



Dr. Blossa Science

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The Science Foundation College
Uganda East Africa
Senior one to senior six
+256 778 633 682, 753 802709
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Sub math matrices

Matrices

A matrix is a regular arrangement of array of numbers called elements or entries. The plural of matrix is matrices.

Matrices are named by capital letters and the elements by small letters. For example

$$P = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

The order of matrix

The order of a matrix is stated by the number of rows and the number of columns. A matrix having p rows and q columns is called a p x q matrix written as p x q. For example

$$O = \begin{pmatrix} a \\ b \end{pmatrix} \text{ has order } 1 \times 2$$

$$P = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \text{ has order } 2 \times 2$$

$$R = \begin{pmatrix} 1 & 3 & 3 \\ 4 & 5 & 6 \end{pmatrix} \text{ has order } 2 \times 3$$

$$S = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \text{ has order } 3 \times 3$$

Types of matrices

(a) **Row matrix** has only one row e.g.

$$(a \ b \ c), (2 \ 1), \text{ etc.}$$

(b) **Column matrix** has only one column e.g.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \text{ etc.}$$

(c) **Square matrix** has the same number of rows as columns

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}, \text{ etc.}$$

(d) Equal matrices have equal corresponding elements, e.g. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(e) A Zero (or null) matrix has all zero elements e.g. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(f) An identity matrix or unity matrix is a square matrix whose every element leading (principal) diagonal is one and whose every other element is zero. It is denoted by I, e.g.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Operation of matrices

$$\text{Given } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} p & q \\ r & s \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) Addition of matrices

$$A + B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$= \begin{pmatrix} a + p & b + q \\ c + r & d + s \end{pmatrix}$$

$$B + A = \begin{pmatrix} p & q \\ r & s \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} p + b & q + b \\ r + c & s + d \end{pmatrix}$$

Note that $A+B = B + A$

(b) Subtraction of matrices

$$A - B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$= \begin{pmatrix} a - b & b - q \\ c - r & d - s \end{pmatrix}$$

$$B - A = \begin{pmatrix} p & q \\ r & s \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} p - a & q - b \\ r - c & s - d \end{pmatrix}$$

Note $A - B \neq B - A$

(c) Multiplication of matrices

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$= \begin{pmatrix} ap + br & bq + ds \\ cp + dr & cq + ds \end{pmatrix}$$

$$BA = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} pa + qc & pb + qd \\ ra + sc & rb + sd \end{pmatrix}$$

Note that $AB \neq BA$

Hence the commutative property does not apply for matrix

(d) Multiplication of a matrix with and identity

$$AI = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} a \times 1 + b \times 0 & a \times 0 + b \times 1 \\ cx1 + d \times 0 & cx0 + dx1 \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$IA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} 1xa + 0xc & 1xb + 0xd \\ 0xa + 1xb & 0xb + 1xd \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Note that $AI = IA = A$

(e) Determinant of a matrix

$$\text{Given that } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Determinant of A is denoted by $\det(A)$ or $|A| = ad - bc$

(f) Inverse of matrix

$$\text{Given that } A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\text{Adjunct of } A = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Inverse of A is denoted by A^{-1}

$$A^{-1} = \frac{\text{Adjunct of } A}{\det(A)}$$

Example 1

$$\text{Given that } A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find

$$(i) \quad A + B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 + 5 & 2 + 6 \\ 3 + 7 & 4 + 8 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

$$(ii) \quad B + A = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 5 + 1 & 6 + 2 \\ 7 + 3 & 8 + 4 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

Note that **Commutative property**: Changing the order of the numbers being added does not affect the result; apply

$$(iii) \quad A - B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} - \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - 5 & 2 - 6 \\ 3 - 7 & 4 - 8 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & -4 \\ -4 & -4 \end{pmatrix}$$

$$\begin{aligned} \text{(iv)} \quad B - A &= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 5-1 & 6-2 \\ 7-3 & 8-4 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad AB &= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \\ &= \begin{pmatrix} 1x5 + 2x7 & 1x6 + 2x8 \\ 3x5 + 4x7 & 3x6 + 4x8 \end{pmatrix} \\ &= \begin{pmatrix} 5 + 14 & 6 + 16 \\ 15 + 28 & 18 + 32 \end{pmatrix} \\ &= \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(vi)} \quad BA &= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 5x1 + 6x3 & 5x2 + 6x4 \\ 7x1 + 8x3 & 7x2 + 8x4 \end{pmatrix} \\ &= \begin{pmatrix} 5 + 18 & 10 + 24 \\ 7 + 24 & 14 + 32 \end{pmatrix} \\ &= \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix} \end{aligned}$$

