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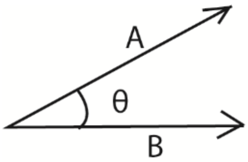
Resultant of forces

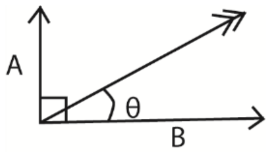
A force is anything which change a body's state of rest or uniform motion in a straight line. Examples are weight, tension, reaction, friction, resistance force.

Resultant of two forces

Consider two forces A and B inclined to each other at an angle θ



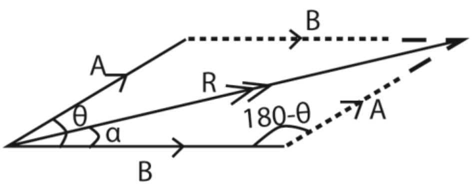
(i) θ is right angle (θ =90^0)



Resultant, R = $\sqrt{A^2 + B^2}$

Direction of resultant, θ =$\tan^{-1}\left(\frac{A}{B}\right)$

(ii) θ is acute ($0^0 \le θ \le 90^0$)

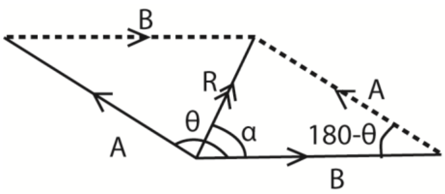


Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

$\alpha = \sin^{-1}\left(\frac{A\sin(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\,(180-\theta)]}$

(iii) θ is obtuse ($90^0 \le θ \le 180^0$)



Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

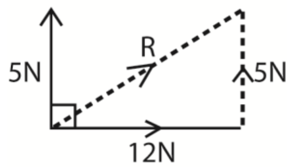
$\alpha = \sin^{-1}\left(\frac{A\sin(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\,(180-\theta)]}$

Example 1

Two forces of magnitude 5N and 12N act on a particle with their direction inclined at 90^0. Find the magnitude and direction of the resultant

Solution



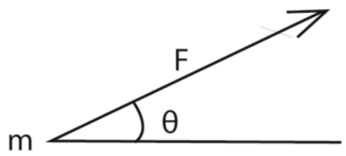
$R = \sqrt{5^2 + 12^2}$ = 13N $\alpha = \tan^{-1}\left(\frac{5}{12}\right) = 22.6^0$

The resultant = 13N at 22.6^0 to 12N force

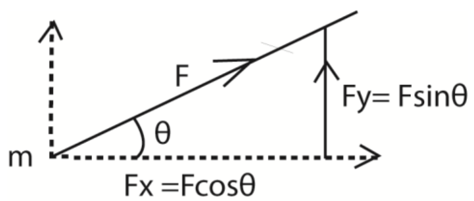
Resolution of forces along vertical and horizontal direction

The component of a vector is the effective value of a vector along a particular direction. The component along any direction is the magnitude of a vector multiplied by the cosine of the angle between its direction and the direction of the component.

Suppose a force F pulls a body of mass m along a truck at an angle θ to the horizontal as shown below



The effective force (Fx) that makes the body move along the horizontal is the component of F along the horizontal



Also

$\cos\theta = \frac{F_x}{F}$;

$F_x = F\cos\theta$

$\sin\theta = \frac{F_y}{F}$;

$F_y = F\sin\theta$

Resultant $F_R = \sqrt{{F_x}^2 + {F_y}^2}$

Direction, $\alpha = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

Note

When a vector is inclined at an angle θ to the horizontal then;

- along the horizontal, the component of the vector is cosθ
- along the vertical, the component of the vector is sinθ

When a vector is inclined at an angle θ to the horizontal then;

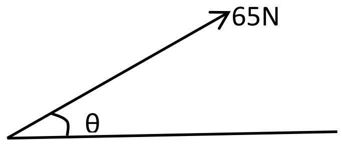
- ❖ along the horizontal, the component of the vector is sinθ
- ❖ along the vertical, the component of the vector is cos θ

Example 2

(a) A force of 65N is inclined at an angle of θ to horizontal. The horizontal component of the force is 25N.

Calculate the

Let the angle be θ



i) angle θ (03marks)

65 cos θ = 25

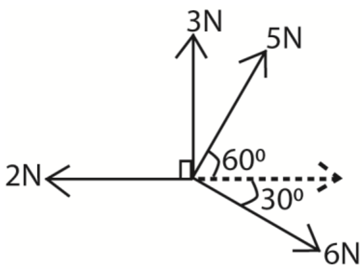
$\theta = \cos^{-1}\frac{25}{65} = 67.38$

ii) vertical component of the force. (02marks)

Vertical component of force = 65sin67.38 = 60N

(b) Forces of magnitude 3N, 2N, 6N and 5N act from a point in the direction 090^0, 180^0, 330^0 and 060^0 respectively

(i) magnitude of the resultant force. (07marks)



Forces acting in x direction = 5cos60 + 0 – 2 + 6cos30

= 2.5 – 2 + 5.2

= 5.7N

Forces acting in y direction = 5sin60 + 3 + 0 - 6sin30

= 4.33 + 3 – 3

= 4.33M

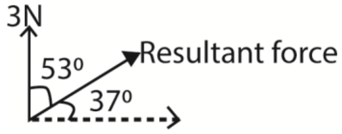
Resultant force = $\sqrt{5.7^2 + 4.33^2}$ = 7.158N

(ii) inclination of the resultant force to the 3N force (02 marks)

Let the angle the resultant makes with horizontal axis = x

$\tan x = \frac{4.33}{5.7}$

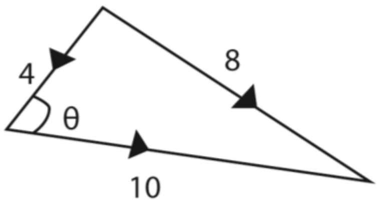
$x = \tan^{-1}\frac{4.33}{5.7} = 37^0$



Hence the angle the resultant makes with 3N (or vertical axis) = 90 – 37= 53^0

Example 3

. (a) Forces P = 10N and Q = 4N act away from point A. The magnitude of their resultant is 8N. Find the angle between P and Q. (05marks)



Using: $b^2 = a^2 + c^2 - 2ac\cos B$ (cosine rule)

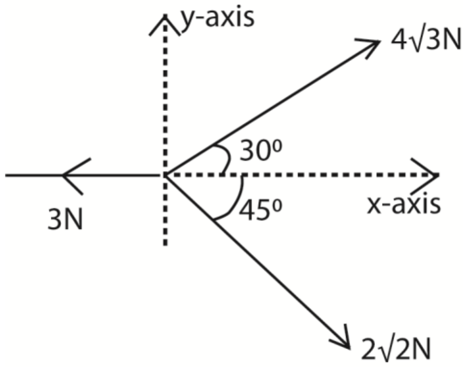
$8^2 = 4^2 + 10^2 – 2 \times 4 \times 10\cos\theta$

$80\cos\theta = 116 – 64 = 52$

$\cos\theta = \frac{52}{80} = 0.56$

$\theta = 49.5^0$

(b) The diagram below shows three forces 3N, $4\sqrt{3}$N and $2\sqrt{2}$N acting on a particle at the origin.



