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### Discrete probability distribution

A probability density function (p.d.f) if it takes on specific values

Properties of discrete probability density functions

- (i)  $P(X=x) \geq 0$
- (ii)  $\sum f(x) = 1$  or  $\sum P(X = x) = 1$
- (iii) The expected value of  $x$  is given by  $E(X) = \sum xP(X \text{ or } x) = \sum xf(x)$
- (iv)  $\text{Var}(X) = E(X^2) - [E(x)]^2$  where  $E(X^2) = \sum x^2P(X = x)$  or  $\sum x^2f(x) - (\sum xf(x))^2$
- (v) Standard deviation  $= \sqrt{\text{Var}(X)} = \sqrt{\sum x^2f(x) - (\sum xf(x))^2}$

### Examples 1

A discrete random variable has a probability distribution

$y$	-3	-2	-1	0	1
$P(Y=y)$	0.1	0.25	0.3	0.15	$a$

Find

- (a) value of  $a$   
 $\sum P(Y = y) = 1$   
 $0.1 + 0.25 + 0.3 + 0.15 + a = 1$ ;  
 $a + 0.8 = 1$   
 $a = 0.2$
- (b) mode  
 Mode is the value  $y$  with the highest probability, mode = -1
- (c)  $P(-3 \leq Y < 0)$   
 $P(-3 \leq Y < 0) = P(Y = -3) + P(Y = -2) + P(Y = -1)$   
 $= 0.1 + 0.25 + 0.3$   
 $= 0.65$
- (d)  $P(Y > -1)$   
 $P(Y > -1) = P(Y = 0) + P(Y = 1)$   
 $= 0.15 + 0.2$   
 $= 0.35$
- (e)  $P(-1 < Y < 1)$   
 $P(-1 < Y < 1) = P(Y = 0)$   
 $= 0.15$

### Example 2

A discrete random variable X has a probability distribution

X	1	2	3	4	5
P(X = x)	0.15	0.20	0.15	c	0.1

Find

- (i) the value of c

$$\sum P(X = x) = 1$$

$$0.15 + 0.20 + 0.15 + c + 0.1 = 1$$

$$c + 0.6 = 1$$

$$c = 0.4$$

- (ii) mode

The mode is a value with highest probability = 4

- (iii) P(X < 4)

$$P(X < 4) = P(X=1) + P(X=2) + P(X=3)$$

$$= 0.15 + 0.20 + 0.15$$

$$= 0.5$$

- (iv) P(X ≤ 4)

$$P(X < 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= 0.15 + 0.20 + 0.15 + 0.4$$

$$= 0.9$$

- (v) P(2 ≤ X ≤ 4)

$$P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4)$$

$$= 0.20 + 0.15 + 0.4$$

$$= 0.75$$

- (vi)  $P\left(\frac{X > 2}{X \leq 4}\right)$

$$P\left(\frac{X > 2}{X \leq 4}\right) = \frac{P(X > 2, X \leq 4)}{P(X \leq 4)}$$

$$= \frac{P(X=3) + P(X=4)}{P(X=1) + P(X=2) + P(X=3) + P(X=4)}$$

$$= \frac{0.15 + 0.4}{0.9}$$

$$= 0.6111$$

### Example 3

The table below shows the probability distribution of the number of Compact Discs (CDs) sold.

Number of CDs (x)	0	1	2	3	4
Probability, P(X = x)	0.05	0.28	c	0.22	0.09

Determine the:

- (a) Value of c (03 marks)

$$\sum P(X = x) = 1$$

$$\Rightarrow 0.05 + 0.28 + c + 0.22 + 0.09 = 1$$

$$c = 0.36$$

(b) Probability that at least 2 CD's are sold. (03 marks)

$$\begin{aligned} P(x \geq 2) &= P(x=2) + P(x=3) + P(x=4) \\ &= 0.36 + 0.22 + 0.09 \\ &= 0.67 \end{aligned}$$

