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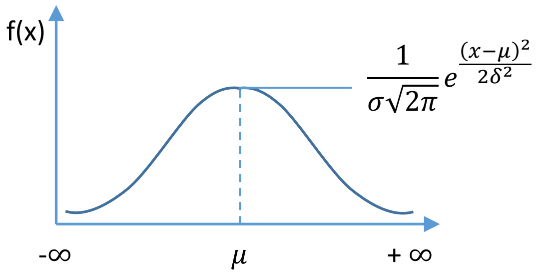
Normal distribution

A continuous random variable X follows a normal distribution with mean, μ and variance, σ^2 if

$X \sim N(\mu, \sigma^2)$ root

Its p.d.f is given by fx = $\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(x-\mu)^2}{2\delta^2}}, -\infty < x < \infty$

A sketch of f(x) gives a normal curve



Properties of the curve

- It is bell shaped
- It is symmetrical about μ
- It extends from $-\infty < x < \infty$

The maximum value of f(x) = $\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{(x-\mu)^2}{2\delta^2}}$

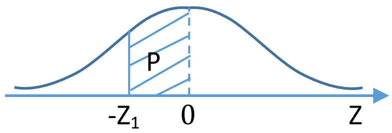
- The total area under the curve = 1

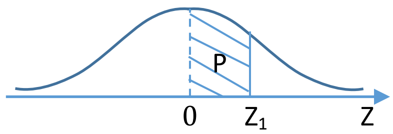
How to read the cumulative normal distribution table

(i) Between 0 and any z value

(a) $P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P (b) $P(-Z_1 \le Z \le 0) = P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P

By symmetrical



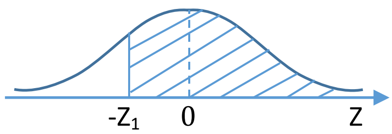


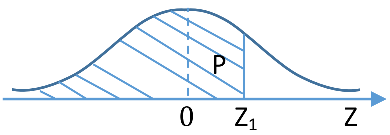
(ii) Less than any positive z value

(a) $P(Z < Z_1) = 0.5 + P(0 \le Z \le Z_1) = \phi(Z_1)$ = region P

(b) $P(Z > -Z_1) = P(Z < Z_1) = 0.5 + P(0 \le Z \le Z_1) = 0.5 + \phi(Z_1)$ = region P

By symmetrical





(iii) Greater than any positive z value

$P(Z > Z_1) = 0.5 - P(0 \le Z \le Z_1) = 0.5 - \phi(Z_1)$ = region P

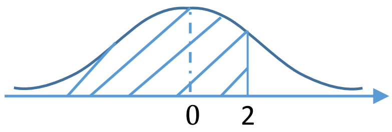
$P(Z < -Z_1) = P(Z > Z_1) = 0.5 - P(0 \le Z \le Z_1) = 0.5 - \phi(Z_1)$ = region P

Example 1

Find

(i) P(Z< 2)

By symmetry

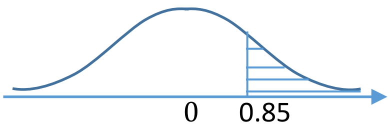


P(Z< 2) = 0.5 + ϕ(2)

= 0.5 + 0.4772

= 0.9772

(ii) P(Z > 0.85)

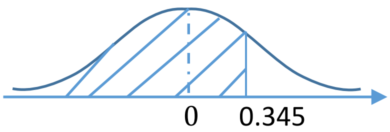


P(Z > 0.85) = 0.5 - ϕ(0.85)

= 0.5- 0.3023

= 0.1977

(iii) P(X< 0.345)



P(X< 0.345) = 0.5 + ϕ(0.345)

= 0.5 + 0.1331 + 0.0019

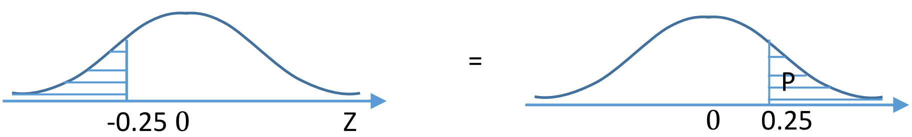
= 0.6350

Example 2

Find

(i) P(Z < -0.25)