Calculate the,

(i) magnitude of the resultant force

Horizontal component of the forces = $4\sqrt{3}\cos 30 – 3 + 2\sqrt{2}\cos 45 = 5$

Vertical component of the forces $=4\sqrt{3}\sin 30 - 2\sqrt{2}\sin 45 = 4.93$

Resultant force $=\sqrt{5^2 + (4.93)^2} = 7.0$N

(ii) angle the resultant force makes with x-axis. (10 marks)

Let the angle be x

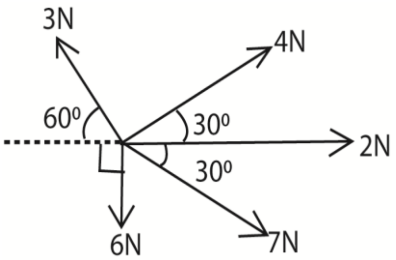
$\tan x = \frac{4.93}{5}$

$x = \tan^{-1}\frac{4.93}{5} = 44.6^0$

Example 4

Find the resultant of the system of forces below

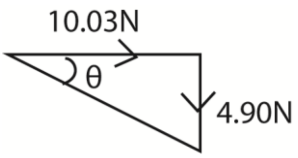
(a)



$$F_R = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}4cos30\\4sin30\end{pmatrix} + \begin{pmatrix}7cos30\\-7sin30\end{pmatrix} + \begin{pmatrix}-3cos60\\3sin60\end{pmatrix} + \begin{pmatrix}0\\-6\end{pmatrix}$$

$$= \begin{pmatrix}10.03\\-4.90\end{pmatrix}$$

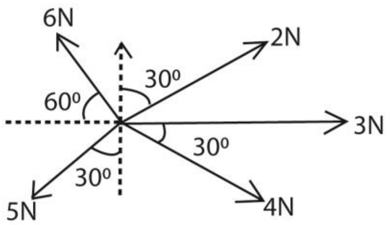
$$F_R = \sqrt{(10.03)^2 + (-4.90)^2} = 11.16\text{N}$$



$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{4.9}{10.03}\right) = 26.04^0$

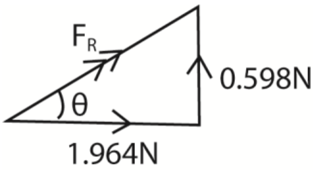
The resultant force is 11.16N at 26.04^0 below the horizontal

(b)



$$F_R = \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}2sin30\\2cos30\end{pmatrix} + \begin{pmatrix}-6\cos 60\\6\sin 60\end{pmatrix} + \begin{pmatrix}-5sin30\\-5cos30\end{pmatrix} + \begin{pmatrix}4cos30\\-4sin30\end{pmatrix} = \begin{pmatrix}1.964\\0.598\end{pmatrix}$$

$$F_R = \sqrt{1.964^2 + 0.598^2} = 2.053\text{N}$$

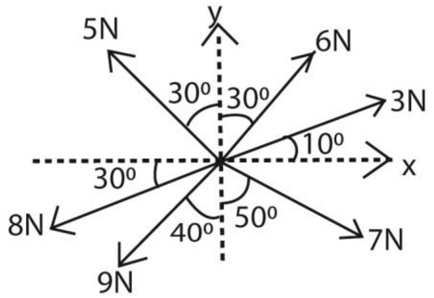


$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{0.598}{1.964}\right) = 16.9^0$

The resultant force is 2.053N at 16.9^0 below the horizontal

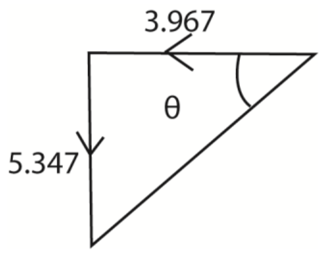
Example 5

Forces of 3N, 6N, 5N, 8N, 9N and 7N act on a particle as shown in the figure below. Find the resultant force.



$F_R = \begin{pmatrix} 3cos10 \\ 3sin10 \end{pmatrix} + \begin{pmatrix} 6sin30 \\ 6cos30 \end{pmatrix} + \begin{pmatrix} -5sin30 \\ 5cos30 \end{pmatrix} + \begin{pmatrix} -8cos30 \\ -8sin30 \end{pmatrix} + \begin{pmatrix} -9sin40 \\ -9cos40 \end{pmatrix} + \begin{pmatrix} 7sin50 \\ -7cos50 \end{pmatrix} = \begin{pmatrix} -3.967 \\ -5.347 \end{pmatrix}$

$F_R = \sqrt{(-3.967)^2 + (-5.347)^2}$ =6.658N

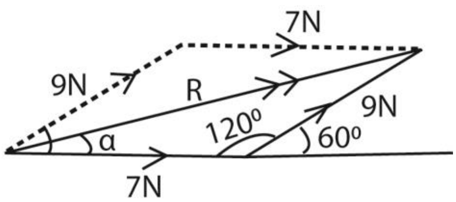


$\theta = \tan^{-1}\left(\frac{5.347}{3.967}\right) = 53.43^0$

Resultant force is 6.658N at 53.43^0 below horizontal

Example 6

Forces of magnitude 7N and 9N act on a particle at an angle of 60^0 between them. Find the magnitude and direction of the resultant.



Resultant, R $=\sqrt{[A^2 + B^2 - 2ABcos\,(180 - \theta)]}$

$=\sqrt{[7^2 + 9^2 - 2x7\;x\;9cos\,(180 - 60)]}$

= 13.89N

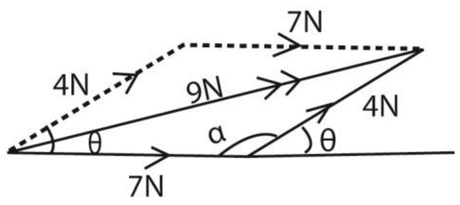
Direction of resultant, $\frac{\sin\alpha}{9} = \frac{\sin(180-\theta)}{13.89}$

$\alpha = \sin^{-1}\left(\frac{9\sin(180-60)}{13.89}\right) = 34.13^0$

Example 7

Find the angle between a force of 7N and 4N their resultant has a magnitude of 9N

Solution



$9^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \times \cos\alpha$

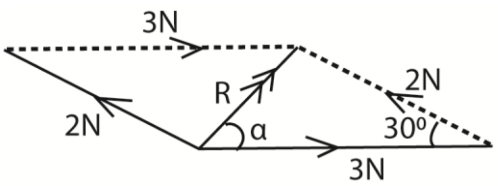
$\alpha = \cos^{-1}\left(-\frac{2}{7}\right) = 106.6^0$

the angle θ between the forces = 180 – 106.6

= 73.4^0

Example 8

Forces of 3N and 2N act on a particle at an angle of 150^0 between them. Find the magnitude and direction of the resultant.