(c) Expectation,  $E(X)$  (03 marks)

$$\begin{aligned} E(X) &= \sum xP(X=x) \\ &= 0.05 \times 0 + 0.28 \times 1 + 0.36 \times 2 + 0.22 \times 3 + 0.09 \times 4 \\ &= 2.02 \end{aligned}$$

(d) Standard deviation. (06marks)

$$\begin{aligned} \text{Var}(X) &= E(X^2) = \sum x^2P(X=x) \\ &= 0.05 \times 0^2 + 0.28 \times 1^2 + 0.36 \times 2^2 + 0.22 \times 3^2 + 0.09 \times 4^2 \\ &= 5.14 \end{aligned}$$

$$\text{S.D} = \sqrt{E(X^2) - (E(x))^2} = \sqrt{5.14 - (2.02)^2} = 1.0294$$

#### Example 4

A discrete random variable  $W$  has a probability distribution shown below

w	-3	-2	-1	0	1
$P(W=w)$	0.1	0.25	0.3	0.15	d

Find

(a) The value of  $d$  (02 marks)

Solution

$$\text{Total probability} = 1 = 0.1 + 0.25 + 0.3 + 0.15 + d$$

$$d + 0.8 = 1$$

$$d = 0.2$$

(b)  $P(-3 \leq W \leq -1)$  (03marks)

$$\begin{aligned} P(-3 \leq W \leq -1) &= P(W = -3) + P(W = -2) + P(W = -1) \\ &= 0.1 + 0.25 + 0.3 \\ &= 0.65 \end{aligned}$$

(c)  $P(W > -1)$  (02 marks)

$$\begin{aligned} P(W > -1) &= P(W = 0) + P(W = 1) \\ &= 0.15 + 0.2 \\ &= 0.35 \end{aligned}$$

(d) (i) the mode (01 mark)

-1

(ii) the mean (01mark)

(iii) the variance of the distribution (05marks)

w	-3	-2	-1	0	1
$w^2$	9	4	1	0	1
$P(W=w)$	0.1	0.25	0.3	0.15	0.2
$w \cdot P(W=w)$	-0.3	-0.5	-0.3	0	0.2
$w^2 \cdot P(W=w)$	0.9	1.0	0.3	0	0.2

$$\text{Mean } E(W) = \sum_{all} wP(W=w) = -0.3 - 0.5 - 0.3 + 0.2 = -0.9$$

$$\begin{aligned} \text{Var}(W) &= \sum_{all} w^2P(W=w) - (\sum_{all} wP(W=w))^2 \\ &= 0.9 + 1.0 + 0.3 + 0.2 - (-0.9)^2 \\ &= 1.5 - 0.81 \\ &= 0.69 \end{aligned}$$

### Example 5

A random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} \frac{x}{10}, & x = 1, 2, 3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate:

(a)  $P(1 \leq X < 3)$ . (03marks)

$$P(1 \leq X < 3) = P(X=1) + P(X=2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10} = 0.3$$

(b) the mean of X,  $E(X)$  (02 marks)

X	1	2	3	4
$X=x$	0.1	0.2	0.3	0.4
$x(P(X=x))$	0.1	0.4	0.9	1.6

$$E(X) = 0.1 + 0.4 + 0.9 + 1.6 = 3$$

### Example 6

A discrete random variable has a probability function  $P(X = x) = \begin{cases} cx^2 & x = 0, 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$

Find the value of c and draw the graph of  $P(X = x)$

Solution

$$\sum P(X = x) = 1$$

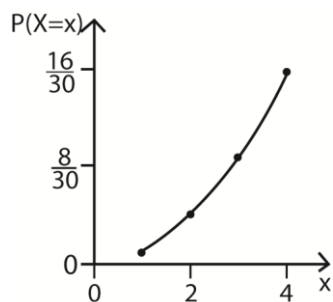
$$c(0^2) + c(1^2) + c(2^2) + c(3^2) + c(4^2) = 1$$

$$c + 4c + 9c + 16c = 2$$

$$c = \frac{1}{30}$$

X	0	1	2	3	4
$P(X = x)$	0	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

Graph



**Example 7**

A discrete random variable has probability function

$$f(x) = \begin{cases} kx, & x = 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}, \text{ find the value of } k \text{ and draw the graph of } f(x)$$