Commutative property does not apply.

$$\begin{aligned} \text{(vii)} \quad BI &= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 5x1 + 6x0 & 5x0 + 6x1 \\ 7x1 + 8x0 & 7x0 + 8x1 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(viii)} \quad IB &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \\ &= \begin{pmatrix} 1x5 + 0x7 & 1x6 + 0x8 \\ 0x5 + 1x7 & 0x6 + 1x8 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \end{aligned}$$

From (vii) and (viii), $BI = IB = B$

$$\begin{aligned} \text{(ix)} \quad \text{Det}(A) \\ A &= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ \text{Det}(A) &= 1 \times 4 - (2 \times 3) \\ &= -2 \end{aligned}$$

$$\begin{aligned} \text{(x)} \quad \text{Det}(B) \text{ given } B &= \begin{pmatrix} 2 & 2 \\ 1 & 4 \end{pmatrix} \\ \text{Det}(B) &= 2 \times 4 - 1 \times 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{(xi)} \quad 2A \\ 2A &= 2 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \end{aligned}$$

Example 2

Given that $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$, find

$$\text{(i)} \quad A^{-1}$$

$$\text{Adjunct of } A = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$$

$$\text{Det}(A) = 1 \times 4 - 2 \times 3 = -2$$

$$A^{-1} = \frac{\text{Adjunct of } A}{\text{det}(A)}$$

$$= -\frac{1}{2} \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\text{(ii)} \quad B^{-1}$$

$$\text{Adjunct of } B = \begin{pmatrix} 8 & -7 \\ -6 & 5 \end{pmatrix}$$

$$\begin{aligned} \text{Det}(B) &= 5 \times 8 - 6 \times 7 \\ &= -2 \end{aligned}$$

$$B^{-1} = \frac{\text{Adjunct of } B}{\text{det}(B)}$$

$$\begin{aligned} B^{-1} &= -\frac{1}{2} \begin{pmatrix} 8 & -7 \\ -6 & 5 \end{pmatrix} \\ &= \begin{pmatrix} -4 & \frac{7}{2} \\ 3 & -\frac{5}{2} \end{pmatrix} \end{aligned}$$

$$\text{(iii)} \quad (BA)^{-1}$$

$$BA = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$\begin{aligned} &= \begin{pmatrix} 5x1 + 6x3 & 5x2 + 6x4 \\ 7x1 + 8x3 & 7x2 + 8x4 \end{pmatrix} \\ &= \begin{pmatrix} 5 + 18 & 10 + 24 \\ 7 + 24 & 14 + 32 \end{pmatrix} \\ &= \begin{pmatrix} 23 & 34 \\ 31 & 46 \end{pmatrix} \end{aligned}$$

$$\text{Adjunct of } BA = \begin{pmatrix} 46 & -34 \\ -31 & 23 \end{pmatrix}$$

$$\text{Det}(BA) = 23 \times 46 - 34 \times 31 = 4$$

$$(BA)^{-1} = \frac{1}{4} \begin{pmatrix} 46 & -34 \\ -31 & 23 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{23}{2} & -\frac{17}{2} \\ -\frac{31}{4} & \frac{23}{4} \end{pmatrix}$$

$$= \begin{pmatrix} 9 & -11 \\ -2 & -2 \end{pmatrix}$$

Comment on your result (05marks)