Direction of resultant, $\frac{\sin\alpha}{2} = \frac{\sin(30)}{1.61}$

$\alpha = \sin^{-1}\left(\frac{2\sin(180-60)}{1.61}\right) = 38.3^0$

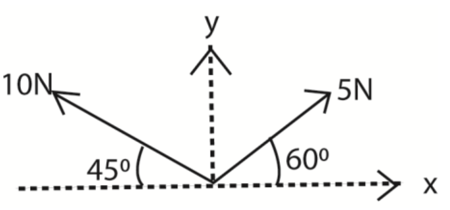
$R^2 = 2^2 + 3^2 - 2 \times 2 \times 2 \times \cos(30)$

R = 1.61N

Revision exercise 1

(Answers are given in brackets at the end of each question)

1. A force of 3N acts at 60^0 to a force of 5N. Find the magnitude and direction of their resultant. [7N at 21.8^0 to the 5N force]
2. A force of 3N at 90^0 to the force of 4N. Find the magnitude and direction of their resultant. [5N at 37^0 to the 4N force]
3. Two coplanar forces act on a point O as shown below



Calculate the magnitude and direction of the resultant force

[12.3N at 68.0 above the horizontal

4. The resultant of two forces pN and 3N is 7N. If the 3N is reversed, the resultant is $\sqrt{17}$N Find the value of p and the angle between the two forces.[$2\sqrt{6}N, 57.02^0$]
5. Forces of 2N, 3N, 5N and 5N act on a particle in the direction 030^0, 090^0, 120^0, 210^0, and 330^0 respectively. Find the magnitude and direction of the resultant force. [1.92N at 7.8^0 to the horizontal]

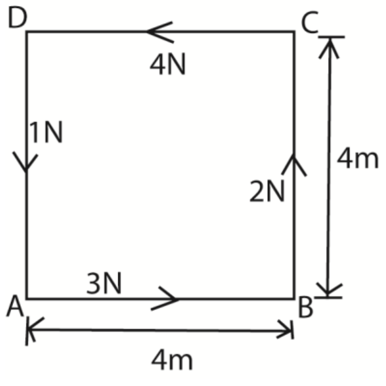
6. Forces of 7N, 2N, 2N and 5N act on a particle in the direction 060^0, 160^0, 200^0, and 315^0 respectively. Find the resultant force. [4.14N at 52.36^0 below the horizontal]
7. Find the magnitude and direction of the resultant of force 10N, 15N and 8N acting in the direction 030^0, 150^0 and 225^0. [12.1N at 55.6^0 below the positive x-axis]
8. Two forces of magnitude 7N and 24N act on a particle with their direction at 90^0. Find the magnitude and direction of the resultant. [25N, 16.26^0 with 24N force]
9. Forces of 5N and 8N act on a particle at an angle of 50^0 between them. Find the magnitude and direction of the resultant. [11.9N at 19^0 with 8N force]
10. Forces of 4N and 6N act on a particle at angle 60^0 between them. Find the magnitude and the direction of the resultant. [5.29N, at 40.9^0 with 6N force]
11. Forces of 9N and 10N act on a particle at angle 40^0 between them. Find the magnitude and the direction of the resultant. [17.9N, at 18.9^0 with 10N force]
12. Forces of 12N and 10N act on a particle at angle 105^0 between them. Find the magnitude and the direction of the resultant. [13.5N, at 45.7^0 with 12N force]
13. Forces of 8N and 3N act on a particle at angle 160^0 between them. Find the magnitude and the direction of the resultant. [5.28N, at 11.2^0 with 8N force]
14. Find the angle between a force of 10N and 4N their resultant has a magnitude of 8N. [130.5^0]
15. The angle between a force αN and a force of 3N is 120^0. If the resultant of the two forces has magnitude 7N, find the value of α. [8N]
16. The angle between a force βN and a force of 8N is 45^0. If the resultant of the two forces has a magnitude 15N, find the value of β. [8.24N]

Magnitude and direction of resultant force of the forces acting along the sides of a polygon

Example 9

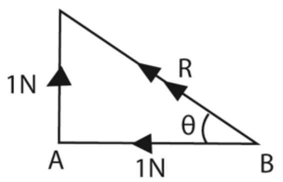
Forces of 3N, 2N, 4N and 1N act along the sides of a square ABCD of side 4m in the direction AB, BC, CD and DA respectively, the direction of force in each case being the order of the letters. Find the magnitude and direction of the resultant force

Solution



$$R = \binom{3}{0} + \binom{0}{2} + \binom{-4}{0} + \binom{0}{-1} = \binom{-1}{1}$$

$$|R| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}N$$



Direction: $\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45^0$ to AB

Revision exercise 2

(Answers are given in brackets at the end of each question)

1. Five forces of magnitude 3N, 4N, 4N, 3N and 5N act along AB, BC, CD, DA and AC respectively of squares of side 1m. The directions of the forces being given in the order of the letters Taking AB and AD as horizontal and vertical respectively. Find the magnitude and direction of the resultant force [$5.1961N$, 60.8^0 to AB]

2. In a rectangle ABCD, AB = 4 and BC = 3m. Forces of magnitude 3N, 10N, 4N, 6N and 5N act in the direction of the letters AB, BC, CD, DA and AC respectively. Taking AB as horizontal, find the magnitude and direction of the resultant force. [7.6158N, 23.2^0 to AB)

3. ABCD is a rectangle with AB = DC = 5m and AD = BC = 6m. Forces of magnitude 8N, 5N, 3N, 6N and 10N act along AB, BC, CD, AD and DB of the rectangle respectively. The directions of the forces being given by the order of the letters. Find the magnitude and the direction of a single force that could replace this system of force. [11.8739N, 16.2^0 to AB]

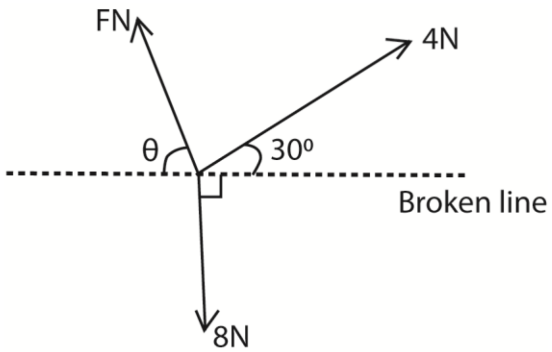
Equilibrium of forces

Several forces acting on a particle are said to be in equilibrium when the resultant force is equal to zero

i.e. $F_R = \binom{0}{0}$

Example 10

The diagram below shows three forces FN, 4N and 8N acting on a particle



If the forces are in equilibrium, find the values of

(i) θ

Both horizontal and vertical component of force = 0

Fcosθ = 4 cos 30 ……………….(i)

Fsinθ = 8 – 4sin30 ……………… (ii)

(ii)÷ (i)

tanθ =$\frac{8-4sin30}{4cos30} = \frac{6}{3.464}$ = 1.732

$\theta = \tan^{-1} 1.732 = 60^0$

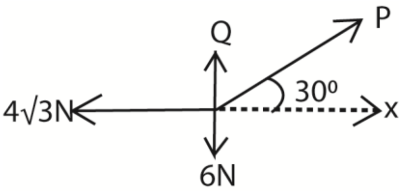
(ii) F (06marks)

From eqn. (i)

F = $\frac{4cos30}{cos60}$ =6.93N

Example 11

In the diagram below, the particle is in equilibrium, find the values of P and Q.