By symmetry



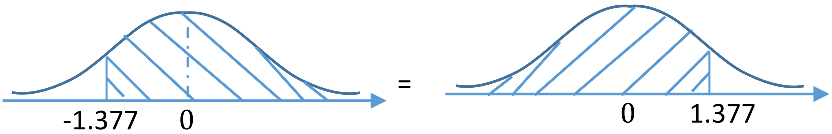
P(Z < -0.25) = P(Z > 0.25) = 0.5 - ϕ(0.25)

= 0.5 – 0.0987

= 0.4013

(ii) P(Z > -1.377)

By symmetry



P(Z > -1.377) = P(Z < 1.377)

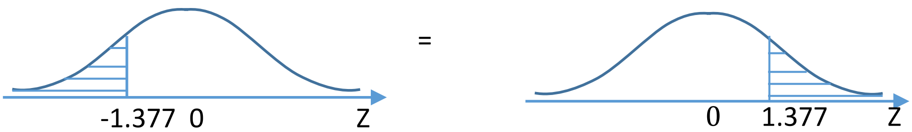
= 0.5 + ϕ(1.377)

= 0.5 + 0.4147 + 0.0011

= 0.9158

(iii) P(Z < -1.377)

By symmetry



P(Z < -1.377) = P(Z > 1.377

= 0.5 - ϕ(1.377)

= 0.5 – (0.4147 + 0.0011)

= 0.0842

Standardizing a random variable X

If a random variable X follows a normal distribution with mean, μ and variance, σ^2, then X ~N(μ, σ^2) and can be standardized using the equation below and read from a cumulative normal table

Z = $\frac{X-\mu}{\sigma}$

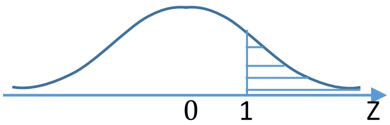
***Example* 3**

Given that the random variable X is X ~N(300, 25). Find

(i) P(X > 305)

P(X> 305) $= P\left(Z < \frac{305-300}{5}\right)$

= P(X > 1)



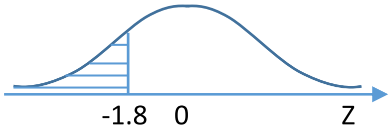
= 0.5 – ϕ(1)

= 0.5 – 0.3413 = 0.1587

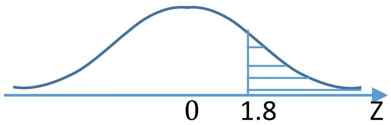
(ii) P(X < 291)

P(X < 291) $= P\left(Z < \frac{291-300}{5}\right)$ = P(Z < -1.8)

By symmetry



=



P(Z < -1.8) = P(Z >1.8)

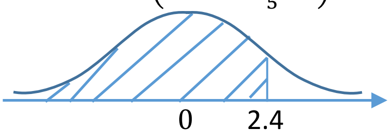
= 0.5 - ϕ(1.8)

= 0.5 – 0.4641

= 0.0359

(iii) P(X < 312)

P(X < 312) $= P\left(Z < \frac{312-300}{5}\right)$= P(X < 2.4)

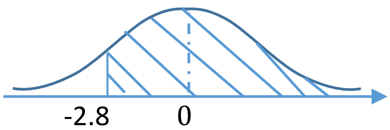


= P(X < 2.4) = 0.5 + ϕ(2.4)

= 0.5 – 0.4918 = 0.9918

(iv) P(X > 286)

P(X > 286) $= P\left(Z < \frac{286-300}{5}\right)$ = P(Z < -2.8



= P(Z < 2.8)

= 0.5 + ϕ(2.8)

= 0.5 + 0.4974= 0.9974

Applications

Example 4

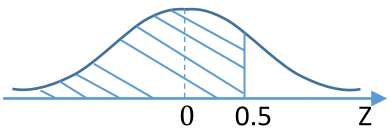
A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

μ = 1000, σ = 40

Let x be the weight of bread

$P(x\leq 1020) = P(z \leq \frac{1020-1000}{40} = P(z \leq 0.5)$



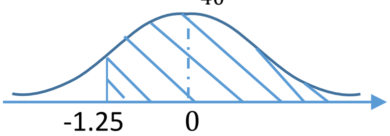
$P(z \leq 0.5) = 0.5 + P(0\leq x\leq 0.5)$

$= 0.5+0.1915$

$= 0.6915$

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

$P(x\leq 950) = P(z \geq \frac{950-1000}{40} = P(z \geq -1.25)$



$P(z \geq -1.25) = P(-1.25\leq z\leq 0) + 0.5$

$= (0 \leq z \leq 1.25) + 0.5$

$= 0.5 + 0.3944$

$= 0.8944$

Number of loaves = 0.8944 x 10,500 = 9391

Example 5

A factory sells animals food in bags. The weights of the bags are normally distributed with mean weight 50kg and standard deviation 2.8kg.

