Surds

These are irrational numbers which cannot be expressed in terms of $\frac{a}{b}$ where a and b are rational. Irrational numbers may be defined as square roots of prime numbers.

Examples are $\sqrt{2}, \sqrt{3}, \sqrt{5}$ ….

Expression of root of numbers in surd form

Example 1

Write the following as the simplest surds

(i) $\sqrt{32}$ $(ii)\sqrt{50}$ $(iii)\sqrt{8}$ $(iv)\sqrt{27}$

Solution

(i) $\sqrt{32} = \sqrt{16\,x\,2} = \sqrt{16}\,x\,\sqrt{2} = 4\sqrt{2}$
(ii) $\sqrt{50} = \sqrt{25\,x\,2} = \sqrt{25}\,x\,\sqrt{2} = 5\sqrt{2}$
(iii) $\sqrt{8} = \sqrt{4\,x\,2} = \sqrt{4}\,x\,\sqrt{2} = 2\sqrt{2}$
(iv) $\sqrt{27} = \sqrt{9\,x\,3} = \sqrt{9}\,x\,\sqrt{3} = 3\sqrt{3}$

Addition and subtraction of surds

This is done by expressing the surds in their simplest form

Example 2

(i) $\sqrt{75} - 3\sqrt{27} + 2\sqrt{12}$

$= \sqrt{25\,x\,3} - 3\sqrt{9\,x\,3} + 2\sqrt{4\,x\,3}$

$= \sqrt{25}\,x\,\sqrt{3} - 3x\sqrt{9}x\,\sqrt{3} + 2\,x\,\sqrt{4}x\sqrt{3}$

$= 5\,x\,\sqrt{3} - 3\,x\,3\,x\,\sqrt{3} + 2\,x\,2\,x\sqrt{3}$

$(5 - 9 + 4)\sqrt{3} = 0$

(ii) $\sqrt{50} + \sqrt{2} - 3\sqrt{18} + 2\sqrt{8}$
$= \sqrt{25\,x\,2} + \sqrt{2} - 3\sqrt{9x\,2} + 2\sqrt{4\,x\,2}$
$=5\sqrt{2} + \sqrt{2} - 3\,x\,3\sqrt{2} + 2x\,2\sqrt{2}$
$= (5 + 1 - 9 + 4)\sqrt{2} = \sqrt{2}$

Multiplication of surds

Finding the product of two surd numbers is the same as finding the root of the product of two numbers.

i.e. $\sqrt{a}\,x\,\sqrt{b} = \sqrt{ab}$

Example 3

Find the value of the following and give your answers in the simplest form

(i) $\sqrt{2}\,x\,\sqrt{2}$
(ii) $\sqrt{2}\,x\,\sqrt{20}$
(iii) $\left(3\sqrt{2} - 2\sqrt{3}\right)^3$

Solution

(i) $\sqrt{2}\,x\,\sqrt{2} = \sqrt{2x2} = \sqrt{4} = 2$
(ii) $\sqrt{2}\,x\,\sqrt{30} = \sqrt{2x30} = 2\sqrt{15}$
(iii) $\left(3\sqrt{2} - 2\sqrt{3}\right)^3$

Using Pascal's triangle; the coefficients of the terms in the expansion $(a + b)^3$ are 1 3 3 1

$\left(3\sqrt{2} - 2\sqrt{3}\right)^3 =$

$(3\sqrt{2})^3 + 3(3\sqrt{2})^2(-2\sqrt{3}) + 3(2\sqrt{3})(-2\sqrt{3})^2 + (-2\sqrt{3})^3$

$=(54\sqrt{2}) + 3(18)(-2\sqrt{3}) + 3(12)(3\sqrt{2}) - 24\sqrt{3}$

$= 162\sqrt{2} - 132\sqrt{3}$

Example 3

Express $\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}}$ in form $b\sqrt{c}$ where b and c are integers. (05 marks)

$\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}} = \frac{4(\sqrt{3}-\sqrt{2})+4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{8\sqrt{3}}{1}$

Hence b = 8 and c= 3

Example 4

Show that $\sqrt{\frac{25^3+5^6}{5^7-5^6}} = \frac{\sqrt{2}}{2}$

$$\sqrt{\frac{25^3+5^6}{5^7-5^6}} = \sqrt{\frac{(5^2)^3+5^6}{5^7-5^6}}$$

$$= \sqrt{\frac{5^6+5^6}{5x5^6-5^6}}$$

$$= \sqrt{\frac{2x5^6}{4\,x5^6}}$$

$$= \sqrt{\frac{2}{4}}$$

$$= \frac{\sqrt{2}}{2}$$

Example 5

Without using a calculator, evaluate $\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\,x\,\sqrt{20}}$.

