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Sub math: Quadratic equations

The roots of the equation $2x^2 + 4x - 1 = 0$ are α and β . Find the value of $\alpha^2 + \beta^2$. (05marks)

$$2x^2 + 4x - 1 = 0$$

$$x^2 + 2x - \frac{1}{2} = 0$$

$$\alpha + \beta = -2$$

$$\alpha \beta = -\frac{1}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4 - (-1) = 5$$

The roots of the equation $2x^2 - 6x + 7 = 0$ are α and β . Determine the

(a) values of $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ (12 marks)

Solution

$$\alpha + \beta = 3$$

$$\alpha\beta = \frac{7}{2}$$

$$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$

$$\text{But } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow (\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 4\alpha\beta$$

$$= 3^2 - 4 \times \frac{7}{2}$$

$$= -5$$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{\alpha + \beta}{\alpha^2\beta^2} = \frac{\alpha + \beta}{(\alpha\beta)^2} = \frac{2}{\left(\frac{7}{2}\right)^2} = \frac{3 \times 4}{49} = \frac{12}{49}$$

- (b) Quadratic equation with integral coefficients whose roots $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ (03marks)

$$\text{Sum of root} = -5 + \frac{12}{49} = -\frac{233}{49}$$

$$\text{Product of roots} = -5 \times \frac{12}{49} = -\frac{60}{49}$$

Equation

$$x^2 - \left(-\frac{233}{49}\right)x - \frac{60}{49} = 0$$

$$\text{Hence equation is } x^2 - \frac{233}{49}x - \frac{60}{49} = 0$$

1. (a) Given that α and β are the root of the quadratic equation $x^2 - 3mx + n^2 = 0$, show that $\alpha + \beta = 3m$ and $\alpha\beta = n^2$. (06 marks)

Solution

$$(x - \alpha)(x - \beta) = 0$$

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Comparing

$$\alpha + \beta = 3m \text{ and } \alpha\beta = n^2$$

- (b) If α and β are the root of the equation $x^2 - 9x + 4 = 0$, find the;

- (i) value of $\alpha^2 + \beta^2$. (03marks)

Solution

$$\text{From } (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (9)^2 - 2 \times 4$$

$$= 81 - 8$$

$$= 73$$

- (ii) value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (03marks)

Solution

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{73}{4^2} = \frac{73}{16} = 4.5625$$

(iii) quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. (03marks)

Solution

Sum of the root, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{73}{16}$

Product of the roots, $\frac{1}{\alpha^2} \cdot \frac{1}{\beta^2} = \frac{1}{(\alpha\beta)^2} = \frac{1}{16}$

Hence quadratic equation

$$x^2 - \frac{73}{16}x + \frac{1}{16} = 0$$



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Quadratic equations

These are equations expressed in the form $ax^2 + bx + c = 0$ where a, b and c are constants and $a \neq 0$

They have at most two roots which may be real or complex.

Example of quadratic equations

$2y^2 + 3y + 5 = 0$; a = 2, b = 3 and c = 5

$x^2 + 4x - 10 = 0$; a = 1, b = 4, c = -10

Forming quadratic equations

Suppose that the roots of a quadratic equation are α and β ,

then $x - \alpha = 0$ and $x - \beta = 0$

When forming a quadratic equation

$(x - \alpha)(x - \beta) = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

This means that if the roots of a quadratic equation are given, its equation in terms of x is $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

Example 1

Form quadratic equation in terms of x with roots

(a) (2, 3)

Solution

Let $x = 2$ or $x = 3$
 $x - 2 = 0$ or $x - 3 = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$x^2 - 5x - 6 = 0$

(b) (p, q)

Solution

Let $x = p$ or $x = q$
 $x - p = 0$ or $x - q = 0$

$\Rightarrow (x - p)(x - q) = 0$

$x^2 - (p + q)x - pq = 0$

(c) (-3, -2)

Solution

Let $x = -3$ or $x = -2$
 $x + 3 = 0$ or $x + 2 = 0$

$$\Leftrightarrow (x+3)(x+2) = 0$$

$$x^2 + 5x + 6 = 0$$

Example 2

The roots of the equation $4x^2 + 9x - k = 0$ are α and 2 . Find the values of α and k .

