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Connected particles

Simple connections

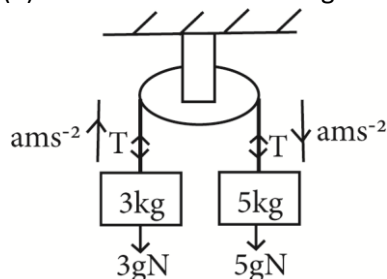
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is taut, the following must be observed.

- acceleration of the particles is the same
- tension in the uninterrupted string is constant
- tensions in interrupted strings are different.

Example 1

Two particles of masses 5kg and 3kg are connected by a light inextensible string passing over a smooth fixed pulley. Find

- acceleration of the particle
- the tension in the string



For 5kg mass: $5g - T = 5a$ (i)

For 3kg mass: $T - 3g = 3a$ (ii)

(i) and (ii)

$$2g = 8a$$

$$a = \frac{2 \times 9.8}{8} = 2.45 \text{ms}^{-2}$$

(ii) tension in the string

$$T - 3g = 3a$$

$$T = 3 \times 2.45 + 3 \times 9.8 = 36.78 \text{N}$$

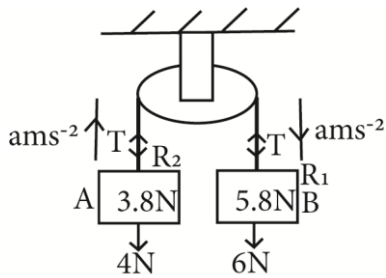
(iii) Force on the pulley

$$R = 2T = 3 \times 36.78 = 73.56 \text{N}$$

Example 2

An inextensible string attached to two scale A and B each of weight 20g passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and B respectively. If the system is released from rest (take $g = 10 \text{ms}^{-2}$). Find the

- Tension in the string
- Reaction of the scale pan holding the 3.8N weight



$$\text{Weight of the scale pan} = \frac{20}{1000} \times 10 = 0.2N$$

$$\text{Total weight of A} = 3.8 + 0.2 = 4N$$

$$\text{Total weight of B} = 5.8 + 0.2 = 6N$$

$$\text{For 6N: } 6 - T = 0.6a \dots\dots (i)$$

$$\text{For 4N: } T - 4 = 0.4a \dots\dots (ii)$$

Adding (i) and (ii)

$$a = 2ms^{-2}$$

$$T = 4 + 0.4 \times 2 = 4.8N$$

$$\text{For scale pan A } R_2 - 3.8 = 0.38a$$

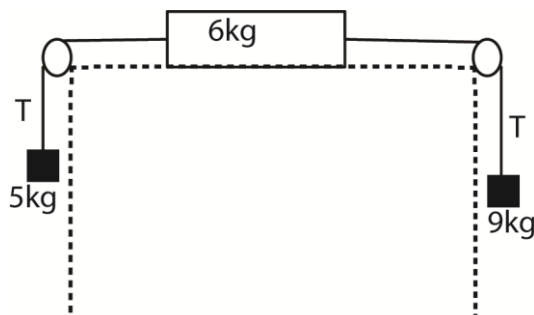
$$R_2 = 3.8 + 2 \times 0.38 \times 2 = 4.56N$$

Example 3

A mass of 6kg is lying on a smooth horizontal table. The mass is connected by two light inextensible strings passing over smooth pulleys at the edges of the table, two masses of 5kg and 9kg on opposite sides of the table. With the two strings taut and the masses hanging vertically, the system is released from rest.

