Integration (A-level)

It the reverse of differentiation; thus the topic should be done after studying differentiation.

During integration the following concepts should be considered.

(a) Polynomial functions;

$\int x^n dx = \frac{1}{n+1}x^{n+1} + 1\ where\ n \neq -1$

i.e. increase the power by 1 and divide the term by the new power

According to NCDC, n ≥ 0

Example 1

(i) $\int 1dx$

Solution

$\int 1dx = \int (x^0)dx$

$= \frac{x^{0+1}}{0+1}$

$= x$

(ii) $\int xdx$

Solution

$\int xdx = \int x^1 dx$

$= \frac{1}{1+1}x^{1+1}$ + c

$= \frac{1}{2}x^2$

(iii) $\int x^4 dx$

Solution

$\int x^4 dx = \frac{1}{4+1}x^{4+1} = \frac{1}{4}x^5$

(iv) $\int 4x^3 dx$

Solution

$\int 4x^3 dx = 4\int x^3 dx$

$= \frac{4}{(3+1)}x^4$

$= x^4$

(v) $\int x^{-3} dx$

Solution

$\int x^{-3} dx = \frac{1}{-3+1}x^{-3+1}$

$= -\frac{1}{2}x^{-2}$

$= \frac{-1}{2x^2}$

Example 2

Evaluate

(a) $\int_{-1}^{2} \frac{2x^4 - x^5}{x^5} dx$

Solution

$\int_{-1}^{2} \frac{2x^4 - x^5}{x^2} dx = \int_{-1}^{2} (2x^2 - x^3) dx$

$= \left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_{-1}^{2}$

$= \left(\frac{16}{3} - 4\right) - \left(-\frac{2}{3} - \frac{1}{4}\right)$

$= \frac{4}{3} + \frac{11}{12}$

$= \frac{27}{12}$

$= 2.25$

(b) $\int_{1}^{2} \frac{x^4 - 1}{x^2} dx$

Solution

$\int_{1}^{2} \frac{x^4 - 1}{x^2} dx = \int_{1}^{2} \left(x^2 - \frac{1}{x^2}\right) dx$

$= \int_{1}^{2} (x^2 - x^{-2})$

$= \left[\frac{x^3}{3} + \frac{1}{x}\right]_{1}^{2}$

$= \left(\frac{8}{3} + \frac{1}{2}\right) - \left(\frac{1}{3} + 1\right)$

$= \left(\frac{19}{6} - \frac{4}{3}\right)$

$= \frac{11}{6}$

Revision exercise 1

(Answers are given in square brackets)

1. Find $\int y dx$ for each of the following
   (a) y = $x^3(2 - x^2)$
   $\left[\frac{x^4}{2} + \frac{x^6}{6} + c\right]$
   (b) y = (x+3)(x+5)
   $\left[\frac{x^3}{3} + 4x^2 + 15x + c\right]$
   (c) y = $(x - 3)^2$
   $\left[\frac{x^3}{3} + 3x^2 + 9x + c\right]$

   (d) y = $\frac{4x^3-3x^2}{2x}$
   $\left[\frac{2x^3}{3} - \frac{3x^2}{4} + c\right]$

   (e) y = $\frac{6x-3}{2\sqrt{x}}$
   $\left[2x^{\frac{3}{2}} - 3x^{\frac{1}{2}}\right]$

2. Evaluate
   (a) $\int_4^5 (4x + 3)dx$ $[21]$
   (b) $\int_2^3 (4 - 3x^2)dx$ $[-15]$
   (c) $\int_2^8 \frac{1}{x^2} dx$ $\left[\frac{3}{8}\right]$
   (d) $\int_4^9 \sqrt{x} dx$ $\left[\frac{38}{3}\right]$
   (e) $\int_1^4 \left(3 - \frac{1}{\sqrt{x}}\right) dx$ $[7]$

Application of integration

A. Area under a curve
   When finding the area under the curve, it is advisable to make a sketch the curve first to visualize the required area.

   Area between the curve and x-axis

   Example 3

   Find the area enclosed by the curves

   (a) y = x(x – 4) and x – axis
   Solution

   When x = 0, y = 0

   When x = 4, y = 0

   Hence intercepts are (0, 0) and (4, 0)

   Turning points when $\frac{dy}{dx} = 0$

   ⇨ 2x – 4 = 0
   x = 2

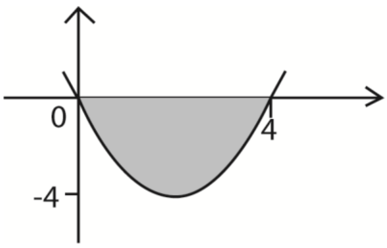
   when x = 2, y = -4

Nature of turning point

$\frac{d^2y}{dx^1} = 2$

Since $\frac{d^2y}{dx^1} > 0$, the point (2, -4) is minimum

Sketch



Area required $= \int_0^4 (x^2 - 4x)dx$

$= \left[\frac{x^3}{3} - 2x^2\right]_0^4$

$= \frac{4^3}{3} - 2x(4)^2$

$= \frac{-32}{3}$

Hence the area under the curve is $\frac{32}{3}$

Revision exercise 2

(Answers are given in square brackets)

1. Find the areas enclosed by the curves and x-axis
   (a) y = $x^2 - 4$ $\left[\frac{32}{3}\right]$
   (b) y = $2x^2 - 5x + 6$ $[1.125]$
   (c) y = $x^2 - 2x - 3$ $[10.7]$
2. Find the area enclosed by the curve y = $x^2 - 4$, the x-axis and line x = 3 $\left[\frac{7}{3}\right]$
3. Find the area encloses between the curve y = $x^2 - 6x + 13$, the x-axis, the line x = 3 and the line x = 5

Thank you

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