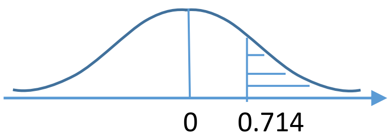
(a) Find the probability that the weight of any bag selected at random;

Let X = random variable for weight of bags

Given μ = 50 and σ = 2.8kg

(i) is more than 53kg (04marks)

$P(X> 52) = P(Z> \frac{52-50}{2.8}) = P(Z > 0.714)$



P(Z > 0.714) = 0.5 – P(0 < Z < 0.714)

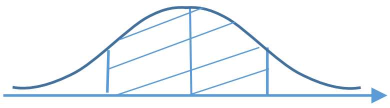
= 0.5 – 0.2623

= 0.2377

(ii) lies between 46 and 55kg (05marks)

P(46 < X < 55) = P$\left(\frac{46-50}{2.8} < Z < \frac{55-50}{2.8}\right)$

= P(-1.429< Z < 1.786)



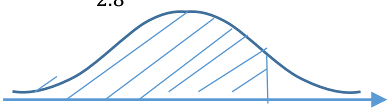
P(-1.429< Z < 1.786) = P(0 <Z< 1.429) + P(0 < Z < 1.789)

= 0.4235 + 0.4630

= 0.8865

(b) Determine the percentage of bags whose weights are less than 54kg. (06marks)

P(X< 54) = P(Z < $\frac{54-50}{2.8}$)= P(Z > 1.429)



P(Z > 1.429) = 0.5 + P(0 < Z < 1.429)

= 0.5 + 0.4235

= 0.9235

Example 6

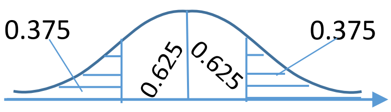
The marks scored by candidates in an examination are normally distributed with a mean score of μ and standard deviation of δ. Given that 37.5% of the candidates scored below 40 and that 12.5% scored above 60, find the;

(i) values of μ and δ. (09marks)

Let X be marks scored

P(X < 40) = $\frac{37.5}{100}$ = 0.375

P(x< 40) = P(Z < Z_0) = 0.375



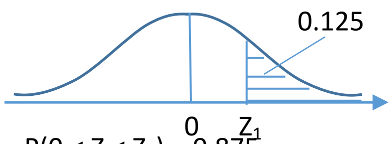
By symmetry, P(Z < Z_0) = P(Z > Z_0)

But Z = $\frac{X-\mu}{\sigma}$

-0.319 = $\frac{40-\mu}{\sigma}$

μ – 0.319σ = 40 ……………………(i)

P(X> 60) = P(Z > Z_1) = $\frac{12.5}{100} = 0.125$



P(0 < Z < Z_1) = 0.875

Z_1 = 1.15

⇨ 1.15 =$\frac{60-\mu}{\sigma}$

μ – 0.319σ = 60 ……………………(ii)

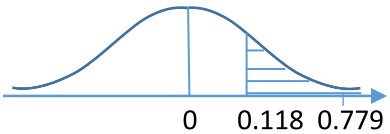
from equation (i) and (ii)

σ = 13.6 and μ= 44.4

(ii) probability that a candidate score between 46 and 55. (06 marks)

P(46< X < 55 =$\left[\frac{46-44.4}{13.6} < Z < \frac{55-44.4}{13.6}\right]$

= P$[0.118 < Z < 0.779\ ]$



= P(0 < Z < 0.779) – P(0<Z<0.118)
= 0.2822 -0.0470
= 0.2352

Example 7

The time taken for a bus to make a journey is normally distributed with mean 3 ½ hours and standard deviation ¾ hours.