Calculate the:

- (i) Acceleration of the masses. (10marks)



Let the acceleration be a

$$T - 5g = 5a$$

$$9g - T = 9a$$

$$4 \times 9.8 = 14a$$

$$A = 2.8 \text{ ms}^{-2}$$

- (ii) Tension in the strings. (05marks)

$$9 \times 9.8 - T = 9 \times 2.8$$

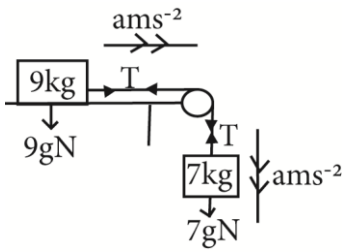
$$T = 63N$$

Example 4

A mass of 9kg resting on a smooth horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely 1.5m above the ground. Find

- (a) common acceleration

- (b) tension in the string
- (c) force on the pulley when the system is allowed to move freely
- (d) time taken for the 7kg mass to hit the ground



$$F = ma$$

$$\text{For 7kg mass: } 7g - T = 7a$$

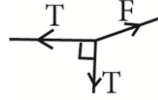
$$\text{For 9kg mass: } T = 9a$$

$$(i) + (ii): 7g = 16a$$

$$a = \frac{7 \times 9.8}{16} = 4.29 \text{ ms}^{-2}$$

$$(b) \text{ Tension: } T = 9a = 9 \times 4.29 = 38.61 \text{ N}$$

(c) The force on the pulley, F:



$$F = \sqrt{T^2 + T^2} = T\sqrt{2} = 38.61\sqrt{2} = 54.603 \text{ N}$$

$$(d) s = ut + \frac{1}{2}at^2$$

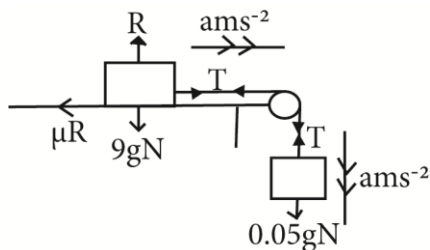
$$1.5 = 0 \times t + \frac{1}{2} \times 4.29 \times t^2$$

$$t = 0.84 \text{ s}$$

Example 5

A mass of 90g resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table attached to a 50g mass hanging freely. The coefficient of friction between the 90g mass and the table is $\frac{1}{3}$ and the system is released from rest, find

- (a) common acceleration
- (b) the tension in the string



$$\text{For 50g mass: } 0.05g - T = 0.05a \dots (i)$$

$$\text{For 90g mass: } T - \mu R = 0.09a$$

$$T - \frac{1}{3} \times 0.09 \times 9.8 = 0.09a \dots (ii)$$

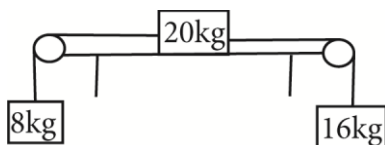
$$(i) + (ii): 0.05g - \frac{1}{3} \times 0.09 \times 9.8 = 0.14a$$

$$a = \frac{0.02g}{0.14} = 1.4 \text{ ms}^{-2}$$

$$(b) 0.05g - T = 0.05a$$

$$T = 0.05 \times 9.8 - 0.05 \times 1.4 = 0.42 \text{ N}$$

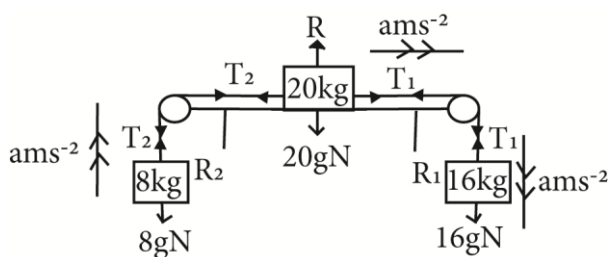
Example 6



The figure shows a block of mass 20kg resting on a smooth horizontal table. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

- (a) Acceleration of 16kg mass
- (b) tension in the string

(c) reaction on each pulley



For 16kg mass: $16g - T_1 = 16a$ (i)

For 20kg mass: $T_1 - T_2 = 20a$ (ii)

For 8kg mass: $T_2 - 8g = 8a$ (iii)

