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Indices

There are five basic rules of indices

(a) $a^p \times a^q = a^{p+q}$

(b) $\frac{a^p}{a^q} = a^{p-q}$

(c) $(a^p)^q = a^{pq}$

(d) $a^{\frac{1}{q}} = \sqrt[q]{a}$

(e) $a^{\frac{p}{q}} = (\sqrt[q]{a})^p$

Example 1

Evaluate the following

(a) $2^2 \times 2^3$

Solution

$$\begin{aligned} 2^2 \times 2^3 &= 2^{2+3} \\ &= 2^5 \\ &= 32 \end{aligned}$$

(b) $\frac{4^3}{4^2}$

Solution

$$\begin{aligned} \frac{4^3}{4^2} &= 4^{3-2} \\ &= 4^1 \\ &= 4 \end{aligned}$$

(c) $(3^2)^3$

Solution

$$\begin{aligned} (3^2)^3 &= 3^{2 \times 3} \\ &= 3^6 \\ &= 729 \end{aligned}$$

(d) $2^{\frac{1}{2}} \times 2^{\frac{1}{2}}$

Solution

$$\begin{aligned} 2^{\frac{1}{2}} \times 2^{\frac{1}{2}} &= 2^{\frac{1}{2} + \frac{1}{2}} \\ &= 2^1 \\ &= 2 \end{aligned}$$

(e) $\sqrt[3]{27}$

Solution

$$\begin{aligned} \sqrt[3]{27} &= (3^3)^{\frac{1}{3}} \\ &= 3^3 \times \frac{1}{3} \\ &= 3^1 \\ &= 3 \end{aligned}$$

(f) $125^{\frac{2}{3}}$

Solution

$$\begin{aligned} 125^{\frac{2}{3}} &= (5^3)^{\frac{2}{3}} \\ &= 5^{3 \times \frac{2}{3}} \\ &= 5^2 \\ &= 25 \end{aligned}$$

Example 2

Evaluate the following

(a) $\left(\frac{125}{27}\right)^{\frac{4}{3}}$

Solution

$$\begin{aligned} \left(\frac{125}{27}\right)^{\frac{4}{3}} &= \left(\frac{(5^3)^{\frac{4}{3}}}{(3^3)^{\frac{4}{3}}}\right) \\ &= \frac{5^4}{3^4} \\ &= \frac{625}{81} \end{aligned}$$

(b) $81^{\frac{3}{4}}$

Solution

$$\begin{aligned} 81^{\frac{3}{4}} &= (3^4)^{\frac{3}{4}} \\ &= 3^{4 \times \frac{3}{4}} \\ &= 3^3 \\ &= 27 \end{aligned}$$

The zero index

$$\text{From } \frac{a^p}{a^p} = a^{p-p} = a^0 = 1$$

∴ Any number raised to power zero = 1

$$\text{i.e. } 100^0 = 529^0 = 83^0 = 1$$

Negative indices

It can be shown that

$$\frac{1}{a} = \frac{a^0}{a^1} = a^{0-1} = a^{-1}$$

Also

$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

Hence a negative index is the inverse of a given number

Example 3

Evaluate the following

(a) $16^{\frac{-3}{2}}$

Solution

$$\begin{aligned} 16^{\frac{-3}{2}} &= (2^4)^{\frac{-3}{2}} \\ &= 2^{4 \times \frac{-3}{2}} \\ &= 2^{-6} \\ &= \frac{1}{2^6} \\ &= \frac{1}{64} \end{aligned}$$

(b) $\left(\frac{64}{27}\right)^{-\frac{2}{3}}$

Solution

$$\text{(a) } \left(\frac{64}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{64}\right)^{\frac{2}{3}} = \left(\frac{\sqrt[3]{27}}{\sqrt[3]{64}}\right)^2 = \frac{9}{16}$$

Example 4

Show that $\sqrt{\frac{25^3+5^6}{5^7-5^6}} = \frac{\sqrt{2}}{2}$

Solution

$$\begin{aligned} \sqrt{\frac{25^3+5^6}{5^7-5^6}} &= \sqrt{\frac{(5^2)^3+5^6}{5^7-5^6}} \\ &= \sqrt{\frac{5^6+5^6}{5 \times 5^6-5^6}} \end{aligned}$$

$$= \sqrt{\frac{2 \times 5^6}{4 \times 5^6}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= \frac{\sqrt{2}}{2} \text{ (as required)}$$