Solution

$\begin{pmatrix}0\\Q\end{pmatrix} + \begin{pmatrix}Pcos30\\Qcos30\end{pmatrix} + \begin{pmatrix}0\\-6\end{pmatrix} + \begin{pmatrix}-4\sqrt{3}\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

0 + Pcos30 + 0 -4$\sqrt{3}$ = 0; => P = 8N

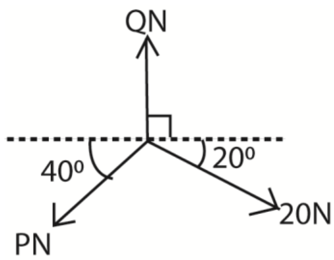
Q + Qcos 30 – 6 + 0 = 0; => Q = 2N

Revision exercise 3

(Answers are given in brackets at the end of each question)

1. Diagram below shows three coplanar forces of magnitude 2N, 3N and PN all acting at point O in the direction shown. Given that the forces are in equilibrium, Find the value of θ and P. [23.413^0, 4.359N]

2.



(i) The diagram above shows three coplanar forces in equilibrium. Find the value of P and Q.

(ii) If the direction of Q s now reversed, find the magnitude and direction of the resultant [(i) 24.5N, 22.6N; (ii) 45.2N]

3. Forces of 6N, 5N, 8N, 5N and 9N act on a particles in the direction N30^{0}E, N30^{0}W, S50^{0}E, N60^{0}W, N80^{0}E and S40^{0}W respectively. Find the additional force that will keep the system of force in equilibrium. [5.358N at 68.920 above the positive axis]

Friction force

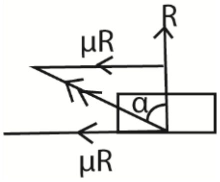
Friction is a force that opposes relative motion or attempted motion between two bodies in contact.

Friction force F = μR where R = normal reaction and μ = coefficient of friction

At limiting equilibrium, the body is at the point of moving (slip or slide) and friction force is maximum.

Angle of friction

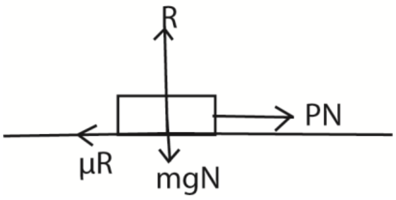
This is the angle between the resultant force and the normal reaction



tanα = $\frac{\mu R}{R} = \mu$

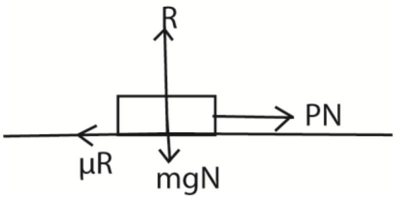
A horizontal plane

(i) at limiting equilibrium (about to slip or slid)



P = μR

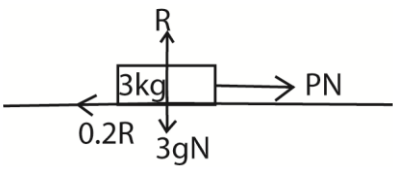
(ii) In motion



P – μR = ma

Example 12

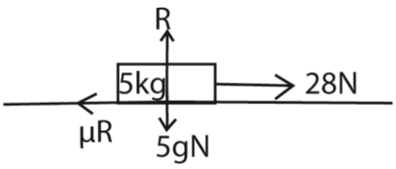
Calculate the maximum frictional force which can act when a block of mass 3kg rests on a rough horizontal surface, the coefficient of friction between the surface being 0.2



F = μR = 0.2 x 3 x 9.8 = 5.88N

Example 13

When a horizontal force of 28N is applied to a body of mass 5kg which is resting on a rough horizontal surface, the body is found to in limiting equilibrium. Find the coefficient of friction between the body and the plane



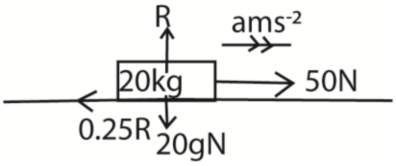
28 = μR

28 = μ x 5 x 9.8

$\mu = 0.57$

Example 3

A block of mass 20kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.25. If a horizontal force of 50N acts on the body, find the acceleration of the body.



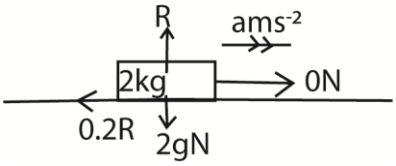
50 - μR = 20a

50 - $(0.25\ x\ 20\ x\ 9.8)$ = 20a

a = 0.05ms^{-2}

Example 4

A block of mass 2kg sliding along a smooth surface at a constant speed of 2ms-1. When the mass encounter a rough surface of coefficient of friction 0.2, it comes to rest. Find the distance the body will move across the rough surface before it comes to rest.



F = ma

0 – μR = 20a

-0.2 x 2 x 9.8 = 20a

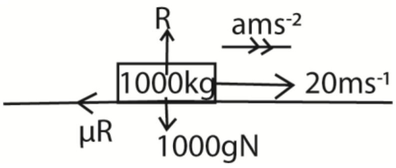
a = -1.96ms^{-2}

s = $\frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2x\ (-1.96)}$ = 1.02m

Example 14

A car of mass 1000kg moving along a straight road with speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 5m. Find the coefficient of kinetic friction between the tyres and the road.

u = $\frac{72\ x\ 1000}{2600}$ = 20ms^{-1}



a = $\frac{v^2 - u^2}{2s} = \frac{0^2 - 20^2}{2x\ 5}$ = -4ms^{-2}

ma = μR

4 x 1000 = 1000 x 9.8 x μ

μ = 0.41

Alternatively

Work done against friction = loss in kinetic energy

μ(mg)x s = $\frac{1}{2}mv^2$

μ x 9.8 x 50 = $\frac{1}{2}x\ 20^2$

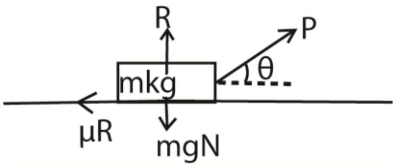
μ = 0.408

Revision exercise 4

(Answers are given in brackets at the end of each question)

1. When a horizontal force of 0.245N is applied to a body of mass 250g which is resting on a rough horizontal plane, the body is found to be in limiting equilibrium. Find the coefficient of friction between the body and the plane. [0.1]
2. A body of mass 40kg is resting on a rough horizontal plane and can just move by a force of 98N acting horizontally. Find the coefficient of friction. [0.25]
3. A block of mass 0.5kg rests on a rough horizontal plane. The coefficient of friction between the block and the table is 0.1. When a horizontal force of 1N acts on the block, find
   (i) friction force experienced by the block. [0.49N]
   (ii) acceleration with which the block will move. [$1.02ms^{-2}$]
4. When a horizontal force of 37N is applied to the body of mass 10kg which is resting on a rough horizontal surface, the body moves along the surface with acceleration $1.25ms^{-2}$. Find the coefficient of friction between the body and the surface. [0.25]

A force inclined at an angle θ to the horizontal



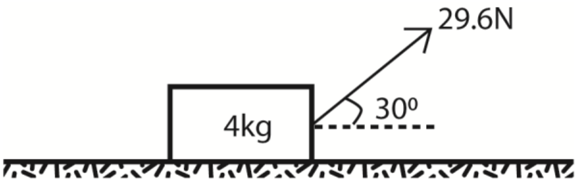
At limiting equilibrium

(→): Pcosθ = μR

(↑): R + Psinθ = mg

Example 15

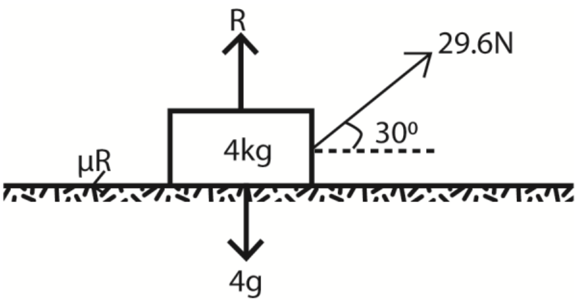
The diagram below shows a block of mass 4kg in limiting equilibrium on a rough horizontal table under the action of a force of 29.6N. The force is inclined at an angle 30^0 to the horizontal.