(i) + (ii) + (iii): $8g = 44a$

$$a = \frac{8 \times 9.8}{44} = 1.782 \text{ms}^{-2}$$

(b) Tension in the string

$$16g - T_1 = 16a$$

$$T_1 = 16 \times 9.8 - 16 \times 1.782 = 128.288 \text{N}$$

$$T_2 - 8g = 8a$$

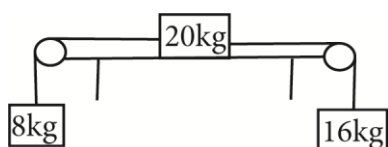
$$T_2 = 8 \times 9.8 + 8 \times 1.782 = 92.656 \text{N}$$

(c) Reaction on each pulley

$$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = \sqrt{2} \times 128.288 = 181.427 \text{N}$$

$$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 92.626 = 131 \text{N}$$

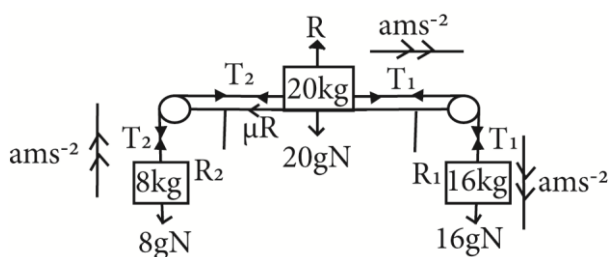
Example 6



The figure shows a block of mass 20kg resting on a rough horizontal table of coefficient of friction 0.21. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

- (a) acceleration of the 16kg mass
- (b) Tension in each string
- (c) reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$ (i)

For 20kg mass: $T_1 - T_2 - 20g\mu = 20a$ (ii)

For 8kg mass: $T_2 - 8g = 8a$ (iii)

(i) + (ii) + (iii): $8g - 20g\mu = 44a$

$$a = \frac{8 \times 9.8 - 20 \times 9.8 \times 0.21}{44} = 0.846 \text{ms}^{-2}$$

(b) Tension in the string

$$16g - T_1 = 16a$$

$$T_1 = 16 \times 9.8 - 16 \times 0.846 = 143.264 \text{N}$$

$$T_2 - 8g = 8a$$

$$T_2 = 8 \times 9.8 + 8 \times 0.846 = 85.168 \text{N}$$

(c) Reaction on each pulley

$$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 2 \times 128.291 = 202.606 \text{N}$$

$$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 85.168 = 120.446 \text{N}$$

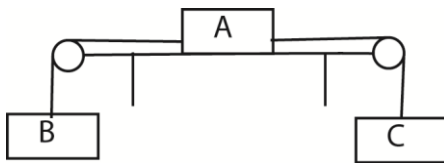
Revision exercise 1

(Answers are given in the square brackets [] at the end of each question)

- Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find
 - acceleration of the particles [3.92ms^{-2}]
 - the tension in the string [41.16N]
 - the force on the pulley [82.32N]
- Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find
 - acceleration of the particles [4.9ms^{-2}]
 - the tension in the string [29.4N]
 - distance moved by the 6kg mass in the first 2s of motion [9.8m]
- A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards.
 - what is the tension in the section of the rope supporting the man [807.06N]
 - what is the acceleration of the bucket [1.73ms^{-2}]
- Two particles of masses 200g and 300g are connected to a light inelastic string passing over a smooth pulley; when released freely find
 - common acceleration [1.96ms^{-2}]
 - the tension in the string [2.352N]
 - the force on the pulley [4.704N]
- The diagram below shows a particles of mass 8kg connected to a light scale pan by a light inextensible string which passes over a smooth fixed pulley. The scale pan holds two blocks A and B of mass 3kg and 4kg, with B resting on top of A. If the system is released from rest find
 - acceleration of the system [0.653ms^{-2}]
 - the reaction between A and B [41.813N]
- A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate
 - the common acceleration of the masses [3.675ms^{-2}]
 - the tension in the string [18.375N]
 - the force acting on the pulley [26N]
- A mass of 3kg on a smooth horizontal table is attached by a light inextensible sting passing over a smooth pulley at the edge of the table, to another mass of 2kg hanging freely 2.1m above the ground; find
 - common acceleration [3.92ms^{-2}]
 - the tension in the string [11.76N]
 - The force on the pulley in the system if it's allowed to move freely. [16.63N]
 - the velocity with which the 2kg mass hits the ground [4.06ms^{-1}]
- A mass of 5kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. the coefficient of friction between the 5kg mass and the table is 0.25 and the system is released from rest find
 - common acceleration [2.144ms^{-2}]
 - tension in the string [22.97N]