(05marks)

$$2 + \alpha = -\frac{9}{4}$$

$$\alpha = -\frac{9}{4} - 2 = -\frac{17}{4} = -4\frac{1}{4}$$

$$2\alpha = -\frac{k}{2}$$

$$2x - \frac{17}{4} = -\frac{k}{2}$$

$$-\frac{k}{2} = -\frac{34}{4}$$

$$k = 34$$

Example 3

The roots of the equation $2x^2 + 4x - 1 = 0$ are α and β . Find the value of $\alpha^2 + \beta^2$. (05marks)

$$2x^2 + 4x - 1 = 0$$

$$x^2 + 2x - \frac{1}{2} = 0$$

$$\alpha + \beta = -2$$

$$\alpha\beta = -\frac{1}{2}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= 4 - (-1) = 5$$

Example 4

The roots of the equation $2x^2 - 6x + 7 = 0$ are α and β . Determine the

(a) values of $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$
(12 marks)

Solution

$$\alpha + \beta = 3$$

$$\alpha\beta = \frac{7}{2}$$

$$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$

$$\text{But } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow (\alpha - \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta - 2\alpha\beta$$

$$= (\alpha + \beta)^2 - 4\alpha\beta$$

$$= 3^2 - 4 \times \frac{7}{2}$$

$$= -5$$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{\alpha + \beta}{\alpha^2\beta^2} = \frac{\alpha + \beta}{(\alpha\beta)^2} = \frac{2}{\left(\frac{7}{2}\right)^2} = \frac{3 \times 4}{49} = \frac{12}{49}$$

(b) Quadratic equation with integral coefficients whose roots $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ (03marks)

Solution

$$\text{Sum of roots} = -5 + \frac{12}{49} = -\frac{233}{49}$$

$$\text{Product of roots} = -5 \times \frac{12}{49} = -\frac{60}{49}$$

Equation

$$x^2 - \left(-\frac{233}{49}\right)x - \frac{60}{49} = 0$$

$$\text{Hence equation is } x^2 - \frac{233}{49}x - \frac{60}{49} = 0$$

Example 5

Determine the possible values of α for which the equation

$2x^2 + (\alpha + 2)x + (\alpha + 2) = 0$ has equal roots
(05marks)

Solution

For equal roots $b^2 = 4ac$

$$(\alpha + 2)^2 = 4(2)(\alpha + 2)$$

$$\alpha^2 + 4\alpha + 4 = 8\alpha + 16$$

$$\alpha^2 - 4\alpha - 12 = 0$$

$$(\alpha - 6)(\alpha + 2) = 0$$

$$\text{Either } \alpha - 6 = 0, \alpha = 6$$

$$\text{Or } \alpha + 2 = 0, \alpha = -2$$

Example 6

2. (a) Given that α and β are the root of the quadratic equation $x^2 - 3mx + n^2 = 0$, show that $\alpha + \beta = 3m$ and $\alpha\beta = n^2$.

Solution

$$(x - \alpha)(x - \beta) = 0$$

$$x^2 - \alpha x - \beta x + \alpha\beta = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Comparing

$$\alpha + \beta = 3m \text{ and } \alpha\beta = n^2$$

- (b) If α and β are the root of the equation $x^2 - 9x + 4 = 0$, find the;

- (i) value of $\alpha^2 + \beta^2$. (03marks)

Solution

$$\text{From } (\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (9)^2 - 2 \times 4$$

$$= 81 - 8$$

$$= 73$$

- (ii) value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

Solution

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} =$$

$$\frac{73}{4^2} = \frac{73}{16} = 4.5625$$

- (iii) quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$.

Solution

$$\text{Sum of the root, } \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{73}{16}$$

$$\text{Product of the roots, } \frac{1}{\alpha^2} \cdot \frac{1}{\beta^2} =$$

$$\frac{1}{(\alpha\beta)^2} = \frac{1}{16}$$

Hence quadratic equation

$$x^2 - \frac{73}{16}x + \frac{1}{16} = 0$$

Exercise 1

(Answers are given in square brackets, [] at the end of each questions

1. Given that the root of the equation $x^2 + px + q = 0$ are α and β , Form quadratic equations with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$
 $[x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$
 Or $qx^2 - (p^2 - 2q)x + q = 0]$

2. Given the equation $x^3 + x - 10 = 0$.

- (a) Show that $x = 2$ is a root of the equation

$$\text{Let } f(x) = x^3 + x - 10$$

Substituting for $x = 2$

$$f(2) = 2^3 + 2 - 10$$

$$= 8 + 2 - 10$$

$$= 10 - 10 = 0$$

Hence $x = 2$ is a root of $x^3 + x - 10 = 0$

- (b) Deduce the values of $\alpha + \beta$ and $\alpha\beta$ where α and β are roots of the equation.