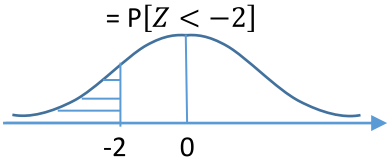
(a) Determine the probability that the bus makes a journey:

(i) in less than 2 hours (05marks)

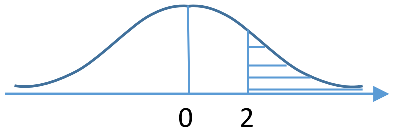
Solution

P(X < 2) = P$\left[Z < \frac{2-3.5}{0.75}\right]$

= P$[Z < -2]$

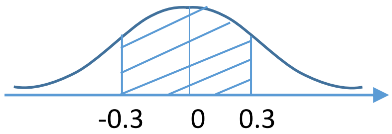


By symmetry, P$[Z < -2]$ = P$[Z > 2]$



P$[Z > 2]$ = 0.5 – (0< Z< 2)
= 0.5 – 0.4772
= 0.0228
Hence the probability = 0.0228

(ii) between 3¼ and 3¾ hours (07 marks)
Solution
P(3.25<X 3.75) = P$\left[\frac{3.25-3.5}{0.75} < Z < \frac{3.75-3.5}{0.75}\right]$
= P$[-0.3 < Z < 0.3\ ]$



P$[-0.3 < Z < 0.3\ ]$ = 2P(0 < Z< 0.3)
= 2 x 0.1179
= 0.2358

(b) If the bus made two hundred journeys, how many of these journeys did it take less than 2 hours? (03 marks)
Number of journeys that take less than 2 hours = 0.0228 x 200 = 4

Example 8

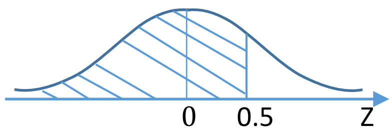
A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

μ = 1000, σ = 40

Let x be the weight of bread

P(x≤ 1020) =P(z ≤ $\frac{1020-1000}{40}$ = P(z ≤ 0.5)



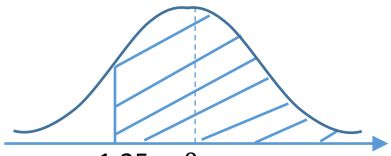
P(z ≤ 0.5)= 0.5 + P(0≤ x≤ 0.5)

= 0.5+0.1915

= 0.6915

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

P(x≤ 950) =P(z ≥ $\frac{950-1000}{40}$ = P(z ≥ -1.25)



P(z ≥ -1.25)= P(-1.25≤ z≤ 0) + 0.5

= (0 ≤ z ≤ 1.25) + 0.5

= 0.5 +0.3944

= 0.8944

Number of loaves = 0.8944 x 10,500 = 9391

Example 5

Given that the random variable X is X ~N(10, 4). Find

Find (i) P(X < 7) (ii) P(X > 12) (iii) P(7 < X < 12) (iv) P(9 < X < 11)

Solution

(i) P(X < 7) $= P\left(Z < \frac{7-10}{2}\right)$ = P(Z < - 0.15) = P(Z > 1.5)
= 0.5 – ϕ(1.5) = 0.5 – 0.4332 0.0668

(ii) P(X > 12) $= P\left(Z > \frac{12-10}{2}\right)$ = P(Z > 1) = 0.5 – ϕ(1) = 0.5 – 0.3413 = 0.1587

(iii) P(7 < X < 12) = $P\left(\frac{7-10}{2} < Z > \frac{12-10}{2}\right)$
= P(-1.5 < Z < 1) = ϕ(1.5) + ϕ(1) = 0.4332 + 0.3413 = 0.7745

(iv) P(9 < X < 11) = $P\left(\frac{9-10}{2} < Z > \frac{11-10}{2}\right)$
= P(-0.5 < Z < 0.5) = ϕ(0.5) + ϕ(0.5) = 2 x 0.1915 = 0.3830

Example 6

Given that the random variable X is X ~N(50, 8). Find

(i) P(48 < X < 54) (ii) P(52 < X < 55) (iii) P(46 < X < 49) (iv) P(|X- 50|< $\sqrt{8}$)

Solution

(i) P(48 < X < 54) = $P\left(\frac{48-50}{\sqrt{8}} < Z > \frac{54-50}{\sqrt{8}}\right)$ = P(-0.707 < Z < 1.414)

= ϕ(1.414) + ϕ(0.707) = 0.4213 + 0.2601 = 0.6814

(ii) P(52 < X < 55) = $P\left(\frac{52-50}{\sqrt{8}} < Z > \frac{55-50}{\sqrt{8}}\right)$ = P(0.707 < Z < 1.768)