Solution

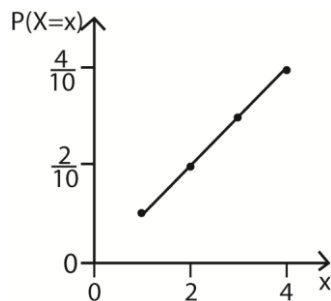
$$\sum f(x) = 1$$

$$k + 2k + 3k + 4k = 1$$

$$k = \frac{1}{10}$$

X	0	1	2	3	4
P(X=x)	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Graph

**Example 8**

A random variable X of a discrete probability distribution given by

$$P(X=1) = 0.2, P(X=2) = P(X=3) = 0.1, P(X=4) = P(X=5) = c$$

Find the value of the constant c and draw the graph of P(X = x)

Solution

$$\sum P(X = x) = 1$$

$$0.2 + 0.1 + 0.1 + c + c = 1; c = 0.3$$

**Example 9**

A discrete random variable has a probability function

$$P(X = x) = \begin{cases} k \left(\frac{2}{3}\right)^x, & x = 1, 2, 3, \dots \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k

Solution

$$k \left(\frac{2}{3}\right)^0 + k \left(\frac{2}{3}\right)^1 + k \left(\frac{2}{3}\right)^2 + k \left(\frac{2}{3}\right)^3 + \dots = 1$$

$$k \left( 1 + \left(\frac{2}{3}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \dots \right) = 1$$

$$\text{Sum to infinity} = S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow k \left( \frac{1}{1-\frac{2}{3}} \right) = 1; k = \frac{1}{3}$$

### Example 10

A discrete random variable X has a probability function

$$f(x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ 0, & \text{otherwise} \end{cases}$$

Find

(i) the value of k

$$\sum P(X = x) = 1$$

$$k + 2k + 3k + 4k + 5k = 1$$

$$15k = 1$$

$$k = \frac{1}{15}$$

(ii)  $P(X = 3)$

$$P(X=3) = 3k = \frac{3}{15} = \frac{1}{5}$$

(iii)  $P(X \geq 3)$

$$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$$

$$= 3k + 4k + 5k$$

$$= 12k$$

$$= 12 \times \frac{1}{15}$$

$$= \frac{4}{5}$$

(iv)  $P(X \leq 3)$

$$P(X \leq 3) = P(X=1) + P(X=2) + P(X=3)$$

$$= k + 2k + 3k$$

$$= 6 \times \frac{1}{15}$$

$$= \frac{2}{5}$$

(v)  $P(1 < X \leq 3)$

$$P(1 < X \leq 3) = P(X=2) + P(X=3)$$

$$= 2k + 3k$$

$$= 5 \times \frac{1}{15}$$

$$= \frac{1}{3}$$

(vi)  $P\left(\frac{X \geq 1}{X < 4}\right)$

$$\begin{aligned}
P(X \geq 2 / X < 4) &= \frac{P(X \geq 2, X < 4)}{P(X < 4)} \\
&= \frac{P(X=2)+P(X=3)}{P(X=1)+P(X=2)+P(X=3)} \\
&= \frac{2k+3k}{k+2k+3k} \\
&= \frac{5k}{6k} \\
&= \frac{5}{6}
\end{aligned}$$

### Example 11

A discrete random variable X can take on values 0, 1, 2, and 3 only. If  $E(X) = 1.4$ ,  $P(X \leq 2) = 0.9$  and  $P(X \leq 1) = 0.5$ . Find (i)  $P(X=1)$  (ii)  $P(X=0)$