Calculate the:

(i) normal reaction exerted by the table on the block. (03 marks)
  Normal reaction R = mg = 4 x 9.8 – 29.6sin 30^0 = 24.4N

(ii) coefficient of friction between the block and the table (03marks)



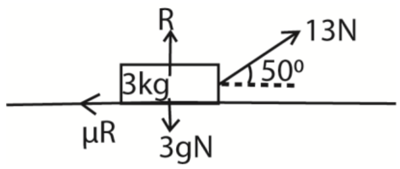
29.6cos30^0 = μR = 24.4μ

$\mu = 1.05$

Example 15

A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 13N inclined at an angle 50^0 to the horizontal, if the particle does not move find the

(i) Normal reaction (ii) coefficient of friction

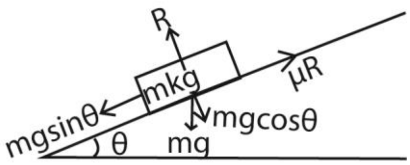


(↑):R = 3 x 9.8 – 13sin 50^0 = 19.4414N

(→): 13cos50^0 = μ x 19.4414

μ = 0.4298

Friction and inclined planes



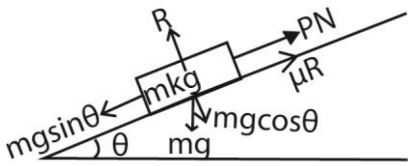
At limiting equilibrium

mgsinθ = μR

mgsinθ = μmgcosθ

μ = tanθ

(ii) A force P applied parallel to and up the plane to just move the particle upwards

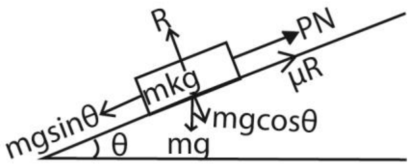


Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ + μR = P

P = mgsinθ + μmgcosθ

(iii) A force P applied parallel to and up the plane so that the particle is on the point of moving downwards (prevent moving downwards)

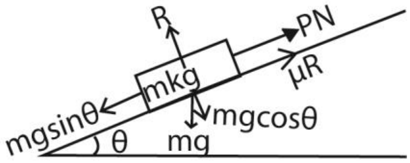


Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ = P+ μR

P = mgsinθ - μmgcosθ

(iv) A force P applied parallel to and up the plane to move the particle upwards



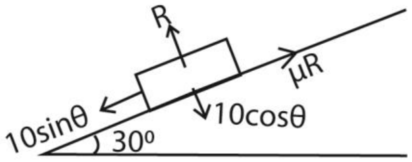
Normal to the plane: mgcosθ = R

Parallel to the plane; P- (mgsinθ + μR) = ma

P – (mgsinθ + μmgcosθ) = ma

Example 16

A particle of weight 10N rests on a rough plane inclined at $30^0$to the horizontal and is just about to slip. Find the value of coefficient of friction between the plane and the particle.



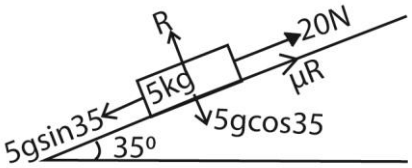
R = 10cos 30 and μR = 10sin30

μ(10cos30) = 10sin30

μ = 0.5774

Example 17

A body of mass 5kg lies on a rough plane which is inclined at 35^0 to the horizontal. When a force of 20N is applied to the body parallel to and up the plane, the body is on the point of moving down the plane. Find the coefficient of friction between the body and the plane.



At limiting equilibrium

R = 5gcos 35

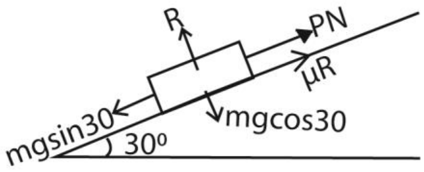
20 +μR = 5gsin35

20 +μ(5gcos35) =5gsin35

μ = 0.2

Example 18

A block of wood of mass 150g rest on an inclined plane. If the coefficient of friction between the surface of contact is 0.3. Find the force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30^0.



At limiting equilibrium

R =0.15 x 9.8cos 30

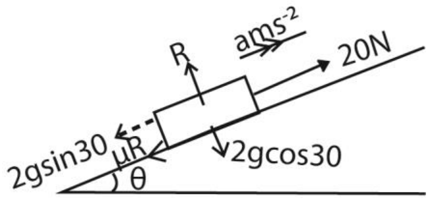
P +μR = 0.15 x 9.8sin30

P +0.3(0.15 x 9.8cos 30) =5gsin30;

P= 0.353N

Example 19

A body of mass 2kg lies on a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$ to the horizontal. A force of 20N is applied to the body, parallel to and up the plane. If the body accelerates up the plane at $1.5ms^{-2}$, find the coefficient of friction between the body and the plane.



Sin θ = $\frac{5}{13}$, cos θ = $\frac{12}{13}$

R = 2 x 9.8 cosθ = 2 x 9.8 x $\frac{12}{13}$ = 18.09N

F = ma

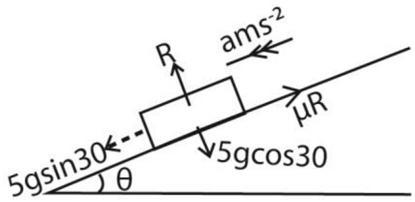
20 – (2gsinθ + μR) = 2a

20 –(2 x 9.8 x $\frac{5}{13}$ + 2 x 9.8 x $\frac{12}{13}$ x μ) = 2 x 1.5

μ = 0.523

Example 20

A body of mass 5kg is released from rest on a rough surface of a plane inclined at 30^0 to the horizontal. If the body takes2.5s to acquire a speed of $4ms^{-2}$ from rest, find the frictional force and coefficient of friction.



v = u + at

4 = 0 + 2.5a

a = $1.6ms^{-2}$

F = ma

5 x 9.8sin30 – μR = 5 x 1.6;

Frictional force, μR = 16.5N

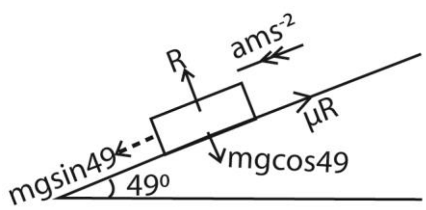
μR = 16.5N

μ = $\frac{16.6}{5 \, x \, 9.8cos30}$ = 0.243

Example 21

A car of mass 500kg moves from rest with engine switched off down a road which is inclined at an angle 49^0 to the horizontal.