= ϕ(1.768) - ϕ(0.707) = 0.4615 -0.2601 = 0.2014

(iii) P(46 < X < 49) = $P\left(\frac{46-50}{\sqrt{8}} < Z > \frac{49-50}{\sqrt{8}}\right)$ = P(-1.414 < Z < -0.354)

=ϕ(1.414) – ϕ(0.354) = 0.4213 – 0.1383 -0.283

(iv) P(|X- 50|< $\sqrt{8}$) = $P\left(\frac{-\sqrt{8}+50-50}{\sqrt{8}} < Z > \frac{\sqrt{8}+50-50}{\sqrt{8}}\right)$ P(-1 < X < 1) = 2 x ϕ(1) = 2 x 0.3413 = 0.6826

Example 6

A random variable X is normally distributed with mean 65 and variance 100, find the probability that X assumes a value between 50 and 90.

P(50 < X < 90) = $P\left(\frac{50-65}{10} < Z < \frac{90-65}{10}\right)$= P(-1.5 < Z < 2.5) = ϕ(1.5) + ϕ(2.5) = 0.4332 + 0.4938 = 0.927

Example 7

Lengths of metal strips produced by a machine are normally distributed with mean length of 150cm and standard deviation of 10cm. find the probability that the length of a randomly selected strip is

(i)shorter than 165 (ii) within 5cm of the mean

Solution

P(X < 165) = $P\left(Z < \frac{165-150}{10}\right)$ =P(Z< 1.5) = 0.5 + ϕ(1.5) = 0.5 + 0.4332 = 0.9332

P(150 – 5 < X < 150 + 5) = $P\left(\frac{-5}{10} < Z < \frac{5}{10}\right)$= P(-0.5 < Z < 0.5) = 2 x ϕ(0.5) = 2 x 0.1915 = 0.383

Example 8

In end of year exams, the marks are normally distributed with a mean mark of 50 and standard deviation 5. If a mark 45 is required to pass the exam, what percentage of the students failed the exam.

P(X < 45) = $P\left(Z < \frac{45-50}{5}\right)$ = P(Z < -1) = P(Z > 1) = 0.5 – ϕ(1) = 0.5 – 0.3413 = 0.1587

Example 9

A bakery supplies bread to a shop every day. The time to deliver bread to the shop is normally distributed with mean 12 minutes and standard deviation of 2 minutes. Estimate the number of days the year when he takes

(i) longer than 17 minutes (ii) less than 10 minutes (iii) between 9 and 13 minutes

Solution

(i) P(X > 17) = $P\left(Z > \frac{17-12}{2}\right)$= P(Z > 2.5) = 0.5 – ϕ(2.5) = 0.5 – 0.4938 = 0.0062

The number of days = 0.0062 x 365 = 2 days

(ii) P(X < 10) = $P\left(Z > \frac{10-12}{2}\right)$= P(Z < -1) = P(Z> 1) = 0.5 – ϕ(1) + 0.5 – 0.3413 0.1587

The number of days = 0.1587 x 365 = 58 days.

(iii) P(9 < X <13) = $P\left(\frac{9-12}{2} < Z < \frac{13-12}{2}\right)$= P(-1.5 < Z < 0.5) = ϕ(1.5) + ϕ(0.5) 0.4332 + 0.1915 = 0.6247

Number of days = 0.6247 x 365 = 228 days.

Example 10

(a) In a certain athletics competition, points are awarded according to level of performance. The average grade was 82 points with standard deviation of 5 points. All competitors whose grades ranged between 88 to 94 points received certificates. If the grades are normally distributed and 8 competitors received certificates. How many participants took part in the competition?

P(88 < X < 94) = $P\left(\frac{88-82}{5} < Z < \frac{94-82}{5}\right)$= P(1.2 < Z < 2.4) =$\frac{8}{n}$

ϕ(2.4) – ϕ(1.2) = 0.4918 – 0.3849 = 0.1069 = $\frac{8}{n}$; n = 74.84

hence 75 participants took part.

(b) If certificates were to be awarded to only those having between 90 and 94 points. What proportion of the participants would acquire certificates.