Solution

$$\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\,x\sqrt{20}} = \frac{6\sqrt{10}+2\sqrt{4x10}}{\sqrt{2}\,x\sqrt{2x10}}$$

$$= \frac{6\sqrt{10}+2\sqrt{4}\,x\sqrt{10}}{\sqrt{2}\,x\sqrt{2}x\sqrt{10}}$$

$$= \frac{6\sqrt{10}+4\,x\sqrt{10}}{2x\sqrt{10}}$$

$$= \frac{10\sqrt{10}}{2x\sqrt{10}}$$

$$= 5$$

Division of surds

There are two types of division of surds;

- A fraction whose denominators has a single term such as $\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{3}}$, etc.
- A fraction whose denominator has double terms, such as $\frac{1}{\sqrt{2}+\sqrt{3}}, \frac{2+\sqrt{3}}{2-\sqrt{3}}, \frac{1}{1+\sqrt{5}}$, etc.
- In both cases, first eliminate the surds from the denominator. The process of eliminating surds from the denominator is called rationalization.
- In the first case rationalize the fraction by multiplying the numerator and denominator by the surd term of the denominator.

Example 5

Rationalize the following

(i) $\frac{2}{3\sqrt{5}}$

(ii) $\frac{1}{\sqrt{2}}$

(iii) $\frac{5}{\sqrt{3}}$

Solution

(i) $\frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{3\sqrt{5}\,x\,\sqrt{5}} = \frac{2\sqrt{5}}{3\,x\,5} = \frac{2\sqrt{5}}{15}$

(ii) $\frac{1}{\sqrt{2}} = \frac{1\,x\sqrt{2}}{\sqrt{2}\,x\,\sqrt{2}} = \frac{\sqrt{2}}{2}$

(iii) $\frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{\sqrt{3}\,x\,\sqrt{3}} = \frac{5\sqrt{3}}{3}$

In the second case rationalize by multiplying the numerator and denominator by the conjugate of the denominator.

Note

(i) The conjugate a + $\sqrt{b}$ is a - $\sqrt{b}$ and that a - $\sqrt{b}$ is a + $\sqrt{b}$

(ii) The product of a surd function and its conjugate is equal to the difference of two squares. i.e. (a + $\sqrt{b}$)(a - $\sqrt{b}$) = $\left(a^2 - \left(\sqrt{b}\right)^2\right)$

Example 7

Rationalize the following

(i) $\frac{2}{2-\sqrt{2}}$

(ii) $\frac{1}{3-\sqrt{5}}$

(iii) $\frac{3-\sqrt{5}}{\sqrt{5-3}}$

solution

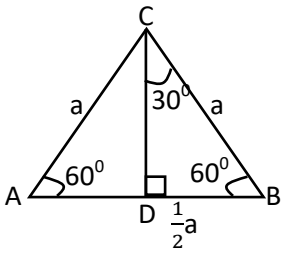
(i) $\frac{2}{2-\sqrt{2}} = \frac{2(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{2(2+\sqrt{2})}{4-2} = (2+\sqrt{2})$

(ii) $\frac{1}{3-\sqrt{5}} = \frac{1(3+\sqrt{5})}{(3-\sqrt{5})((3+\sqrt{5}))} = \frac{(3+\sqrt{5})}{9-5} = \frac{(3+\sqrt{5})}{4}$

(iv) $\frac{3-\sqrt{5}}{\sqrt{5-3}}\,x\,\frac{\sqrt{5+3}}{\sqrt{5+3}} = \frac{(3)^2-(\sqrt{5})^2}{(\sqrt{5})^2-(3)^2} = \frac{9-5}{5-9} = -1$

Set square angle (30^0, 45^0 and 60^0)