$AB \neq BA$; hence the commutative property does not apply for matrix

Example 3

Given the matrices

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \text{ find } (ABC)^{-1}$$

Solution

$$\begin{aligned} ABC &= \begin{pmatrix} 1 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 \times 2 + 3 \times 5 & 1 \times 3 + 3 \times 1 \\ 4 \times 2 + 2 \times 5 & 4 \times 3 + 2 \times 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 17 & 6 \\ 18 & 14 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 17 + 24 & 51 + 12 \\ 18 + 56 & 54 + 28 \end{pmatrix} \\ &= \begin{pmatrix} 41 & 63 \\ 74 & 82 \end{pmatrix} \end{aligned}$$

$$\text{Det}(ABC) = 82 \times 41 - 74 \times 63 = -1300$$

$$\text{Adjunct of } (ABC) = \begin{pmatrix} 82 & -63 \\ -74 & 41 \end{pmatrix}$$

$$\begin{aligned} (ABC)^{-1} &= -\frac{1}{1300} \begin{pmatrix} 82 & -63 \\ -74 & 41 \end{pmatrix} \\ &= \begin{pmatrix} \frac{-82}{1300} & \frac{63}{1300} \\ \frac{74}{1300} & \frac{-41}{1300} \end{pmatrix} \end{aligned}$$

Example 4

$$\text{Given that } A = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}.$$

Find

$$\begin{aligned} \text{(i) } AB &= \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 8 + 0 & 2 + 6 \\ 4 + 0 & 1 - 2 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 8 \\ 4 & -1 \end{pmatrix} \\ \text{(ii) } BA &= \begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 8 + 1 & -12 + 1 \\ 0 - 2 & 0 - 2 \end{pmatrix} \end{aligned}$$

Example 5

The matrix $A = \begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$ and I is a 2×2 Identity matrix.

Determine the matrix B such $A^2 + \frac{1}{2}B = I$ (05marks)

$$\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}^2 + \frac{1}{2}B = I$$

$$\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix} + \frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -9 & -3 \end{pmatrix} + \frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -9 & -3 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 9 & 4 \end{pmatrix}$$

$$B = 2 \begin{pmatrix} 0 & -2 \\ 9 & 4 \end{pmatrix} = \begin{pmatrix} 0 & -4 \\ 18 & 8 \end{pmatrix}$$

$$\text{Hence matrix of } B = \begin{pmatrix} 0 & -4 \\ 18 & 8 \end{pmatrix}$$

Example 6

Given the matrices $A = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$ and

$$B = \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix}, \text{ find}$$

(a) matrix C such that $3A - 2C + B = I$, where I is a 2×2 identity matrix. (03marks)

$$3 \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} - 2C + \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2C = \begin{pmatrix} 9 & 15 \\ -6 & 12 \end{pmatrix} + \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 16 & 12 \\ -10 & 18 \end{pmatrix}$$

$$C = \frac{1}{2} \begin{pmatrix} 16 & 12 \\ -10 & 18 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -5 & 9 \end{pmatrix}$$

(b) the determinant of C. (02 marks)

$$\text{Det}(C) = 8 \times 9 - (-5 \times 6) = 72 + 30 = 102$$

Example 6

The determinant of the matrix

$$P = \begin{pmatrix} 3a & -1 \\ 7 & a \end{pmatrix} = 55$$

(a) Find the value of a. (03 marks)

$$[P] = 3a \times a - (7 \times -1) = 55$$

$$3a^2 + 7 = 55$$

$$3a^2 = 48$$

$$a^2 = 16$$

$$a = 4$$

Using one of the value of a, determine the inverse of P.

$$P = \begin{pmatrix} 12 & -1 \\ 7 & 4 \end{pmatrix}$$

$$\text{Adjunct of } P = \begin{pmatrix} 4 & 1 \\ -7 & 12 \end{pmatrix}$$

$$[P] = 48 - (7 \times -1) = 55$$

$$\text{Inverse of } P^{-1} = \frac{1}{55} \begin{pmatrix} 4 & 1 \\ -7 & 12 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{4}{55} & \frac{1}{55} \\ -\frac{7}{55} & \frac{12}{55} \end{pmatrix}$$

Exercise

(Answers are given in square brackets at the end of each question)