Let  $P(X=0) = a$ ,  $P(X=1)=b$ ,  $P(X=2)=c$   $P(X=3) = d$

$$a + b + c + d = 1 \dots\dots\dots (i)$$

$$P(X \leq 2) = a + b + c = 0.9 \dots (ii)$$

Eqn. (i) and eqn. (ii)

$$d = 0.1$$

$$P(X \leq 1) = a + b = 0.5 \dots\dots (iii)$$

Eqn. (i) and eqn. (iii)

$$0.5 + c + 0.1 = 1$$

$$c = 0.4$$

$$E(X) = 0 \times a + 1 \times b + 2 \times c + 3 \times 0.1 = 1.4$$

$$= b + 2c + 0.3 = 1.4$$

$$b + 2c = 1.1$$

$$b + 2 \times 0.4 = 1.1$$

$$b = 0.3$$

$$a = 0.5 - 0.3 = 0.2$$

Hence, (i)  $P(X=1) = 0.3$  (ii)  $P(X=0) = 0.2$

### Revision exercise 1

(Answers are given in the square brackets, [ ], at end of each question)

1. The discrete random variable Y has a probability distribution is given by

$$P(Y=y) = cy, \quad y = 1, 2, 3, 4$$

$$\text{Find (i) value of } c = 0.1 \quad \text{(ii) } E(X) = \frac{11}{3}$$

2. A discrete random variable has p.d.f

$$P(X = x) = \begin{cases} k2^x, & x = 0, 1, 2, 3, 4, 5, 6 \\ 0, & \text{otherwise} \end{cases},$$

Find

(i) (i) value of  $k \left[ \frac{1}{127} \right]$ ,

(ii) Mean [5.01]

3. A discrete random variable X has a probability distribution

x	0	1	2	3	4	5
P(X=x)	0.11	0.17	0.2	0.13	p	0.09

Find

- (i) the value of p [0.3]
- (ii) Expected value of X [2.61]

4. A random variable X of discrete probability distribution is given by

x	-1	0	1	2
P(X=x)	0.25	0.10	0.45	0.20

Find

- (i)  $P(-1 \leq X < 1)$  [0.35]
- (ii)  $E(X)$  = [0.6]

5. A discrete random variable X has a probability distribution

x	1	2	3	4	5
P(X=x)	0.2	0.25	0.4	0.1	0.05

Find

- (i) Mean [2.55]
- (ii) The variance (1.148)
- (iii) Standard deviation = [1.0714]

6. A random variable X of a discrete probability distribution given by

x	10	20	30
P(X=x)	0.2	0.3	0.5

Find

- (i)  $E(X)$  [22]
- (ii)  $\text{Var}(X)$  [36]
- (iii) Standard deviation [6]

7. A random variable X of discrete probability distribution is given by

x	1	2	3
P(X=x)	0.2	0.3	0.5

Find (i)  $E(X) = 2.3$  (ii)  $E(X^2) = 5.9$  (iii)  $\text{Var}(X) = 0.61$

8. A random variable X of discrete probability distribution is given by

x	-1	0	1	2
P(X=x)	0.25	0.1	0.45	0.2

Find: (i)  $P(-1 \leq X < 2) = 0.8$  (ii)  $E(X) = 0.6$  (iii)  $E(2x + 3) = 4.2$

9. A random variable X of a discrete probability distribution

$$P(X=0) = 0.05, P(X=1) = 0.45, P(X=2) = 0.5$$

Find: (i)  $E(X) = 1.45$ , (ii)  $E(X^2) = 2.45$  (iii)  $\text{Var}(X) = 0.348$

10. A random variable X of discrete probability distribution is given by

$$1) 0.1, P(X=2) = 0.2, P(X=3) = 0.3, P(X=4) = 0.4$$

Find (i)  $E(X) = 3$  (ii)  $\text{Var}(X) = 1$  (iii)  $P(X=2|X \geq 2) = \frac{2}{9}$

P(X =

11. The discrete random variable Y has a probability distribution  $P(Y=y) = k^{-1}, 2, 3, 4, 5, 6$

Find (i) mean,  $\mu = 3.5$  (ii)  $E(3X + 4) = 15\frac{1}{6}$  (iii)  $E(X^2) = 14.5$  (iv) standard deviation = 1.708

12. The discrete random variable R has a probability distribution is given by

$$P(R = r) = \frac{3r+1}{22}; r = 0, 1, 2, 3$$

Find (i) mean,  $\mu = \frac{24}{11}$ ,  $E(R^2) = \frac{61}{11}$  (iii)  $E(3R-2) = \frac{50}{11}$