Solving equations with unknown indices

It involves making appropriate substitution after expressing terms containing powers in simplified form

Example 5

Solve the equation

$$2^{2x+1} - 7(2^x) + 6 = 0$$

Solution

$$2^{2x+1} - 7(2^x) + 6 = 0$$

$$2^1 \cdot 2^{2x} - 7(2^x) + 6 = 0$$

$$2(2^x)^2 - 7(2^x) + 6 = 0$$

Let $p = 2^x$

$$\Rightarrow 2p^2 - 7p + 6 = 0$$

$$(2p - 3)(p - 2) = 0$$

Either $2p - 3 = 0$

$$p = \frac{3}{2}$$

or

$$p - 2 = 0$$

$$p = 2$$

when $p = \frac{3}{2} \Rightarrow 2^x = \frac{3}{2}$

$$\log 2^x = \log \frac{3}{2}$$

$$x \log 2 = \log \frac{3}{2}$$

$$x = \frac{\log \frac{3}{2}}{\log 2} = 0.585$$

When $p = 2$

$$2^x = 2 = 2^1$$

$$x = 1$$

Hence $x = 1$ and $x = 0.585$ (3d.p)

Example 6

Show that

$$\frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x} = 4$$

Solution

$$\frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x} = 4$$

Handling terms on the LHS

$$\begin{aligned} \frac{3(2^{x+1})-4(2^{x-1})}{2^{x+1}-2^x} &= \frac{3(2^x \cdot 2^1)-4(2^x \cdot 2^{-1})}{2^x \cdot 2^1-2^x} \\ &= \frac{3(2^x \cdot 2)-4(2^x \cdot \frac{1}{2})}{2^x \cdot 2-2^x} \\ &= \frac{2^x(3 \cdot 2-4 \cdot \frac{1}{2})}{2^x(2-1)} = \frac{6-2}{1} = 4 \end{aligned}$$

Example 5

Solve $x^{\frac{4}{3}} = 81$

$$x^{\frac{4}{3} \cdot \frac{3}{4}} = 81^{\frac{3}{4}}$$

$$x = (\sqrt[4]{81})^3 = 3^3 = 27$$

Solving equations with squares

Example 6

$$\sqrt{2x + 5} = x + 1$$

Square both sides

$$(\sqrt{2x + 5})^2 = (x + 1)^2$$

$$2x + 5 = x^2 + 2x + 1$$

$$x^2 = 4$$

$$x = \pm 2$$

Testing/checking using -2

$$\sqrt{2x + 5} = x + 1$$

$$\sqrt{2(-2) + 5} = -2 + 1$$

$$1 \neq -1$$

Hence -2 is **not** a solution to the equation

Testing/checking using 2

$$\sqrt{2x + 5} = x + 1$$

$$\sqrt{2 \cdot 2 + 5} = 2 + 1$$

$$3 = 3$$

Hence 2 is the solution to the equation

Example 7

Solve for x: $\sqrt{x + 2} = 4$

Square both sides

$$(\sqrt{x + 2})^2 = 4^2$$

$$x + 2 = 16$$

$$x = 14$$

Finding square roots of terms containing rational and irrational quantities

When finding roots of terms expressed in the form $a + \sqrt{b}$, where a is a rational and b is an irrational quantity, we let the root to be in the form of $\pm(\sqrt{x_1} + \sqrt{x_2})$ where x_1 and x_2 are integers.

Example 8

Find the square root of $6 + 2\sqrt{5}$

Let $\pm(\sqrt{x_1} + \sqrt{x_2})$ be square root of $6 + 2\sqrt{5}$

$$\Rightarrow \pm(\sqrt{x_1} + \sqrt{x_2}) = \sqrt{6 + 2\sqrt{5}}$$

Squaring both sides

$$(\sqrt{x_1} + \sqrt{x_2})^2 = (\sqrt{6 + 2\sqrt{5}})^2$$

$$x_1 + x_2 + 2\sqrt{x_1 \cdot x_2} = 6 + 2\sqrt{5}$$

Comparing terms on the two sides

$$x_1 + x_2 = 6$$

$$x_1 = 6 - x_2 \dots\dots\dots(i)$$

$$x_1 \cdot x_2 = 5 \dots\dots\dots(ii)$$

Substituting eqn. (i) into eqn. (ii)