1. Given that $A = \begin{pmatrix} 0 & 1 \\ 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

(a) $N^2 \left[\begin{pmatrix} 7 & 4 \\ 12 & 7 \end{pmatrix} \right]$

(b) $MN \left[\begin{pmatrix} 3 & 2 \\ 6 & 3 \end{pmatrix} \right]$

(c) The value of scalar k if $N^2 + kN MN$ [-2]

2. Given that $P = \begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$, $Q = \begin{pmatrix} 9 & 9 \\ 1 & 0 \end{pmatrix}$ and

$$R = \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix}$$

Find $(PQR)^{-1} \left[\frac{1}{360} \begin{pmatrix} 144 & -36 \\ -24 & 64 \end{pmatrix} \right]$

3. If $\begin{pmatrix} 4 & 1 \\ x & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, determine x and y
[x = -6 or 2 and y = 28 or -4]

4. Given $A = \begin{pmatrix} 1 & 3 \\ 4 & 11 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 1 & 3 \end{pmatrix}$

Find

(i) $AB \left[\begin{pmatrix} 2 & 11 \\ 7 & 41 \end{pmatrix} \right]$

(ii) $(AB)^{-1} \left[\begin{pmatrix} 41 & -11 \\ 5 & 5 \\ -7 & 2 \\ 5 & 5 \end{pmatrix} \right]$

5. If matrix P and Q are such that

$$P = \begin{pmatrix} 2 & 1 \\ 2 & 3 \end{pmatrix} \text{ and } PQ = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$$

Find $Q \left[\begin{pmatrix} 9 & -3 \\ -6 & 6 \end{pmatrix} \right]$

6. Given the matrix $P = \begin{pmatrix} 4.5 & 1 \\ 0 & 7 \end{pmatrix}$ and

$$Q = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$$

Find matrix R such that $3R - 1I = 2A - B$,

where I is an identity of order 2. $\left[\begin{pmatrix} 3 & 3 \\ -1 & 5 \end{pmatrix} \right]$

7. Given that $P = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$, find

$$PQ - QP. \left[\begin{pmatrix} -4 & 0 \\ 0 & 4 \end{pmatrix} \right]$$

8. If $\begin{pmatrix} 2 & 4 \\ -3 & 3 \end{pmatrix} + k \begin{pmatrix} 3 & 1 \\ 0 & n \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -3 & -1 \end{pmatrix}$, find the values of k and n. [k=2, n = -4]

Using matrices to solve simultaneous equation

Example 7

Using the matrix method, solve the simultaneous equations.

$$3x - y = 16$$

$$x + 2y = 3 \quad (05 \text{ marks})$$

Solution

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 16 \\ 3 \end{pmatrix}$$

$$\text{Let } A = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$$

$$\text{Adjunct } A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

$$\text{Det}(A) = (3 \times 2) - (-1 \times 1) = 6 + 1 = 7$$

$$A^{-1} = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

Pre-multiply both sides with the inverse

$$\frac{1}{7} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 16 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 32 + 3 \\ -16 + 9 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 35 \\ -7 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$$

Hence $x = 5$ and $y = -1$

Example 8

Using the matric method, solve the simultaneous equations.

Example 9

A family bought the following items for three successive days. The first day it bought three bunches of matooke, two kilogram of rice, five kilograms meat and two kilogram of sugar. The second day it bought one kilogram of sugar. The third the family bought a bunch of matooke and two kilogram of rice. A bunch of matooke costs shs. 15,000. A kilogram of rice, meat and sugar cost 3,300, shs 8,000 and shs 3,000 respectively.