13. The discrete random variable R has a probability distribution given by

$$P(R = r) = \begin{cases} \frac{2r+1}{20}; & r = 0, 1, 2, 3 \\ \frac{11-r}{20}, & r = 4, 5 \end{cases}$$

Find (i)  $E(R) = 2.55$ , (ii)  $\text{Var}(R) = 1.45$

14. The discrete random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, 4, 5 \\ k(10 - x), & x = 6, 7, 8, 9 \end{cases}$$

Find (i) constant,  $k = 0.04$ , (ii)  $E(X) = 5$  (iii)  $\text{Var}(X) = 4$

15. The discrete random variable X has a probability distribution is given by

$$P(X = x) = kx, \quad x = 1, 2, 3, \dots, n; \text{ where } k \text{ is a constant}$$

Show that  $k = \frac{2}{n(n+1)}$ , hence find in terms of n the mean  $X = \frac{1}{3}(2n + 1)$

16. A random variable X of a discrete probability distribution given by

$$P(X = 0) = P(X = 1) = 0.1, P(X = 2) = 0.2, P(X = 3) = P(X = 4) = 0.3. \text{ Find } \text{Var}(X) = 1.64$$

17. A random variable X of a discrete probability distribution given by

$$P(X = 2) = 0.1; P(X = 4) = 0.3; P(X = 6) = 0.5; P(X = 8) = 0.1. \text{ Find } \text{Var}(X) = 2.56$$

## Continuous random variable

A probability density function (p.d.f) is continuous if it takes on values between intervals.

Properties of a continuous probability density functions

(i)  $f(x) \geq 0$

(ii)  $\int f(x) dx = 1$

(iii)  $E(X) = \int xf(x) dx$

(iv)  $E(X^2) = \int x^2 f(x) dx$

(v)  $\text{Var}(X) = E(X^2) - [E(X)]^2$

(vi) **Standard deviation**  $= \sqrt{\text{Var}(X)} = \sqrt{\int x^2 f(x) dx - (\int f(x) dx)^2}$

### Example 12

A continuous random variable X has a probability density function given by,

$$f(x) = \begin{cases} \frac{kx}{6}, & 1 \leq x \leq 2, \\ 0 & \text{Otherwise} \end{cases}$$

Where k is a constant

(a) Find

(i) The value of k (04marks)

$$\int f(x)dx = 1$$

$$\frac{k}{6} \int_1^2 x dx = \frac{k}{6} \left[ \frac{x^2}{2} \right]_1^2 = 1$$

$$\frac{k}{6} \left( 2 - \frac{1}{2} \right) = \frac{k}{6} \left( \frac{3}{2} \right) = 1$$

$$k = 4$$

(ii)  $P(X \geq 1.5)$  (04marks)

$$P(X \geq 1.5) = \frac{4}{6} \left[ \frac{x^2}{2} \right]_{1.5}^2 = \frac{2}{3} \left( 2 - \frac{2.25}{2} \right) = 0.58$$

(iii) The mean of X,  $E(X)$  (03 marks)

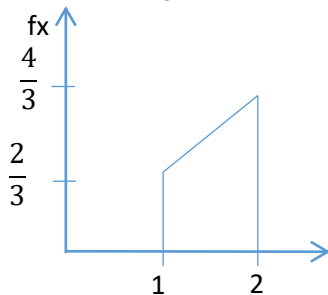
$$E(X) = \int xf(x)dx = \frac{4}{6} \int_1^2 x^2 dx = \frac{4}{6} \left[ \frac{x^3}{3} \right]_1^2 = \frac{4}{6} \left( \frac{8}{3} - \frac{1}{3} \right) = \frac{2}{3} \cdot \frac{7}{3} = \frac{14}{9}$$

