Motion in straight line

Distance and displacement

Distance is a length between 2 fixe points

Displacement is the distance covered in a specific direction

Speed and velocity

Speed is the rate of change of distance with time

Velocity is the rate of change of displacement with time

Average speed = $\frac{total\ distance}{total\ time\ taken}$

Average velocity = $\frac{total\ displacement}{total\ time\ taken}$

Example 1

Find the distance travelled in 5s by a body moving with a constant speed of $3.2ms^{-1}$

Solution

Average speed = $\frac{total\ distance}{total\ time\ taken}$ | $3.2 = \frac{total\ distance}{5}$ | distance = 16m

Example 2

John ran 1500m in 3minutes and 33s, find his average speed.

Average speed = $\frac{total\ distance}{total\ time\ taken}$ | speed = $\frac{1500}{(3\ x\ 60+33)} = 7.04ms^{-1}$

Acceleration

It is the rate of change of velocity

Acceleration = $\frac{change\ in\ velocity}{total\ time\ taken}$ | $a = \frac{v-u}{t}$ where v = final velocity, u = initial velocity, t = time

Uniform acceleration

This is the constant rate of change of velocity with time

Equations of uniform acceleration

1st equation

Suppose a body moving in a straight line with uniform acceleration a, increases its velocity from u to v in time t, then from the definition of acceleration

a = $\frac{v-u}{t}$	at = v -u	**v = u + at** ……..1

2nd equation

Suppose an object with velocity u moves with uniform acceleration a time t and attains a velocity v, the distance s travelled by the object is given by: s = average velocity x time

s = $\left(\frac{v+u}{2}\right) t$ but v = u + at	s = $\left(\frac{2ut+at^2}{2}\right)$
s = $\left(\frac{u+u+at}{2}\right) t$	**s = ut +** $\frac{1}{2}$$\boldsymbol{at^2}$ ……2

3rd equation

s = average velocity x time

s = $\left(\frac{v+u}{2}\right) t$ but t = $\frac{v-u}{a}$	s = $\left(\frac{v^2-u^2}{2a}\right)$
s = $\left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right)$ =	$\boldsymbol{v^2 = u^2 - 2as}$ ……3

Example 3

A car is initially at rest at a point O. The car moves from O in a straight line with an acceleration of 4ms^{-2}. find how far the car

(i) is from O after 2s

From **s = ut +** $\frac{1}{2}$$\boldsymbol{at^2}$**;** **s =** 0 x 2 + $\frac{1}{2}x\ 4\ x\ 2^2$ **=** 8m

(ii) is from O after 3s

s = 0 x 2 + $\frac{1}{2}x\ 4\ x\ 2^2$ **= 1**8m

(iii) distance travelled in the third second = 18 – 8 = 10m

Example 4

A body at O moving with a velocity 10ms^{-2} decelerates at 2ms^{-2}.

(a) find the displacement of the body from O after 7s

From s = ut + $\frac{1}{2}at^2$

s = 10 x 7 + $\frac{1}{2}x-2\ x\ 7^2$ = 21m

(b) how far from O does the body come to rest and how long does it take

s = $\left(\frac{v^2-u^2}{2a}\right)$ = $\frac{0^2-10^2}{2\ x-2}$ = 25m

t = $\frac{v-u}{a}$ = $\frac{0-10}{-2}$ = 5s

Example 5

A taxi approaching a stage runs two successive half kilometres in 16s and 20s respectively. Assuming the retardation is uniform, find

(i) Initial speed of the taxi

$s = ut + \frac{1}{2}at^2$

For the first half kilometre or 500m

$500 = 16u + \frac{1}{2}a(16)^2$ …… (i)

for the kilometre or 1000m

$1000 = 36u + \frac{1}{2}a(36)^2$…… (ii)

from eqn. (i) and eqn. (ii)

$a = \frac{25}{72}$ and $u = 34.028 ms^{-1}$

(ii) the further distance, the taxi runs before stopping

$s = \left(\frac{v^2 - u^2}{2a}\right) = s = \left(\frac{0^2 - (34.028)^2}{2\left(\frac{25}{72}\right)}\right) = 1667.3m$

Extra distance = 1667.3 – 1000 = 667.3m

Example 6

An overloaded taxi travelling at constant velocity of 90km/h overtakes a stationary traffic police car. 2s later, the police car sets in pursuit, accelerating at a uniform rate of $6ms^{-2}$. How far does the traffic car travel before catching up with the taxi?