(a) calculate the normal reaction
(b) if the coefficient of friction between the tyres and the surface of the road is 0.32. Find the acceleration of the car.



(a) R =mgcos49 = 500 x 9.8cos49 = 3217.97N

(b) F = ma

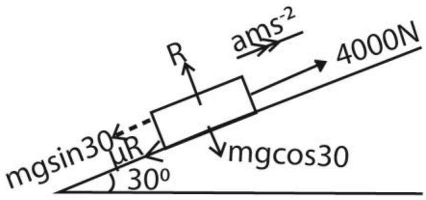
mgsin49 – μR = 500a

500 x 9.8sin49 – 0.32 x 3217.97 = 500a

a = $5.34ms^{-2}$

Example 22

A car of mass 1000kg climbs a plane which is inclined at 30^0 to the horizontal. The speed of the car at the bottom of the incline is $36kmh^{-1}$. If the coefficient of friction between the plane and the car tyres is 0.3 and the engine exerts a force of 4000N, how far up the incline does the car move in 5s



u = $36kh^{-1}$ = $\frac{36\ x\ 1000}{3600}$ = $10ms^{-1}$

F = ma

4000 – (mgsin30 + μR) = ma

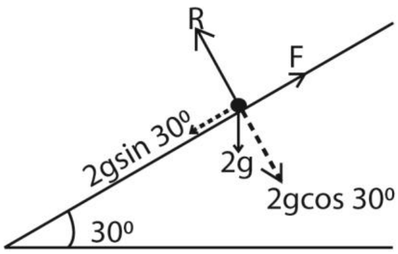
4000 – (1000 x 9.8sin30 + 0.3 x 1000 x 9.8cos30) = 1000a

a = $-3.45ms^{-2}$

s = ut + $\frac{1}{2}at^2$ = 10 x 5 + $\frac{1}{2}$ $x\ (-3.45)x\ 5^2$ = 6.9m

Example 23

A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.



R = 2gcos 30^0
F = 2gsin 3^0
μR = 2gsin30^0
$\mu[2gcos\ 30^0]$ = 2gsin30^0
$\mu = \frac{2gsin\ 30^0}{2gcos\ 30^0}$ = tan 30^0 = 0.57735

Example 24

A car of mass 2000kg ascends on an incline of $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal.

The resistance force to motion of the car is 1000N. The power of the car engine is 59,200W.

Calculate the maximum speed of the car. (05 marks)

Power = (R + mgsinθ)v

59,200 = (1000 + 2000 x 9.8x$\frac{1}{10}$)v

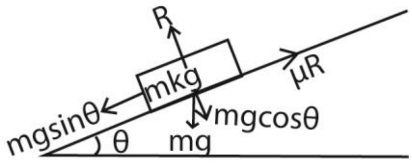
v = $\frac{59200}{2960}$ = $20ms^{-1}$

Example 24

A particle of mass 5kg rests in limiting equilibrium on a rough plane inclines at 20^0 to the horizontal.

Calculate the

(a) Normal reaction



R = mgcosθ = 5 x 9.8 cos20 = 4.7N

(b) Coefficient of friction between the particle and the plane. (02 marks)

mgsin20 = μR

$\mu = \frac{5\ x\ 9.8\ sin20}{5x\ 9.8cos20} = tan20 = 0.364$

Example 25

(a) A brick of mass 750 g is dragged by a horizontal force at a uniform speed along a rough horizontal surface, through a distance of 20m. The work done against friction is 49.8J. Calculate the coefficient of friction between the brick and the surface. (06 marks)

Work = force x distance

Force = $\frac{49.8}{20} = 2.49 =$ μR =μmg

750g = $\frac{750}{1000} = 0.75kg$

μ = $\frac{2.49}{0.75x9.8} = 0.339$

(b) A truck of mass 8 tonnes has a maximum speed of $20ms^{-1}$ up an incline of aresin $\frac{1}{50}$ when the engine is working against resistances of 30,000N. Calculate the maximum power of the engine. (09 marks)

Power = (F + Mgsinθ) v

= (30,000 + 8,000 x 9.8 x $\frac{1}{50}$) x 20

= 631,360W

Example 25

2. A bullet of mass 50g is fired toward a stationary wooden block and enters the block when travelling horizontally with a speed of $500ms^{-1}$. The wooden block provides a constant resistance of 36,000N. Find how far into the block the bullet will penetrate. (05marks)

F = ma

36000 = $\frac{50}{1000}x\ a$

$a = \frac{36000x\ 1000}{50}$ = 720,000ms^{-2}

From $v^2 = u^2 - 2as$, v = 0

s $= \frac{500^2}{2x720,00}$ =0.1736m

Example 26

A car of mass 1200kg has a maximum speed of 180kmh^{-1} on level road when the power of the engine is 50kW. When the car ascends an incline of 1 in 5 with the same engine power, the resulting force is 1648N.

Determine the;

(a) resistance force along the level road (05marks)

at constant speed, the engine power is balanced by resistance force,

Power = force x velocity

180kmh^{-1} = $\frac{180\ x\ 1000}{3600}$ = 50ms^{-1}

Thus 50,000 = R x 50

F = 1000N

(b) maximum speed of the car up the incline. (06marks)

$\frac{\boldsymbol{Power}}{\boldsymbol{V}}$= (R + mgsinθ + ma)

50000= (1000 + 1200 x 9.8 x $\frac{1}{5}$ +1648)V

V = 10ms^{-1}

(c) acceleration of the car up the plane when its speed is 8ms^{-1}. (04 mark)

$\frac{\boldsymbol{Power}}{\boldsymbol{V}}$ = R + mgsinθ + ma

$\frac{50000}{8}$= (1000 + 1200 x 9.8 x $\frac{1}{5}$ +1200a) x 8

6250 = 3,352 + 1200a

1200a = 2,898

a = 2.415ms^{-2}

Work done

W = Fs

Example 27

Find the work done against gravity when a body of mass 5kg is moved through a vertical distance of 2m.

Solution

W = Fs = mgs | W = 5 x 9.8 x 2 = 98J

Example 28

A man building a wall lifts 50 bricks through a vertical distance of 3m. If each brick has a mass 4 kg. how much does the man do against gravity

Solution

W = Fs = mgs | W = 50 x 4 x 9.8 x 3 = 5880J

Example 29

A body of mass 2kg is moved vertically upwards at constant speed of $5ms^{-1}$. Find how much work is done against gravity in each second.

Solution

W = Fs = mgs | W = 2 x 9.8 x 5 x 1 = 98J

Example 30

A horizontal force pulls a body of mass 5kg a distance of 8m across a rough horizontal surface, coefficient of friction 0.25. The body moves with uniform velocity, find the work done against friction.

Solution

W = Fs = mgs | W = 0.25 x 5 x 9.8 x 8 = 98J

Example 31

A man of mass 9kg carries a 50kg bag of cement for a distance of 7m up a slope. The slope is inclined an angle 30^0 to the horizontal.