(b) Sketch the graph of  $f(x)$  (04 marks)

$$f(x) = \frac{2}{3}x$$

$$\text{At } x = 1, fx = \frac{2}{3}$$

$$\text{At } x = 2, fx = \frac{4}{3}$$



### Example 12

A random variable X has a probability density function,  $f(x)$ , defined by

$$f(x) = \begin{cases} kx(x+2), & 0 \leq x \leq 2, \\ 0, & \text{otherwise} \end{cases}$$

where k is constant

Determine

(a) value of k (04marks)

$$\int f(x)dx = 1$$

$$\Rightarrow k \int_0^2 (x^2 + 2x) dx = 1$$

$$k \left[ \frac{x^3}{3} + x^2 \right]_0^2 = 1$$

$$k \left( \frac{8}{3} + 4 \right) = 1 \Rightarrow k = \frac{3}{20}$$

(b)  $P(1 \leq X \leq 1.5)$  (03marks)

$$P(1 \leq X \leq 1.5) = \frac{3}{20} \left[ \frac{x^3}{3} + x^2 \right]_1^{1.5} = \frac{3}{20} \left[ \left( \frac{8}{3} + 4 \right) - \left( \frac{1}{3} + 1 \right) \right] = 0.306$$

(c) Expectation,  $E(X)$  (04marks)

$$\begin{aligned} E(X) &= \int x f(x) dx = \frac{3}{20} \int_0^2 (x^3 + 2x^2) dx \\ &= \frac{3}{20} \left[ \frac{x^4}{4} + \frac{2x^3}{3} \right]_0^2 \\ &= \frac{3}{20} \left( \frac{16}{4} + \frac{16}{3} \right) \\ &= 1.4 \end{aligned}$$

(d) Variance,  $\text{Var}(X)$  (04marks)

$$\begin{aligned} E(X^2) &= \int x^2 f(x) dx = \frac{3}{20} \int_0^2 (x^4 + 2x^3) dx \\ &= \frac{3}{20} \left[ \frac{x^5}{5} + \frac{2x^4}{4} \right]_0^2 \\ &= \frac{3}{20} \left( \frac{32}{5} + \frac{32}{4} \right) \\ &= 2.16 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(x))^2 \\ &= 2.16 - (1.4)^2 = 0.2 \end{aligned}$$

#### Example 14

A cumulative random variable  $X$ , has a probability density function (pdf) given by

$$f(x) = \begin{cases} k(x^2 + 6), & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

Where  $k$  is a constant

Determine the:

(i) value of  $k$  (04 marks)

$$\begin{aligned} \int_0^3 f(x) dx &= 1 \\ k \int_0^3 (x^2 + 6) dx &= 1 \\ k \left[ \frac{x^3}{3} + 6x \right]_0^3 &= 1 \\ k \left( \frac{3^3}{3} + 6 \cdot 3 \right) &= 1 \\ 27k &= 1 \\ k &= \frac{1}{27} \end{aligned}$$

(ii)  $P(X > 1)$ . (04 marks)

$$\frac{1}{27} \left[ \frac{x^3}{3} + 6x \right]_1^3 = \frac{1}{27} \left[ 27 - 6 \frac{1}{3} \right] = \frac{62}{81} = 0.7654$$

(iii) expectation,  $E(X)$  (03marks)

$$\begin{aligned} E(X) &= \frac{1}{27} \int_0^3 x f(x) dx \\ &= \frac{1}{27} \int_0^3 x(x^2 + 6) dx = \int_0^3 (x^3 + 6x) \\ &= \frac{1}{27} \left[ \frac{x^4}{4} + 3x^2 \right]_0^3 = \frac{1}{27} \left( \frac{81}{4} + 27 \right) \end{aligned}$$

$$= 1.75$$

(iv) variance,  $\text{Var}(X)$  (04 marks)

$$\begin{aligned} E(X^2) &= \frac{1}{27} \int_0^3 x^2 f(x) dx \\ &= \frac{1}{27} \int_0^3 x^2 (x^2 + 6) dx = \int_0^3 (x^4 + 6x^2) \\ &= \frac{1}{27} \left[ \frac{x^5}{5} + 2x^3 \right]_0^3 = \frac{1}{27} \left( \frac{243}{5} + 54 \right) \\ &= 3.8 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= 3.8 - (1.75)^2 \\ &= 0.7375 \end{aligned}$$