$$(6 - x_2)x_2 = 5$$

$$x_1^2 - 6x_2 + 5 = 0$$

$$x_1^2 - x_2 - 5x_2 + 5 = 0$$

$$x_2(x_2 - 1) - 5(x_2 - 1) = 0$$

$$(x_2 - 1)((x_2 - 5) = 0$$

$$\text{Either: } x_2 - 1 = 0 \Rightarrow x_2 = 1$$

$$\text{Or } x_2 - 5 = 0 \Rightarrow x_2 = 5$$

$$\text{When } x_2 = 1: x_1 = 6 - 1 = 5$$

$$\text{When } x_2 = 5: x_1 = 6 - 5 = 1$$

Hence the square root of $6 + 2\sqrt{5}$ is $\pm(1 + \sqrt{5})$

Example 9

Find the square root of $8 - 2\sqrt{15}$

Let $\pm(\sqrt{x_1} - \sqrt{x_2})$ be square root of $8 - 2\sqrt{15}$

$$\pm(\sqrt{x_1} + \sqrt{x_2}) = \sqrt{8 - 2\sqrt{15}}$$

Squaring both sides

$$(\sqrt{x_1} - \sqrt{x_2})^2 = (\sqrt{8 - 2\sqrt{15}})^2$$

$$x_1 + x_2 - 2\sqrt{x_1 \cdot x_2} = 8 - 2\sqrt{15}$$

Comparing terms on the two sides

$$x_1 + x_2 = 8$$

$$x_1 = 8 - x_2 \dots\dots\dots(i)$$

$$x_1 \cdot x_2 = 15 \dots\dots\dots(ii)$$

Substituting eqn. (i) into eqn. (ii)

$$(8 - x_2)x_2 = 15$$

$$x_1^2 - 8x_2 + 15 = 0$$

$$x_1^2 - 3x_2 - 5x_2 + 15 = 0$$

$$x_2(x_2 - 3) - 5(x_2 - 3) = 0$$

$$(x_2 - 3)((x_2 - 5) = 0$$

$$\text{Either: } x_2 - 5 = 0 \Rightarrow x_2 = 5$$

$$\text{Or } x_2 - 3 = 0 \Rightarrow x_2 = 3$$

$$\text{When } x_2 = 5: x_1 = 8 - 5 = 3$$

$$\text{When } x_2 = 3: x_1 = 8 - 3 = 5$$

Hence the square root of $8 - 2\sqrt{15}$ is $\pm(\sqrt{5} - \sqrt{3})$

Revision exercise

1. Simplify

$$(i) \quad 9a^2 \div 27a^{-4} \left[\frac{2}{3} a^6 \right]$$

$$(ii) \quad (6a^{-3}) \div (9a^{-4})^2 \left[\frac{2}{27} a^5 \right]$$

$$(iii) \quad \frac{2a^{-3}b^2}{7c^{-4}d^2} \left[\frac{2b^2c^4}{7a^3d^2} \right]$$

$$(iv) \quad (x^4yz^{-3})^2 \times \sqrt{x^{-5}y^2z} \div (xz)^{\frac{1}{2}} [x^5yz^{-6}]$$