P(90 < X < 94 = $P\left(\frac{90-82}{5} < Z < \frac{94-82}{5}\right)$=P(1.6 < Z < 2.4 = ϕ(2.4) – ϕ(1.6)

= 0.4918 – 0.4452 = 0.0466

= 0.0466 x 100% = 4.66%

Revision exercise 1

1. The amount of meat sold by a butcher is normally distributed with mean 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg. (0.7333)
2. Given that a random variable X is X~N(2, 2.89). Find P(X<0)(0.1198)
3. In a school of 800 students their average weight is 54.5kg and standard deviation 6.8kg. given that the weight of students are normally distributed, find
   (i) Probability that the weight of any student randomly selected is 52.8 kg or less = 0.4014
   (ii) Number of students who weigh over 75kg = 1
   (iii) Weight of the middle 56% of the students (49.251< X < 59.750)
4. A sugar factory sells sugar in bags of mean weight 50kg and standard deviation 2.5kg. given that the weight of the bags is normally distributed, find the
   (i) Probability that the weight of any bag of sugar randomly selected lies between 51.5kg and 53kg = 0.1592
   (ii) Percentage of bags whose weight exceeds 54kg = 5.48%
   (iii) Number of bags that will be rejected out of 1000 bags purchased for weighing below 45.0kg = 23
5. A certain maize firm sells maize in bags of mean weight 40kg and standard deviation 2kg, given that the weight of the bags are normally distributed, find
   (i) Probability that the weight of any bag of maize randomly selected lies between 41.0 and 42.5kg = 0.2029
   (ii) Percentage of bags whose weight exceeds 43kg = 6.68%

(iii) Number of bags that will be rejected out of 500 bags purchased for weighing below 38.5kg = 113

6. Given that the random variable X is X~N(300, 25) Find
   (i) P(X>308) = 0.0548 (ii) P(X> 311.5) = 0.0107 (iii) P(X < 294) = 0.8849
   (iv) P(X < 290.5) = 0.9713 (v) P(X > 302) = 0.6554 (vi) P(X > 312) = 0.9918
7. If X~N(50, 20). Find
   (i) P(X > 60.3) = 0.0106 (ii) P(X < 47.3) = 0.273 (iii) P(X > 48.9) = 0.5972
   (iv) P(X > 53.5) = 0.2831 (v) P(X < 59.8) = 0.9857 (vi) P(X < 62.3) = 0.9970
8. If X~N(-8, 12). Find
   (i) P(X < -9.8) = 0.1587 (ii) P(X > 0) = 0.8413 (iii) P(X < -3.4) =0.9079
   (iv) P(X > -5.7) =0.2533 (v) P(X < 10.8) = 0.2097 (vi) P(X > -1.6) = 0.0323
9. If X~N(α, α^2) . Find
   (i) P(X < 0) = 0.1587 (ii) P(X > 0) = 0.8413 (iii) P(X < 0.5α) = 0.6915 P(X > 0.5α) = 0.3085
10. If X~N(100, 80) . Find
    (i) P(85 < X < 112) = 0.8634 (ii) P(105 < X < 115) = 0.2413
    (iii) P(85 < X < 92) = 0.1388 (iv) P(|X|< $\sqrt{80}$ = 0.6826
11. If X~N(84, 12) . Find
    (i) P(80 < X < 89)= 0.8014 (ii) P(X <79 or X > 92) = 0.085 (iii) P(76 < X < 82) = 0.2714
    (iv) P(|X- 84|> 2.9) = 0.4028 (v) P(87 < X < 93) = 0.1886
12. The masses of packages from a particular machine are normally distributed with a mean of 200g and standard deviation of 2g, find the probability that a randomly selected package from the machine weighs
    (i) less than 197g = 0.0668
    (ii) more than 200.5g = 0.4013
    (iii) between 198.5g and 199.5g = 0.1747
13. The heights of boys at a certain school follow a normal distribution with mean = 150.3cm and variance 25cm, find the probability that a boy picked at random from the group has a height;
    (i) less than 153cm = 0.7054
    (ii) more than 158cm = 0.018
    (iii) between 150 cm and 158 cm= 0.4621
    (iv) more than 10cm difference from the mean height = 0.0046
14. The masses of a certain type of cabbages are normally distributed with mean of 1000g and standard deviation of 0.15kg. In a batch of 800 cabbages, estimate how many have a mass between 750g and 1290g = 740
15. Cartons of milk from quality super market are advertised as containing 1 litre, but in fact the volume of the content is normally distributed with a mean of 1012ml and standard deviation of 15ml.
    (i) Find the probability that a randomly chosen carton contains more than 1010ml =0.6554
    (ii) In a batch of 1000 cartons, estimate the number of cartons containing less than the advertised volume of milk =8
16. A random variable X is such that X~N(-5, 9). Find the probability that;
    (i) A randomly chosen item from the population will have positive value = 0.0478
    (ii) Out of 10 items chosen randomly, exactly 4 will have a positive value = 0.00082
17. The life of a laptop is normally distributed with a mean of 2000 hours and standard deviation of 120 hours. Estimate the probability that the life of such a laptop will be
    (i) greater than 2150 hours = 0.1056
    (ii) greater than 1910hours = 0.7734
    (iii) within a range 1850 hours to 2090 hours = 0.6678