### Example 15

A random variable of continuous p.d.f is given by  $f(x) = \begin{cases} kx^2 & 0 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) value of  $k$  and sketch  $f(x)$  (ii)  $P(1 \leq x \leq 3)$

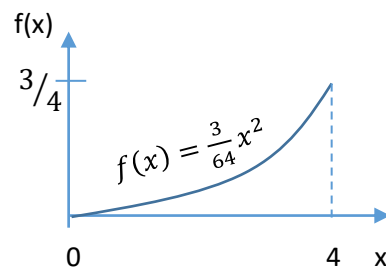
Solution

$$\begin{aligned} \text{(i)} \quad \int_0^4 kx^2 dx &= 1 \\ k \left[ \frac{x^3}{3} \right]_0^4 &= k \left[ \frac{4^3}{3} - \frac{0^3}{3} \right] = 1 \\ k &= \frac{3}{64} \end{aligned}$$

$$\text{When } x = 0, f(x) = \frac{3}{64} 0^2 = 0$$

$$\text{When } x = 4, f(x) = \frac{3}{64} 4^2 = \frac{3}{4}$$

Sketch



$$\begin{aligned} \text{(ii)} \quad P(1 \leq x \leq 3) &= \frac{3}{64} \int_1^3 kx^2 dx = 1 \\ &= \frac{3}{64} \left[ \frac{x^3}{3} \right]_1^3 = \frac{3}{64} \left[ \frac{3^3}{3} - \frac{1^3}{3} \right] = 0.4063 \end{aligned}$$

### Example 16

A random variable  $X$  of a continuous p.d.f is given by  $f(x) = \begin{cases} k(x^2 + 1) & 0 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$

Find

(i) value of k and sketch f(x)

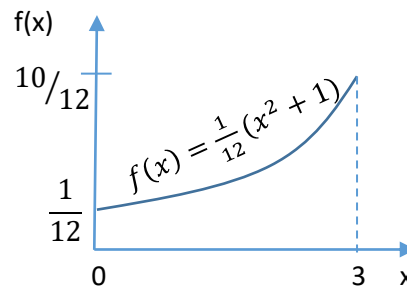
$$\int_0^3 k(x^2 + 1)dx = 1$$

$$k \left[ \frac{x^3}{3} + x \right]_0^3 = k \left[ \left( \frac{3^3}{3} + 3 \right) - \left( \frac{0^3}{3} + 0 \right) \right] = 1$$

$$k = \frac{1}{12}$$

When  $x = 0$ ,  $f(x) = \frac{1}{12}(0^2 + 1) = \frac{1}{12}$

When  $x = 3$ ,  $f(x) = \frac{1}{12}[3^2 + 1] = \frac{10}{12}$



(ii)  $P(1 \leq x \leq 3)$

$$\frac{1}{12} \int_1^3 (x^2 + 1)dx = \frac{1}{12} \left[ \frac{x^3}{3} + x \right]_1^3 = \frac{1}{12} \left[ \left( \frac{3^3}{3} + 3 \right) - \left( \frac{1^3}{3} + 1 \right) \right] = 0.8889$$

**Example 17**

A random variable X of continuous p.d.f is given by  $f(x) = \begin{cases} k, & 0 \leq x \leq 2 \\ k(2x - 3), & 2 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) value of k and sketch f(x) (ii)  $P(X < 1)$  (iii)  $P(X > 2.5)$  (iv)  $(0 \leq X \leq 2 / X \geq 1)$