(a) Find the work done against gravity. (03marks)

w = F x s = 50 x 9.8 x 7 = 3,430J

(b) The man took 42 second to do the work. Calculate the power developed. (02mark)

P = $\frac{W}{t} = \frac{3,430}{42}$ = 81.6W

Revision exercise 5

1. Find the work done against gravity when a body of mass 1kg is raised through a vertical distance of 3m. [29.4J]
2. Find the work done against gravity when a person of mass 80kg climbs a vertical distance of 25m. [19600J]
3. A body of mass 200g is moved vertically upwards at a constant speed of 2ms^{-1}. Find how much work is done against gravity in each second. [3.92J]
4. A body of mass 10kg is pulled a distance of 20m across a horizontal surface against resistance totalling 40N. If the body moves with a uniform velocity, find the work done against the resistance. [800J]
5. A horizontal force pulls a body of mass 3kg a distance of 20m across a rough horizontal surface, coefficient of friction$\frac{2}{7}$. The body moves with a uniform velocity and the only resistance is that due to friction. Find the work done.[168J]
6. A horizontal force drags a body of mass 4kg a distance of 10m across a rough horizontal floor at a constant speed. The work done against friction is 49J. Find the coefficient of friction between the body and the surface. [0.125]
7. A block of mass 15kg rests on a smooth plane inclined at an angle of 30^0 to the horizontal. The block is pulled at a uniform speed a distance of 10m up the line of greatest slope. Find the work done against gravitation force. [735J]

Work – energy theorem

It state that the work done by the **net force** acting on a body is equal to the change in its energy. Consider a body of mass m accelerated from velocity, u by a constant force, F so that in a distance, s it gains velocity, v.

a = $\frac{v^2-u^2}{2s}$

resultant force = ma = $\frac{m(v^2-u^2)}{2s}$

But work done = F x s

W = $\frac{m(v^2-u^2)}{2s} x\ s$ = $\frac{m(v^2-u^2)}{2}$

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ work – energy theorem

Example 32

A constant force pushes a mass of 4kg in a straight line across a smooth horizontal surface. The body passes a point A with speed of 5ms^{-1} and then through a point B with a speed of 8ms^{-1}. B is 6m from A. Find the magnitude of force acting on the mass.

a = $\frac{v^2-u^2}{2s}$ = $\frac{8^2-5^2}{2\ x\ 6s}$ = 3.25ms^{-2}

F = ma = 4 x 3.25 = 13N

Alternatively

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

F x 6 = $\frac{1}{2}\ x\ 4\ (8^2 - 5^2)$

F = 13N

Example 33

A car of mass 1000kg moving at $50ms^{-1}$ skids to rest in 4s under a constant retardation. Calculate the magnitude of the work done by the force of friction

Using v = u + at

0 = 50 + 4a

a = $-12.5ms^{-2}$

Frictional force = ma

= 1000 x -12.5 = 1250N

s = ut + ½ at2

s = 50 x 4 + ½ x -12.5 x 4^2

= 100m

W = F x s = 1250 x 100

= 125 000J

Alternatively

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

W = $\frac{1}{2}\ x\ 1000\ (50^2 - 0^2)$

Work done =125 000J

Example 34

A body of mass 4kg is moving with an initial velocity of $4ms^{-1}$ on a plane. The kinetic energy of the body is reduced by 16J in a distance of 40m. Find the deceleration of the body

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

16 = $\frac{1}{2}\ x\ 4(5^2 -\ v^2)$

v^2 = 17

a = $\frac{v^2-u^2}{2s}$ = $\frac{17-5^2}{2\ x\ 40}$ = $-0.1ms^{-2}$

Example 35

A body of mass 5kg moves in a straight line across a horizontal surface against a constant resistance of magnitude 10N. The body passes through point A and then comes to rest at point B, 9m from A. Find the speed of the body when it is at A.

F = ma

-10 = 5a

a = $-2ms^{-2}$

$v^2 = u^2 + 2a$

$0^2 = u^2$ + 2 x -2 x 9

u = $6ms^{-2}$

Or

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

10 x 9 = $\frac{1}{2}\ x\ 5(u^2 -\ 0^2)$

u = $6ms^{-2}$

Example 36

A body of mass 5kg slides over a rough horizontal surface. in sliding 5m, the speed of the body decreases from 8ms-1 to 6ms-1, find

(a) Frictional force

W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

F x 5 = $\frac{1}{2}\ x\ 5(8^2 -\ 6^2)$

F = 14N

F = μR

μ = $\frac{14}{5\ x\ 9.8}$ = 0.286

Alternatively; $v^2 = u^2 + 2as$

a = $\frac{v^2-u^2}{2s} = \frac{6^2-8^2}{2\ x\ 5}$

= $-2.8ms^{-2}$

F= ma = 5 x 2.8 = 14N

Example 37

A bullet of mass 15g is fired towards a fixed wooden block and enters the block when travelling horizontally at 400ms^{-1}. It comes to rest after penetrating a distance of 25cm. Find the

(i) work done against resistance of the wood
W = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$ = $\frac{1}{2}\ x0.015(400^2 -\ 0^2)$ = 1200J

(ii) magnitude of resistance
W = F x s
1200 = F x 0.25
F = 4800N

Revision exercise 6

1. A carton of mass 0.4kg is thrown across a table with velocity of 25ms-1. The resistance of the table to its motion is 50N. How far will it travel before coming to rest? What must the resistance be if it travels only 2m? [2.5m, 62.5N]
2. A and B are two points 4m apart on a smooth horizontal surface. A body of mass 2kg is initially at rest at A and is pushed by a force of constant magnitude acting in direction from a to B. The body reaches B with speed of 4ms^{-1}. Find the magnitude of force [4N]
3. A and B are two points 3m apart on a smooth horizontal surface. A body of mass 6kg is initially at rest at A and is pushed towards B with a constant force of 9N. Find the speed of the body when it reaches B. [3ms^{-1}]
4. A constant force of magnitude 8N pushes a body of mass 4kg in a straight line across a smooth horizontal surface. The body passes through a point a with speed of 4ms^{-1} and then through appoint B 5m from A. Find the speed of the body at B. [6ms^{-2}]
5. A particle of mass 100g moves in a straight line across a horizontal surface against of constant magnitude. The particle passes through a point A with a speed of 7ms^{-1} and then through B with speed of 3ms^{-1}, B being 2m from A. Find the magnitude of resistance. [1N]
6. A and B are two points 15m apart in the same vertical line, with A above B. A body of mass 5kg is released from rest at A and falls vertically against constant resistance of 25N. Find the speed of the body when it passes B. [12ms^{-1}]
7. A particle of mass 6kg is released from rest and falls freely under gravity. Find its speed when it has fallen a distance of 90m. [42ms-1]
8. A particle of mass 6kg is released from rest and falls freely under gravity. Find the distance it has fallen when its speed is 7ms^{-1}. [2.5m]
9. A body of mass 3kg is projected vertically upwards from a point A with speed 4ms^{-1}. The body passes through a point B 5m below A. Find the speed at B. [10.7ms^{-1}]
10. A particle of mass 2kg falls vertically against a constant resistance of 14N. The particle passes through two points A and B with a speed of 3ms^{-1} and 10ms^{-1} respectively. The distance AB. [16.25m

Power

Power is the rate of doing work

Power = $\frac{work\ done}{time}$

P = $\frac{F\ x\ d}{t}$

P = $F\ x\ \frac{d}{t}$

P = F x v

F = $\frac{P}{v}$

Example 38

What is the average rate at which work must be done in lifting a mass of 100kg a vertical distance of 5m in 7s.