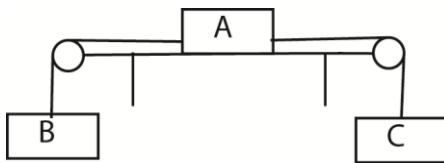
9. A mass of 11kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to 500g mass hanging freely. The coefficient of friction between the 1kg mass and the table is 0.1 and the system is released from rest find
- common acceleration [2.61ms^{-2}]
 - the tension in the string [3.593N]
10. The objects of mass 3kg and 5kg are attached to ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3kg mass touching the floor and the 5kg mass at 4m above the floor and then release, what is
- the acceleration of the system [2.45ms^{-2}]
 - tension in the chord [36.75N]
 - the time that will elapse before the 5kg object hits the floor [1.81s]

11.



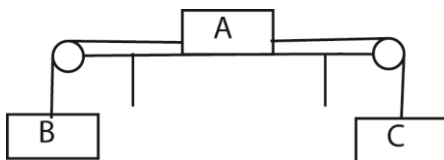
- The diagram shows a particle A of mass 2kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 5kg and C of mass 3kg by light inextensible strings hanging over smooth pulleys. If the system is released from rest find the
- common acceleration [0.98ms^{-2}]
 - the tension of each string [12.37N , 44.15N]

12.



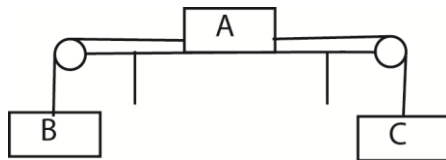
- The diagram shows a particle A of mass 3kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 4kg and C of mass 6kg by light inextensible strings hanging over smooth pulleys. If the system is released from rest find the
- common acceleration [0.75ms^{-2}]
 - the tension of each string [54.277N , 31.662N]

13.



- The diagram shows a particle A of mass 5kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 3kg and C of mass 2kg by light inextensible strings hanging over smooth pulleys. If the system is released from rest body B descend with an acceleration of 0.28ms^{-2} , find the coefficient of friction between the body A and the surface of the table. [0.143]

14.



The diagram shows a particle A of mass 10kg resting on a smooth horizontal table. It is attached to the particle B of mass 4kg and C of mass 7kg by light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

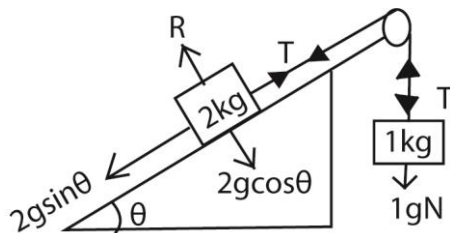
- (c) common acceleration [1.4ms^{-2}]
- (d) the tension of each string [44.8N , 58.8N]

Connected particles on inclined planes

Example 7

A mass of 2kg lies on a smooth plane of inclination 1 in 3. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its end. If the system is released from rest, find the

- (i) acceleration of the masses
- (ii) tension in the string
- (iii) distance each particle travels in the first 2s.