18. Height of female students at particular college are normally distributed with mean 169cm and standard deviation 9cm.
    If X stands for the height of students in cm. find
    (i) $P(X < X_0) = 0.8$ [176.578]
    (ii) P(X>v1) = 0.6 [166.723]
19. The marks of 500 candidates in an examination are normally distributed with mean 45 marks and standard deviation 20 marks.
    (i) Given that the pass mark is 41, estimate the number of students who passed the examination. [290]
    (ii) If 15% of the candidates obtained a distinction by scoring mark x or more, estimate the value of x. [77.9]
20. The mass of soap powder in certain packets is normally distributed with mean 842 grams and variance 225 grams^2.
    Find the probability that a random sample of 120 packets has mass
    (i) Between 844 grams and 846 [0.0702]
    (ii) Less than 843 grams [0.7673]

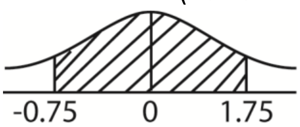
Solutions to revision questions 1

1. The amount of meat sold by a butcher is normally distributed with mean 43kg and standard deviation 4kg. Determine the probability that the amount of meat sold is between 40kg and 50kg.

   X ~N(43,4)

   P(40<x<50) = $P\left(\frac{40-43}{4} < Z < \frac{50-43}{4}\right)$

   = P(- 0.75 < Z < 1.75)



   P(40<x<50) = P(-0.75 < Z < 0) + P(0 < Z < 1.75)

   By property of symmetry

   P(40<x<50) = P(-0.75 < Z < 0) + P(0 < Z < 1.75)

   = 0.2735 + 0.4599

   = 0.733

2. Given that a random variable X is X~N(2, 2.89). Find P(X<0)

   μ = 2, σ = $\sqrt{2.89}$ = 1.7

   P(X < 0) = $P(Z < \frac{0-2}{1.7})$ = P(Z < - 1.176) = P(X> 1.176)

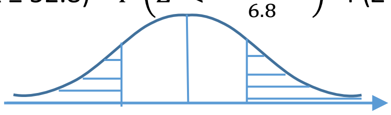
   = 0.5 – P(0 < Z < 1.176)

   = 0.5 – 0.3802 = 0.1198

3. In a school of 800 students their average weight is 54.5kg and standard deviation 6.8kg. given that the weight of students are normally distributed, find
   (i) Probability that the weight of any student randomly selected is 52.8 kg or less

   Let x be the weight of the student

   P(x ≤ 52.8) = $P\left(Z < \frac{52.8-54.5}{6.8}\right)$= P(Z <-0.25)

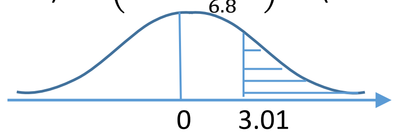


-0.25 0 0.25

$= P(Z > 0.25) = 0.5 - P(0 < Z < 0.25) = 0.5 - 0.0987 = 0.4013$

(ii) Number of students who weigh over 75kg = 1

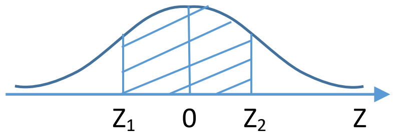
$P(Z > 75) = P\left(Z > \frac{75-54.5}{6.8}\right) = P(Z > 3.01)$



$P(Z > 3.01) = 0.5 – P(0 < Z < 3.01) = 0.5 – 0.4990 = 0.001$

Number of students who weigh more than 75g = 800 x 0.001 =1

(iii) Weight of the middle 56% of the students



$P(X_1 < X < X_2) = P(Z_1 < Z < Z_2) = 0.56$

But $P(0 < Z < Z_2) = 2.8$; $Z_2 = 0.772$ and $Z_1 = -0.772$

$Z_1 = \frac{x_1 - 54.5}{6.8}$

$-0.772 = \frac{x_1-54.5}{6.8}$; $x_1 = 49.251$

$Z_2 = \frac{x_1 - 54.5}{6.8}$

$0.772 = \frac{x_1-54.5}{6.8}$; $x_2 = 59.750$

Hence the weight range of the middle 56% of students of the school is $49.251 < X < 59.750$