(a) Consider an equilateral triangle ABC of each side = a units



$ED^2 = a^2 - \left(\frac{1}{2}a\right)^2 = \frac{3a^2}{4}$

EC=$\frac{a\sqrt{3}}{2}$

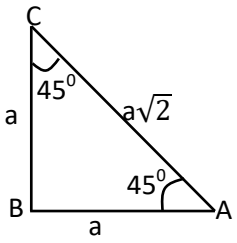
cos 60^0 = $\frac{a}{2a} = \frac{1}{2}$; sin$60^0$ =$\frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$

tan 60^0= $\frac{\sqrt{3}}{2}\ x\frac{2}{1} = \sqrt{3}$

cos 30^0 = $\frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$; sin$60^0$ =$\frac{a}{2a} = \frac{1}{2}$

tan 60^0= $\frac{1}{2}\ x\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

(b) Given a right angled isosceles triangle ABC with equal perpendicular sides each of length a units



$AC^2 = a^2 + a^2 = 2a^2$

$AC = a\sqrt{2}$

cos 45^0 = sin45^0 = $\frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

tan 45^0 = $\frac{a}{a} = 1$

Example 8

Express without a surd in the denominator each of the following

(a) $\frac{1+\tan 30^0}{1-\tan 30^0}$

(b) $\left(\frac{1+\cos 45^0}{2-\sin 60^0}\right)^2$

Solution

(a) $\frac{1+\tan 30^0}{1-\tan 30^0} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}} = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)\left(\frac{\sqrt{3}+1}{\sqrt{3}+1}\right)$

$= \frac{3+2\sqrt{3}+1}{3-1} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$

(b) $\left(\frac{1+\cos 45^0}{2-\sin 60^0}\right)^2 = \left(\frac{1+\frac{\sqrt{2}}{2}}{2-\frac{\sqrt{3}}{2}}\right)^2 = \left(\frac{2+\sqrt{2}}{4-\sqrt{3}}\right)^2$

$= \frac{4+4\sqrt{2}+2}{16-8\sqrt{3}+3}$

$= \left(\frac{6+4\sqrt{2}}{19-8\sqrt{3}}\right)\left(\frac{19+8\sqrt{3}}{19+8\sqrt{3}}\right)$

$= \frac{114+48\sqrt{3}+76\sqrt{2}+32\sqrt{6}}{169}$

Revision exercise

1. Simplify
   (a) $\sqrt{48}$ $[4\sqrt{3}]$
   (b) $\sqrt{162}$ $[9\sqrt{2}]$
   (c) $\sqrt{28}$ $[2\sqrt{7}]$
   (d) $\sqrt{45}$ $[3\sqrt{5}]$
   (e) $\sqrt{125}$ $[5\sqrt{5}]$
   (f) $\sqrt{147}$ $\left[7\sqrt{3}\right]$
2. Simplify the following suds
   (a) $\sqrt{8}+\sqrt{200}-4\sqrt{18}$ $[2\sqrt{2}]$
   (b) $5\sqrt{20}+2\sqrt{45}+2\sqrt{5}$ $[18\sqrt{5}]$
   (c) $3\sqrt{50}+2\sqrt{32}-2\sqrt{75}+2\sqrt{12}-\sqrt{27}$ $[23\sqrt{2}-7\sqrt{3}]$
3. Given that $4\sqrt{20}+3\sqrt{5}-5\sqrt{125} = x\sqrt{5}$, find the value of x [-14]
4. Find the value of the following simplifying the answer as much as possible
   (a) $\left(5\sqrt{2}-\sqrt{5}\right)\left(3\sqrt{5}-2\sqrt{2}\right)\left[5+13\sqrt{10}\right]$
5. Express each of the following in the form $\frac{a\sqrt{b}}{c}$ where a, b and c are integers
   (a) $\frac{2}{\sqrt{7}}$ $\left[\frac{2\sqrt{7}}{7}\right]$
   (b) $\frac{3}{\sqrt{2}}$ $\left[\frac{3\sqrt{2}}{2}\right]$
   (c) $\frac{14\sqrt{5}}{\sqrt{7}}$ $\left[2\sqrt{35}\right]$

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Thanks

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