Solution

$$\int_0^2 k dx + \int_2^3 k(2x - 3) dx = 1$$

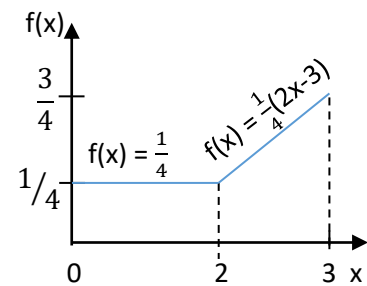
$$k[x]_0^2 + k[x^2 - 3x]_2^3 = 1$$

$$k = \frac{1}{4}$$

When  $x = 0$ ,  $f(x) = k = \frac{1}{4}$

When  $x = 2$ ,  $f(x) = k = \frac{1}{4}$

When  $x = 3$ ,  $f(x) = \frac{1}{4}(2 \times 3 - 3) = \frac{3}{4}$



(ii)  $P(X < 1) = \frac{1}{4} \int_0^1 dx = \frac{1}{4} [x]_0^1 = \frac{1}{4}$

(iii)  $P(X > 2.5) = \frac{1}{4} \int_{2.5}^3 (2x - 3) dx = \frac{1}{4} [x^2 - 3x]_{2.5}^3 = 0.3125$

(iv)  $P(0 \leq X \leq 2 / X \geq 1) = \frac{P(0 \leq X \leq 2)}{P(X \geq 1)} = \frac{P((0 \leq X \leq 2) \cap (X \geq 1))}{P(X \geq 1)} = \frac{\frac{1}{4} \int_1^2 dx}{\frac{1}{4} \int_1^2 dx + \frac{1}{4} \int_2^3 (2x - 3) dx} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} = \frac{1}{3}$

## Revision exercise 2

(Answers are given in the square brackets, [ ], at end of each question)

- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} kx^2, & 0 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

(a) Sketch  $f(x)$

(b) Find (i) value of  $k (= \frac{3}{64})$  (ii)  $E(X) = 3$  and  $\text{var}(X) = 0.6$  (iii)  $P(1 < X < 2) = \frac{7}{64}$
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} kx, & 0 \leq x \leq 1 \\ k(2-x), & 1 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) constant  $k = 1$  (ii)  $E(X) = 1$  (iii)  $\text{var}(X) = \frac{1}{6}$  (iv)  $P(0.75 < X < 1.5) = \frac{19}{32}$  (v) mode = 1
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} \frac{1}{27}x^2, & 0 \leq x \leq 3 \\ \frac{1}{3}, & 3 \leq x \leq 5 \\ 0, & \text{elsewhere} \end{cases}$

(a) Sketch  $f(x)$

(b) Find (i)  $E(X) = 3417$  (ii) standard deviation = 1.008
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} \frac{k}{x(4-x)}, & 1 \leq x \leq 3 \\ 0, & \text{elsewhere} \end{cases}$

(i) Show that  $k = \frac{3}{\ln x}$

(ii) Find (i)  $E(X) = 2$  (ii)  $\text{Var}(X) = 4 - \frac{4}{\ln x}$
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} k(ax - x^2), & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$

(i) Show that  $k = \frac{8}{6a-8}$

(ii) Given that  $E(X) = 1$ , find the values of  $a (=2)$  and  $k (=0.75)$

(iii) For the above values of  $a$  and  $k$ , find  $\text{Var}(X) = 0.2$
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} 12(x^2 - x^3), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$

Find the (i) mean = 0.6 (ii) standard deviation = 0.2
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} \frac{k}{\beta}, & 0 \leq x \leq \beta \\ 0, & \text{elsewhere} \end{cases}$

Find (i) value of  $k (=1)$  (ii) mean =  $\frac{\beta}{2}$  (iii) standard deviation =  $\frac{\beta}{2\sqrt{3}}$
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} \frac{1}{8}(x+1), & 2 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) mean =  $\frac{37}{12}$  (ii)  $\text{var}(X) = \frac{47}{144}$  (iii)  $P(2.5 < x < 3) = 0.234$
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} k(1-x)^2, & 2 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) constant  $k = \frac{3}{26}$  (ii) mean =  $\frac{1}{4}$  (iii) standard deviation = 0.94
- A random variable  $x$  of a continuous p.d.f is given by  $f(x) = \begin{cases} kx, & 0 \leq x \leq 2 \\ k(4-x), & 2 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) value of  $k = \frac{1}{4}$  (ii)  $E(X) = 2$  (iii)  $\text{Var}(X) = \frac{2}{3}$  (iv)  $P(X < 1) = \frac{1}{8}$  (v)  $P(X < X < 3) = \frac{3}{8}$