4x)}{2(x+1)^{\frac{1}{2}}}$

$= \frac{(5x+6)(x+2)}{2(x+1)^{\frac{1}{2}}}$

(ii) $\frac{2x^2+3x}{(x-4)^2}$

Solution

$\frac{dy}{dx} = \frac{(4x+3)(x-4)^2-2(2x^2+3x)(x-4)}{(x-4)^4}$

$= \frac{(4x+3)(x-4)-2((2x^2+3x)}{(x-4)^3}$

OR

Iny = In$(2x^2+3x)$ – 2In(x - 4)

$\frac{1}{y}\frac{dy}{dx} = \frac{4x+3}{2x^2+3x} - \frac{2}{(x-4)}$

$\frac{dy}{dx} = \frac{(4x+3)(x-4)}{2(2x^2+3x)}.\frac{2x^2+3x}{(x-4)^2}$

$= \frac{(4x+3)(x-4)-2((2x^2+3x)}{(x-4)^3}$

(b) The base radius of a right circular cone increases and the volume changes by 2%. If the height of the cone remains constant, find the percentage increase in the circumference of the base.

Solution

$\frac{\Delta V}{V} = 2\%$, V=$\frac{1}{3}\pi r^2h$

$\Delta V = 2\%V = 2\%\ x\frac{1}{3}\pi r^2h$

$\frac{dV}{dr} = \frac{2}{3}\pi rh$

$\frac{\Delta V}{\Delta r} = \frac{dV}{dr}$

$\Delta r = \Delta V.\frac{dr}{dV} = \frac{2\%\ x\frac{1}{3}\pi r^2h}{\frac{2}{3}\pi rh} = 1\%r$

c = 2πr

$\frac{dc}{dr} = 2\pi$

$\frac{\Delta c}{\Delta r} = \frac{dc}{dr}$

$\Delta c = \frac{dc}{dr}.\Delta r = \frac{2\pi\ x1\%r}{2\pi r} = 1\%$

16. (a) Solve the differential $\frac{dy}{dx} = \frac{sin^2x}{y^2}$, given that y = 1 when x = 0.

Solution

$y^2dy=sin^2xdx$

$\int y^2dy = \int\frac{1}{2}(1-cos2x)dx$

$\frac{y^3}{3} = \frac{x}{2} - \frac{1}{4}sin2x + c$

Substituting y =1 and x = 0

$\frac{1}{3} = \frac{0}{2} - \frac{1}{4}sin0 + c$ => c = $\frac{1}{3}$

Hence

$\frac{y^3}{3} = \frac{x}{2} - \frac{1}{4}sin2x + \frac{1}{3}$

$4y^3 = 6x - 3sin2x + 4$

(b) It is observed that the rate at which a body cools is proportional to the amount by which its temperature exceeds that of its surroundings. A body at 78^0Cis placed in a room at 20^0C and after 5minutes the body has cooled to 65^0C. what will be its temperature after further 5 minutes?

Solution

$\frac{d\theta}{dt} = -kt$

$\int \frac{d\theta}{\theta} = -\int kdt$

$In\theta = -kt + c$

At t=0, θ = 78 – 20 = 58

c = In58

At t = 5, θ = 65 – 20 =45

In 45 = -5k + In58

5k =$In\frac{58}{45}$

k = $\frac{1}{5}In\frac{58}{45}$

At t = 10,

$In\theta = -\frac{1}{5}In\frac{58}{45}x10 + In58$

θ = 34.90

The temperature will be = 20 + 34.9^0

= 54.9^0

Thank you

Dr. Bbosa Science