Hence form a quadratic equation whose

roots are α^2 and β^2 .

$$\Rightarrow x^3 + x - 10 = (x-2)(x^2 + 2x + 5)$$

Either $x - 2 = 0$

Or $(x^2 + 2x + 5) = 0$

$$\alpha + \beta = 2$$

$$\alpha\beta = 5$$

$$\text{Sum of roots} = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-2)^2 - 2(5) = 4 - 10 = -6$$

$$\text{Product} = \alpha^2\beta^2 = (\alpha\beta)^2 = 5^2 = 25$$

The equation become

$$x^2 - (-6)x + 25 = 0$$

$$x^2 + 6x + 25 = 0$$

Solving quadratic equations

Quadratic equations may be solved by

- (a) Factorization method
- (b) Completing square method

(a) Factorization method

It is used for quadratic equations that are easy to factorise

Example 7

(a) $4x^2 + 7x + 3 = 0$

Solution	side work
$4x(x + 1) + 3(x + 1) = 0$	Product = $4 \times 3 = 12$
$(x + 1)(4x + 3) = 0$	Sum = 7
Either $x + 1 = 0$; $x = -1$	Factor = (4, 3)
Or $4x + 3 = 0$; $x = -\frac{3}{4}$	

(b) $2x^2 + 5x + 3 = 0$

Solution	side work
$2x(x + 1) + 3(x + 1) = 0$	Product = $3 \times 2 = 6$
$(x + 1)(2x + 3) = 0$	Sum = 5
Either $x + 1 = 0$; $x = -1$	Factors 3, 2

Or $2x + 3 = 0$; $x = -\frac{3}{2}$

(c) $x^2 + x - 20 = 0$

Solution	Side work
$x(x - 4) + 5(x - 4) = 0$	Product = -20
$(x - 4)(x + 5) = 0$	Sum = 1
Either $x - 4 = 0$; $x = 4$	Factors (5, -4)
Or $x + 5 = 0$; $x = -5$	

(d) $10x^2 + x - 3 = 0$

Solution	Side work
$5x(2x - 1) + 3(2x - 1) = 0$	product $10 \times -3 = -30$
$(5x + 3)(2x - 1) = 0$	sum = 1
Either $5x + 3 = 0$; $x = -\frac{3}{5}$	Factors (6, -5)
Or $2x - 1 = 0$; $x = \frac{1}{2}$	

(b) Method II: Completing squares approach

The idea is to create a perfect square on one side of the equation:

Given the equation $ax^2 + bx + c = 0$

- Dividing the equation by a and transposing the constant term to the RHS

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

- Marking the LHS a perfect square, add a half the coefficient of x squared on both sides

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

- Factorise the terms on the LHS
- $\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$
- Taking square root on both sides of the equation

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\text{- Solving } x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\text{- } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is the general quadratic equation formula for finding the square root of any quadratic equation. This formula is locally known as **bull dozer formula**

Example 8

Solve the following equations by completing squares

(a) $2x^2 - x - 3 = 0$

Solution

$$2x^2 - x - 3 = 0$$

$$x^2 - \frac{1}{2}x = \frac{3}{2}$$

$$x^2 - \frac{1}{2}x + \left(-\frac{1}{4}\right)^2 = \frac{3}{2} + \left(-\frac{1}{4}\right)^2$$

$$\left(x - \frac{1}{4}\right)^2 = \frac{25}{16}$$

$$x - \frac{1}{4} = \sqrt{\frac{25}{16}} = \pm \frac{5}{4}$$

Either $x = \frac{5}{4} + \frac{1}{4} = \frac{6}{4} = \frac{3}{2}$

Or $x = -\frac{5}{4} + \frac{1}{4} = \frac{-4}{4} = -1$

Hence $x = -1$ and $x = \frac{3}{2}$

(b) $18x^2 + 7x - 1 = 0$

Solution

$$18x^2 + 7x - 1 = 0$$

$$x^2 + \frac{7}{18}x = \frac{1}{18}$$

$$x^2 + \frac{7}{18}x + \left(\frac{7}{36}\right)^2 = \frac{1}{18} + \left(-\frac{7}{36}\right)^2$$

$$\left(x + \frac{7}{36}\right)^2 = \frac{1}{18} + \frac{49}{1296} = \frac{121}{1296}$$