P = $\frac{F\ x\ d}{t}$ = $\frac{100\ x\ 9.8\ x\ 5}{5}$ = 700W

Example 39

What is the rate at which work must be done lifting mass of 500kg at constant speed of 3ms^{-1}?

P = F x v = 500 x 9.8 x 3 = 14700W

Motion of cars

Consider a car being driven along a road, the forward or tractive force F_T moves the car is supplied by the engine working at constant rate of P watts.

Power = $\frac{work\ done}{time}$

P = $\frac{F_T\ x\ d}{t}$

P = $F_T\ x\ \frac{d}{t}$

P = F_T x v

F_T = $\frac{P}{v}$

Example 40

A cyclist travels along a road at constant speed of 8ms^{-1}. If the resistance to motion is 50N, find the rate at which the cyclist is working.

Solution

F =ma at constant speed a = 0ms-2

F – R = m x 0

$\frac{P}{8} - 50 = 0$

P = 400W

Example 41

A car of mass 800kg is driven along a level road against a constant resistance to motion of 200N. With the engine working at a steady rate of 14kW, find

(i) acceleration of the car when its speed is 10ms-1
(ii) maximum speed at which the car can move

Solution

(i) F = ma

$F_T - R = m \times a$

(ii) F = ma at maximum speed a = 0

$F_T - R = m \times a$

$\frac{14000}{v} - 200 = 800 \times 0$

$v = 70ms^{-1}$

$\frac{14000}{10} - 200 = 800a$

a = $1.5ms^{-1}$

Example 42

A car of mass 500kg has an engine of maximum power 2.5kW

(a) Calculate the force resisting the motion of the car when it is travelling at its maximum speed of $72kmh^{-1}$ on a level road.

$72kmh^{-1} = \frac{72 \times 1000}{3600} = 20ms^{-1}$

F = ma

$F_T - R = m \times a$

at maximum speed a = 0

$\frac{2500}{20} - R = 500 \times 0$

R = 125N

(b) If the resistance remains unaltered, find the acceleration of the car when travelling at $36kmh^{-1}$ on a level road with the engine working at the same rate.

$F_T - R = m \times a$

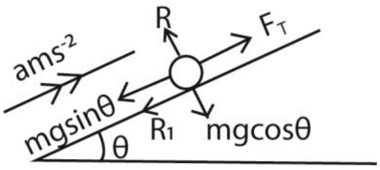
$\frac{2500}{10} - 125 = 500 \times a$

a = $0.25ms^{-2}$

Inclined planes

Example 43

A train of mass 20,000kg moves at constant speed of $72kmh^{-1}$ up a straight inclined plane against a frictional force of 128N. The incline is such that the train rise vertically 1m for every 100m travelled along the incline. Calculate the necessary power developed by the train



$72kmh^{-1} = \frac{72 \times 1000}{3600} = 20ms^{-1}$

$\sin\theta = \frac{1}{100}$

at constant speed, a = 0

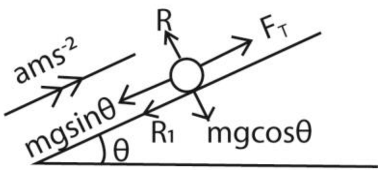
Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = ma$

$\frac{P}{20} - \left(20000 x\ 9.8 \times \frac{1}{100} + 128\right) = 20000 \times 0$

P= 41760W

Example 44

A car of mass 1.5metric tonnes moves at constant speed of 6ms^{-1} up a slope inclined at $\sin^{-1}\frac{1}{7}$. Given that the engine of the car is working at constant rate of 18kW. Find the resistance to the motion.



sinθ = $\frac{1}{7}$

at constant speed, a = 0

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = ma$

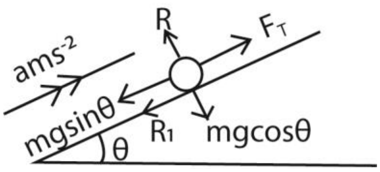
$\frac{P}{6} - \left(1500x\ 9.8\ x\ \frac{1}{7} + R_1\right) = 1500 \text{ x } 0$

R_1 =900N

Example 45

A car of mass 800kg with engine working at constant rate of 15kW climbs a hill of inclination 1 in 98 against a constant resistance to motion of 420N. Find the

(i) the acceleration when the speed is 10ms^{-1}
(ii) maximum speed of the car up the hill



Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = ma$

$\frac{P}{10} - \left(800x\ 9.8\ x\ \frac{1}{98} + 420\right) = 800 \text{ x a}$

a =1.25ms^{-2}

(ii) at constant speed, a = 0

$\frac{15000}{v} - \left(8000x\ 9.8\ x\ \frac{1}{98} + 420\right) = 800 \text{ x } 0$

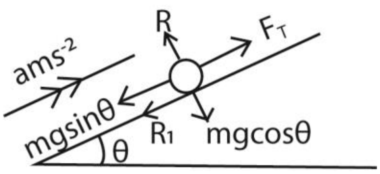
v = 30ms^{-1}

Example 46

A car of mass 1000kg has a maximum speed of 40ms^{-1} on a level road and the engine s working at 32kW against constant resistance

(i) find the resistance to motion
$F_T - R_1 = ma$; at constant speed a = 0
$\frac{32000}{40} - R_1 = 1000\ x\ 0$
$R_1 = 800N$

(ii) Given that the resistance in both cases varies as the speed, find the rate at which the engine must work for the car to ascend a slope of 1 in 98 at constant speed of 20ms^{-1}.



R_1 = kv

k = $\frac{800}{40} = 20$

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = ma$

at constant speed a =0

$\frac{P}{20} - \left(1000x\ 9.8\ x\ \frac{1}{98} + 20\ x\ 20\right) = 1000 \text{ x } 0$

P = 10,000W

Example 47

A car of mass 1000kg has maximum speed of $150kmh^{-1}$ on a level road and working at 60kW

(i) find the coefficient of friction between the car and the road if all resistance is due to friction

$150kmh\text{-}1 = \frac{150000}{3600} = 41.67ms^{-1}$

F = ma

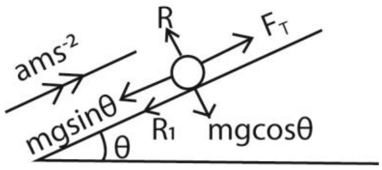
$F_T - \mu R = ma$

At constant speed, a = 0

$$\frac{60000}{41.67} - \mu(1000\ x\ 9.8) = 1000\ x\ 0$$

μ = 0.147

(ii) Given that the tractive force remains unaltered and the non-gravitation resistance in both cases varies as square of the speed, find the greatest slope on which a speed of $120kmh^{-1}$ could be maintained.



$\mu R = kv^2$

$k = \frac{0.147\ x\ 1000\ x\ 9.8}{(41.67)^2} = 0.8297$

$120kmh^{-1} = \frac{120\ x1000}{3600} = 33.33ms^{-1}$

Force, $F_T = \frac{P}{v} - (mgsin\theta + R_1) = ma$

at constant speed a =0

$$\frac{60000}{41.67} - (1000x9.8sin\theta + 0.8297(33.33)^2 = 1000\ x0$$

$\theta = 3.04^0$

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Thanks

Dr. Bbosa Science