$$\sin\theta = \frac{1}{3} \quad F = ma$$

$$\text{For 2kg mass: } T - 2g\sin\theta = 2a \dots (i)$$

$$\text{For 1kg mass: } 1g - T = 1a \dots (ii)$$

$$(ii) + (i): = 1g - 2g\sin\theta = 3a$$

$$a = \frac{9.8 - 2 \times 9.8 \times \frac{1}{3}}{3} = 1.089\text{ms}^{-2}$$

$$(ii) \text{ Tension: } 1g - T = 1a$$

$$T = 9.8 - 1.089 = 8.71\text{N}$$

$$(iii) s = ut + \frac{1}{2}at^2$$

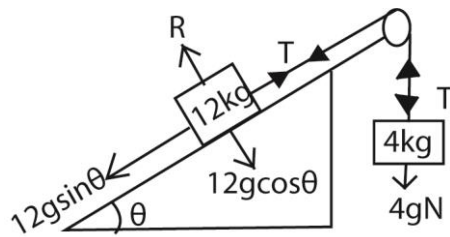
$$s = 0 \times 2 + \frac{1}{2} \times 1.089 \times 2^2 = 2.178\text{m}$$

Example 8

A mass of 12kg lies over a smooth incline plane 6m long and 1m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane to freely suspended mass of 4kg at its other end. If the system is released from rest, find the

- (a) acceleration of the system
- (b) velocity with which the 4kg mass hits the ground
- (c) time the 4kg mass takes to hit the ground.

Solution



$$\sin\theta = \frac{1}{6} \quad F = ma$$

For 12kg mass: $T - 12g\sin\theta = 2a \dots (i)$

For 4kg mass: $4g - T = 4a \dots (ii)$

$$(ii) + (i): = 4g - 12g\sin\theta = 16a$$

$$a = \frac{4 \times 9.8 - 12 \times 9.8 \times \frac{1}{6}}{16} = 1.225 \text{ms}^{-2}$$

(ii) Tension: $4g - T = 4a$

$$T = 4 \times 9.8 - 4 \times 1.225 = 34.3 \text{N}$$

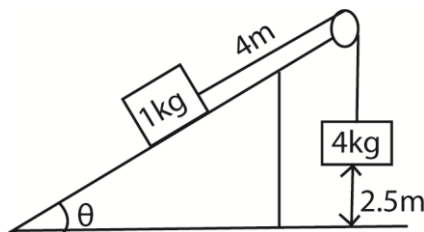
(iii) $s = ut + \frac{1}{2}at^2$

$$1 = 0 \times t + \frac{1}{2} \times 1.225 \times t^2$$

$$t = 1.28 \text{s}$$

Example 9

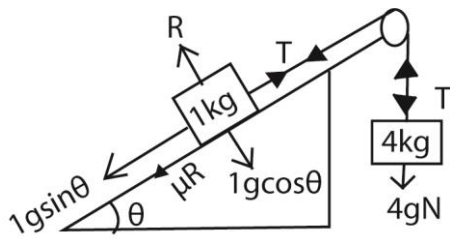
A mass of 1kg lies on a rough plane with coefficient of friction 0.25. One end of a light inextensible string is attached to 1kg mass and passes up the line of greatest slope over a smooth fixed pulley at the top of the plane and the other end of a string is tied to a mass of 4kg hanging freely.



The plane makes an angle θ with the horizontal where $\sin\theta = \frac{3}{5}$. When the system is released from rest, find:

- (i) the acceleration of the system
- (ii) tension in the string
- (iii) velocity with which the 4kg mass hits the floor
- (iv) velocity with which the 1kg mass hits the pulley