4. A sugar factory sells sugar in bags of mean weight 50kg and standard deviation 2.5kg. given that the weight of the bags is normally distributed, find the

(i) Probability that the weight of any bag of sugar randomly selected lies between 51.5kg and 53kg

$P(51.5 < X < 53) = \frac{51.5-50}{2.5} < Z < \frac{53-50}{2.5} = P(0.6 < Z\ 1.2)$

$= \phi(1.2) – \phi(0.6) = 0.3849 – 0.2257 == 0.1592$

(ii) Percentage of bags whose weight exceeds 54kg

$P(X > 54) = P(Z > \frac{54-50}{2.5}) = P(Z > 1.6) = 0.5 - \phi(1.6) = 0.5 – 0.4452 = 0.0548$

$= 0.0548 \times 100 = 5.48\%$

(iii) Number of bags that will be rejected out of 1000 bags purchased for weighing below 45.0kg

$P(X < 45) = P(Z < \frac{45-50}{2.5}) = P(Z < -2) = P(Z < 2) = 0.5 - \phi(2) = 0.5 - 0.4772 = 0.0228$

Number of bags rejected = 0.0228 x 1000 = 22.8 ≈23

5. A certain maize firm sells maize in bags of mean weight 40kg and standard deviation 2kg, given that the weight of the bags are normally distributed, find

(i) Probability that the weight of any bag of maize randomly selected lies between 41.0 and 42.5kg

$P(41.0 < X < 42.5) = \frac{41-40}{2} < Z < \frac{42.5-40}{2} = P(0.5 < Z\ 1.25)$

$= \phi(1.25) – \phi(0.5) = 0.3944 - 0.1915 = 0.2029$

(ii) Percentage of bags whose weight exceeds 43kg

$P(X > 43) = P(Z > \frac{43-40}{2}) = P(Z > 1.5) = 0.5 - \phi(1.5) = 0.5 – 0.4332 = 0.0668$

$= 0.0668 \times 100 = 6.68\%$

(iii) Number of bags that will be rejected out of 500 bags purchased for weighing below 38.5kg

P(X < 38.5) = P($Z < \frac{38.5-40}{2}$) $= P(Z < -0.77) = P(Z < 0.75)$ = 0.5 –ϕ(0.75)

= 0.5- 0.2734 = 0.2266

Number of bags rejected = 0.2266 x 500 = 113

7. If X~N(50, 20). Find

(i) P(X > 60.3)

P(X > 60.3) = P(Z > $\frac{60.3-50}{\sqrt{20}}$) = P(Z > 2.303) = 05 –ϕ(2.303)

= 0.5 –(0.4893 + 0.0001) = 0.0106

(ii) P(X < 47.3)

P(X < 47.3) P(Z > $\frac{47.3-50}{\sqrt{20}}$) = P(Z <-0.6037) =P(Z> 0.6037) = 05 –ϕ(0.6037)

= 0.5 –(0.2257 + 0.0013) = 0.273

(iii) P(X > 48.9)

P(X < 48.9) = P(Z > $\frac{48.9-50}{\sqrt{20}}$) = P(Z <-0.246) = P(Z < 0.246) = 0.5 +ϕ(0.246)

= 0.5 + 0.0948 + 0.0022 = 0.597

(iv) P(X > 53.5)

P(X > 53.5) = P(Z > $\frac{53.5-50}{\sqrt{20}}$) = P(Z > 0.783) =0.2823 + 0.0008 = 0.2831

(v) P(X < 59.8)

P(X < 59.8) = P(Z < $\frac{59.8-50}{\sqrt{20}}$) = P(Z < 2.191) = 0.5 + ϕ(2.191)

= 0.5 + 0.4826 +0.0001 =0.9857

(vi) P(X < 62.3)

P(X < 62.3) = P(Z < $\frac{62.3-50}{\sqrt{20}}$) = P(Z < 2.750) = 0.5 + ϕ(2.730

= 0.5 + 0.4970 = 0.9970

Thanks

Dr. Bbosa Science