$$x + \frac{7}{36} = \sqrt{\frac{121}{1296}} = \pm \frac{11}{36}$$

Either $x = \frac{11}{36} - \frac{7}{36} = \frac{4}{36} = \frac{1}{9}$

Or $x = -\frac{11}{36} - \frac{7}{36} = -\frac{18}{36} = -\frac{1}{2}$

Hence $x = \frac{1}{9}$ or $x = -\frac{1}{2}$

(c) $3x^2 + 7x + 2 = 0$

Solution

$$3x^2 + 7x + 2 = 0$$

$$x^2 + \frac{7}{3}x = -\frac{2}{3}$$

$$x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 = -\frac{2}{3} + \left(\frac{7}{6}\right)^2$$

$$\left(x + \frac{7}{6}\right)^2 = -\frac{2}{3} + \frac{49}{36} = \frac{25}{36}$$

$$x + \frac{7}{6} = \sqrt{\frac{25}{36}} = \pm \frac{5}{6}$$

Either $x = \frac{5}{6} - \frac{7}{6} = -\frac{2}{6} = -\frac{1}{3}$

Or $x = -\frac{5}{6} - \frac{7}{6} = -\frac{12}{6} = -2$

Hence $x = -2$ and $x = -\frac{1}{3}$

Example 8

Solve the following equations by using the quadratic formula

(a) $7x^2 - 5x - 2 = 0$

Solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{-5^2 - 4 \times 7 \times -2}}{2 \times 7} = \frac{5 \pm 9}{14}$$

Either $x = \frac{14}{14} = 1$ or $x = \frac{-4}{14} = -\frac{2}{7}$

Hence $x = 1$ and $x = -\frac{2}{7}$

(b) $3x^2 - 7x - 6 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{-7^2 - 4 \cdot 3 \cdot x - 6}}{2 \cdot 3} = \frac{7 \pm 11}{6}$$

Either $x = \frac{18}{6} = 3$ or $x = \frac{-4}{6} = -\frac{2}{3}$

Hence $x = 3$ and $x = -\frac{2}{3}$

Revision exercise 2

[Answers are given in square brackets]

- Given that the roots of the equation $x^2 + px + q = 0$ are α and β , find the values of:
 - $\alpha^2 + \beta^2 [p^2 - 2q]$
 - $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} \left[\frac{p^2 - 2q}{q} \right]$
 - $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} \left[\frac{p}{q} (3q - p^2) \right]$
 - $\alpha - \beta \left[\pm \sqrt{p^2 - 4q} \right]$
 - $\alpha^2 - \beta^2 \left[-p \sqrt{(p^2 - 4q)} \right]$
 - $\alpha^3 + \beta^3 [p(3q - p^2)]$
- If the roots of the equation $x^2 + 2x + 3 = 0$ are α and β , form an equation whose roots are $\alpha^2 - \beta$ and $\beta^2 - \alpha$. [$x^2 + 2 = 0$]
- Given that the roots of the equation $x^2 - 2x + 10 = 0$ are α and β , form an equation whose roots are $\frac{1}{(2+\alpha)^2}$ and $\frac{1}{(2+\beta)^2}$ [$324x^2 + 1 = 0$]
- If α and β are roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\frac{\alpha^3 - 1}{\alpha}$ and $\frac{\beta^3 - 1}{\beta}$

$$[qx^2 - (p^2q - 2q^2 - p)x + (q^3 - p^3 + 3pq) + 1 = 0]$$

- The roots of the equation $3x^2 - ax + 6b = 0$ are α and β . Find the condition for one root to be
 - Twice the other [$81b = a^2$]
 - The cube of the other [$a^4 - 648(b - 1)b^2 - 18(4a^2 + 9)b = 0$]
- By Factorization method solve the following quadratic equations
 - $x^2 + 9x + 14$ [$x = -7, x = -2$]
 - $x^2 + 2x - 8 = 0$ [$x = -4, x = 2$]
 - $2x^2 + x - 10 = 0$ [$x = 2, x = \frac{5}{2}$]
 - $6x^2 - 19x + 10$ [$x = \frac{5}{2}, x = \frac{2}{3}$]
- Solve the following equations by completing squares
 - $2x^2 + 5x + 3 = 0$ [$x = -1$ and $x = -\frac{3}{2}$]
 - $x^2 + 9x + 20 = 0$ [$x = -5$ and $x = -4$]
 - $x^2 + x - 20 = 0$ [$x = -5$ and $x = 4$]
 - $x^2 - x - 20 = 0$ [$x = 4$ and $x = 5$]
- Solve the following equations using the quadratic formula
 - $2x^2 + 5x + 3 = 0$ [$-\frac{3}{2}, -1$]
 - $x^2 + 9x + 20$ [-5, -4]
 - $x^2 + x - 20$ [-5, 4]

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Thanks

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