Solution



$$\sin\theta = \frac{3}{5}; \cos\theta = \frac{4}{5} \quad F = ma$$

$$\text{For 1kg mass: } T - 1g\sin\theta - 0.25R = 1a \dots (i)$$

$$\text{For 4kg mass: } 4g - T = 4a \dots (ii)$$

$$(ii) + (i): 4g - 1g\sin\theta - 0.25R = 5a$$

$$a = \frac{4 \times 9.8 - 1 \times 9.8 \times \frac{3}{5} - 0.25 \times 1 \times 9.8 \times \frac{4}{5}}{5} = 6.272 \text{ms}^{-2}$$

$$(ii) \text{ Tension: } 4g - T = 4a$$

$$T = 4 \times 9.8 - 4 \times 6.272 = 14.112 \text{N}$$

$$(iii) v^2 = u^2 + 2as \text{ but } u = 0$$

$$v = \sqrt{2 \times 6.272 \times 2.5} = 5.6 \text{ms}^{-1}$$

(iv) When a 4kg mass hits the floor, the 1kg mass has still to move $4 - 2.5 = 1.5\text{m}$ before hitting the pulley. It will experience a retarding force due to gravity and friction

$$F = 1a = 1g\sin\theta + 0.25R$$

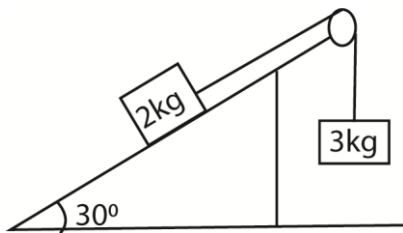
$$= (1 \times 9.8 \times \frac{3}{5} + 0.25 \times 1 \times 9.8 \times \frac{4}{5})$$

$$a = -7.84 \text{ms}^{-2}$$

$$v^2 = u^2 + 2as$$

$$v = \sqrt{5.6^2 - 2 \times 7.84 \times 1.5} = 2.8 \text{ms}^{-1}$$

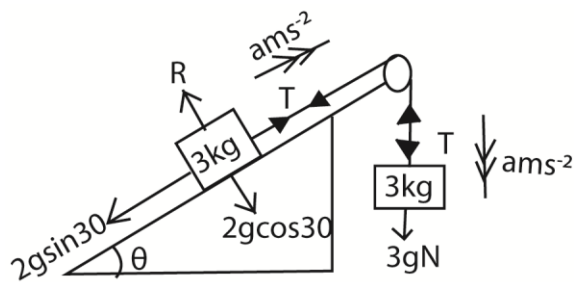
Example 10



A particle of mass 2kg on a rough plane inclined at 30° to the horizontal is attached by means of light inextensible string passing over a smooth pulley at the top edge of the plane to a particle of mass 3kg which hangs freely. If the system is released from rest with above parts of the strings taut, the 3kg mass travels a distance of 0.75m before attains a speed of 2.25ms^{-1} . Calculate

- (a) acceleration
- (b) coefficient of friction between the plane and 2kg mass
- (c) reaction of the pulley on the string

Solution



(i) $v^2 = u^2 + 2as$

$$a = \frac{2.25^2 - 0^2}{2 \times 0.75} = 3.375 \text{ms}^{-2}$$

(ii) $F = ma$

For 2kg mass: $T - 2g \sin \theta - \mu R = 2a$ (i)

For 4kg mass: $3g - T = 3a$ (ii)

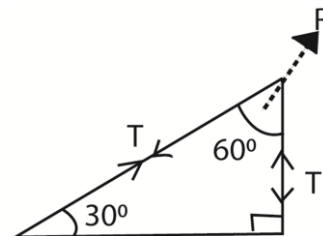
(ii) + (i): $3g - 2g \sin \theta - \mu(2g \cos \theta) = 5a$

$$\mu = \frac{(3 \times 9.8) - (2 \times 9.8 \sin 30 + 5 \times 3.375)}{2 \times 9.8 \times \cos 30} = 0.161$$

(iii)

Tension: $3g - T = 3a$

$$T = 3 \times 9.8 - 3 \times 3.375 = 19.275 \text{N}$$



Using parallelogram law of force

$$R^2 = T^2 + T^2 + 2 \times T \cos 60 = 3T^2$$

$$R = 19.275\sqrt{3} = 33.4 \text{N}$$

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Thanks

Dr. Bbosa Science