- (i) represent the family's requirements in a 3 x 4 matrix

Purchase

$$\begin{matrix} & \text{D} & \text{B} & \text{R} & \text{M} & \text{S} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} & = & \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} \end{matrix}$$

$$2x - 3y = 12$$

$$x + 2y = -1$$

Forming matrices

$$\begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

$$\text{Let } P = \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix}$$

$$\text{Det}(P) = (2 \times 2) - (1 \times -3) = 7$$

$$\text{Adjunct of } P = \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

$$P^{-1} = \frac{1}{7} \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix}$$

Pre-multiply both sides with the inverse

$$\frac{1}{7} \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 2 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

Hence $x = 3$, $y = -2$

(ii) write down the cost of each item as column matrix

Item costs

$$\begin{matrix} B \\ R \\ M \\ S \end{matrix} \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix} = \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

(iii) Use the matrices in b(i) and b(ii) to find the family's total expenditure for the three days.
(10 marks)

$$\begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

$$= 3 \times 15000 + 2 \times 3300 + 5 \times 8000 + 2 \times 3000 + 1 \times 3000 + 1 \times 15000 + 2 \times 3300$$

$$= 45000 + 6600 + 40,000 + 6000 + 3000 + 15000 + 6600$$

$$= 122,200$$

Example 10

Given that $M = \begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix}$, $N = \begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$, $K = \begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix}$ and $K = M + N$,

find the value of x and y. (07marks)

Solution

$$\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix} = \begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$$

$$y = 4x - 1 \dots\dots\dots(i)$$

$$12 = -2x + 3y \dots\dots\dots(ii)$$

Substitution of (i) in (ii)

$$12 = -2x + 3(4x - 1)$$

$$12 = -2x + 12x - 3$$

$$10x = 15$$

$$x = 1.5$$

Using equation (i)

$$y = 4 \times 1.5 - 1 = 5$$

Therefore $x = 1.5$ and $y = 5$

(b) In a football tournament, three teams Arsenal, Chelsea and Liverpool had the following results

- Arsenal won two matches, drew once and lost one match
- Chelsea won two matches and lost two matches
- Liverpool won 1 match, drew twice and lost one match.

The teams are awarded 3 points of a win, 1 point for a draw and no point for a loss.

(i) Write a 3×3 matrix for the results and a column matrix for points. (04marks)

$$\begin{matrix} & \begin{matrix} W & D & L \end{matrix} \\ \begin{matrix} A \\ C \\ L \end{matrix} & \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \end{matrix} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \text{ and } \begin{matrix} W \\ D \\ L \end{matrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

(ii) By matrix multiplication, determine the winner of the tournament. (04 marks)

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \times 2 + 1 \times 1 + 1 \times 0 \\ 2 \times 2 + 0 \times 1 + 2 \times 0 \\ 1 \times 2 + 2 \times 1 + 1 \times 0 \end{pmatrix} = \begin{pmatrix} 6 + 1 + 0 \\ 6 + 0 + 0 \\ 3 + 2 + 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$$

Arsenal won with 7 points

Revision Exercise 2

(Answers are given in the square, [] brackets at the end of each question)

1. Use matrix method to find the values of x and y

(a) $2x - 3y = 12$

$$x + 2y + 1 = 0 \quad [x = 3, y = -2]$$

(b) $2y - 4x + 2 = 0$

$$3x - 2y = 5 \quad [x = -3, y = -7]$$

(c) $2x - 3y + 5 = 0$

$$x + 2y - 8 = 0 \quad [x = 2, y = 3]$$

(d) $4x + 5y = 13$

$$3x - 2y = 4 \quad [x = 2, y = 1]$$

(e) $4y - 3x = 9$

$$x - 2y = -5 \quad [x = 1, y = 3]$$

(f) $3x + 2y = 8$

$$3y + 4x = 11 \quad [x = 2, y = 1]$$

2. Given that matrix $A = \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix}$ find a matrix B such that $AB = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$. Hence or otherwise find the inverse of matrix A $\left[B = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}, A^{-1} = \begin{pmatrix} \frac{5}{7} & \frac{2}{7} \\ \frac{4}{7} & \frac{3}{7} \end{pmatrix} \right]$
3. Given that matrix, $P = \begin{pmatrix} 1 & 3 & 7 \\ -1 & 3 & 2 \end{pmatrix}$ and $Q = \begin{pmatrix} 1 & -1 \\ 0 & 3 \\ 5 & 3 \end{pmatrix}$, find matrix R such that $R = PQ$. $\left[\begin{pmatrix} 8 & 14 \\ 9 & 14 \end{pmatrix} \right]$

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Thanks

Dr. Bbosa Science