$$(v) \quad \sqrt[4]{y^3x} \sqrt{y^{\frac{1}{2}}} [y^{\frac{5}{4}}]$$

2. Evaluate

$$(a) \quad (64)^{-\frac{3}{2}} [16]$$

$$(b) \quad \left(\frac{8}{27}\right)^{-\frac{1}{3}} \left[\frac{3}{2}\right]$$

$$(c) \quad \left(\frac{1}{25}\right)^{\frac{1}{2}} \left[\frac{1}{5}\right]$$

$$(d) \quad \left(\frac{8}{27}\right)^{\frac{2}{3}} \left[\frac{4}{9}\right]$$

$$(e) \quad \left(\frac{243}{512}\right)^{-\frac{2}{3}} [1.6445]$$

3. Solve the following equations

$$(a) \quad 98x^2 = 2 [x = 0.1429]$$

$$(b) \quad x^{-3} = 8 \left[x = \frac{1}{2} \right]$$

$$(c) \quad \frac{1}{32} x^3 = 8x^{-1} [x = 4]$$

$$(d) \quad \frac{9}{25} x = \frac{5}{3} x^{-2} \left[x = \frac{5}{3} \right]$$

$$(e) \quad \frac{2}{14} x^{-2} + 14x = 0 [x = -0.2169]$$

4. Solve for x

$$(a) \quad 3^{2x+1} + 3 = 10(3^x) [x = 1 \text{ or } x = -1]$$

$$(b) \quad 2^{2x-1} + \frac{3}{2} = 2^{x+1} [x = 0, x = 1.585]$$

$$(c) \quad 7^x = 3^{1-x} [x = 0.3608]$$

$$(d) \quad 7x^{\frac{1}{2}+2} = 0 \left[x = \frac{4}{49} \right]$$

$$(e) \quad 5x^{\frac{2}{3}} = x^{-\frac{1}{3}} \left[x = \frac{1}{5} \right]$$

$$(f) \quad 4x^{-\frac{1}{3}} = 5x^{\frac{1}{6}} \left[x = \frac{16}{25} \right]$$

$$(g) \quad 6x^{\frac{2}{3}} - \frac{2}{3} x^{-\frac{1}{2}} = 0 [x = 0.077]$$

$$(h) \quad 8x^{-2} = 343x^{\frac{1}{2}} [x = 0.003562]$$

5. Show that

$$(a) \quad \frac{(2^{2x} - 3 \cdot 2^{2x-2})(3^x - 2 \cdot 3^{x-2})}{3^{x-4}(4^{x+3} - 2^{2x})} = \frac{1}{4}$$

$$(b) \quad \frac{(1+a)^{\frac{1}{2}} - \frac{1}{3}a(1+a)^{-\frac{2}{3}}}{(1+a)^{\frac{2}{3}}} = \frac{3+2a}{3(1+a)^{\frac{4}{3}}}$$

$$(c) (a - a^{-1}) \left(a^{\frac{4}{3}} - a^{\frac{2}{3}} \right) = \frac{a^2 - a^{-2}}{a^{-\frac{1}{3}}}$$

$$(d) \frac{a^{\frac{1}{2}} + ab}{ab - b^2} - \frac{\sqrt{a}}{\sqrt{a-b}} = \sqrt{\frac{a}{b}}$$

6. Solve

$$(a) x^{\frac{1}{3}} - 3 = 28x^{-\frac{1}{3}} \quad [x = -64, x = 343]$$

$$(b) 2x^{\frac{1}{4}} = 9 - 4x^{-\frac{1}{4}} \quad [x = \frac{1}{16}, x = 256]$$

$$(c) x^3 + 8 = 9x^{\frac{3}{2}} \quad [x = 1, x = 4]$$

$$(d) 2x^{\frac{1}{3}} = \frac{81}{8}x^{-1} \quad [x = 8.6967]$$

$$(e) 49x^{-\frac{5}{6}} - \frac{8}{81}x^{\frac{7}{6}} = 0 \quad [x = 22.2739]$$

$$(f) x^{\frac{2}{3}} - x^{\frac{1}{3}} - 2 = 0 \quad [x = -1]$$

$$(g) x^{\frac{1}{2}} - 5x^{\frac{1}{4}} + 6 = 0 \quad [x = -1]$$

$$(h) 6x^{\frac{1}{3}} + 5 + x^{-\frac{1}{3}} = 0 \quad \left[x = \frac{1}{2}, x = \frac{1}{3} \right]$$

7. Solve for x

$$(a) \sqrt{x+2} - x = 0 \quad [x = 2]$$

$$(b) \sqrt{1+x} = 1 + \sqrt{1-x} \quad \left[x = \frac{\sqrt{3}}{2} \right]$$

$$(c) (3-x)^{\frac{1}{2}} = (1+x)^{\frac{1}{2}} + (2-x)^{\frac{1}{2}} \quad [x = -0.92665]$$

$$(d) \sqrt{x+6} = \sqrt{1-3x} - \sqrt{4-x} \quad [-5]$$

8. Without using mathematical tables or calculators, find the value of

$$\frac{(\sqrt{5}+2)^2 - (\sqrt{5}-2)^2}{8\sqrt{5}} \quad [1]$$

9. Find the square root of the following

$$(a) 6 + 2\sqrt{5} \quad [\pm(1 + \sqrt{5})]$$

$$(b) 18 - 2\sqrt{12} \quad [\pm(\sqrt{0.695} - \sqrt{17.303})]$$

$$(c) 18 - 2\sqrt{2} \quad [\pm(\sqrt{0.1118} - \sqrt{17.8882})]$$

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Thanks

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