Solution

t_1 = time taken by the taxi

t_2 = time taken by the police car

$t_1 = 2 + t_2$

speed of the taxi in m/s

$90km/h = \frac{90\ x\ 1000}{3600} = 25ms^{-1}$

$s = ut + \frac{1}{2}at^2$

$s_T = 25t_1$

$s_C = 0 \times t_2 + \frac{1}{2}x\ 6\ x\ {t_2}^2 = 3\ {t_2}^2$

For the car to catch taxi; $s_T = s_C$

$25t_1 = 3\ {t_2}^2$

$25(2 + t_2) = 3\ {t_2}^2$

t = 10s or $t = \frac{4}{3}$s

the car leaves 2s later then 10s is the correct time since it gives positive distance

$s_C = 3\ {t_2}^2 = 3\ x\ 10^2 = 300m$

Example 7

A lorry starts from a point A and moves along a straight horizontal road with a constant acceleration of $2ms^{-2}$. At the same time a car moving with a speed of $20ms^{-1}$ and a constant acceleration of $3ms^{-1}$ is 400m behind the point A and moving in the same direction as the lorry. find:

(a) how far from A the car overtakes the lorry.

a car over takes the lorry; both move in the same time, t

$s = ut + \frac{1}{2}at^2$

distance moved by the car = 400 + distance moved by the lorry

$20t + \frac{1}{2}\ x\ 3x\ t^2 = 400 + \frac{1}{2}\ x\ 2\ x\ t^2$

$t^2 + 40t - 800 = 0$; t = 14.64s or t = -54.64s

Hence t = 14.64s

sL = $\frac{1}{2}\ x\ 2\ x\ (14.64)^2$= 214.33m

(b) the speed of the lorry when it is being overtaken

v = u + at

= 0 + 2 x 14.64 = 29.28ms^{-1}

Example 8

The seed of a taxi decreases from 90kmh^{-1} to 18kmh^{-1} in a distance of 120 metres. Find the speed of the taxi when it had covered a distance of 50metres. (05marks)

Given u = 90kmh^{-1}, v = 18kmh^{-1}, s = 120m = 0.12km
Using $v^2 = u^2 + 2as$
$18^2 = 90^2 + 2a(0.12)$
a = -32400kmh^{-2}
When s = 50m = 0.05km, u = 90kmh^{-1}, a = -32400kmh^{-2}
Using $v^2 = u^2 + 2as$
$v^2 = 90^2 - 2 \times 32400 \times 0.05 = 4860$
$v = \sqrt{4860} = 69.71kmh^{-1}$

Example 9

(a) Show that the final velocity v of a body which starts with an initial velocity u and moves with uniform acceleration a consequently covering a distance x, is given by $v = [u^2 + 2ax]^{\frac{1}{2}}$

x = average velocity x time

$x = \left(\frac{v+u}{2}\right) t$ but $t = \frac{v-u}{a}$

$x = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right) = \left(\frac{v^2-u^2}{2a}\right)$

$\boldsymbol{v^2 = u^2 + 2ax}$

$v = [u^2 + 2ax]^{\frac{1}{2}}$

(b) Find the value of x in (a) if v = 300m/s, u = 10m/s and a = 5m/s

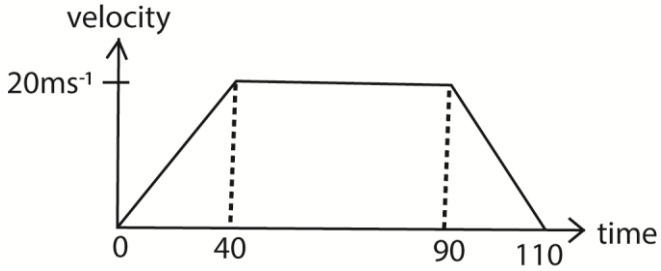
$30 = [10^2 + 2x\ 5x]^{\frac{1}{2}}$
900 = 100 + 10x
x =80m

Velocity-time graphs

Example 10

A car started from rest and attained a velocity of 20m/s in 40s. It then maintained the velocity attained for 50s. After that it was brought to rest by a constant breaking force in 20s.

(i) Draw a velocity-time graph for the motion



(ii) using the graph, find the total distance travelled by the car

Total distance = total area under the graph

$= \frac{1}{2}bh + lw + \frac{1}{2}bh$

$= \frac{1}{2}x\ 40\ x\ 20 + 50x\ 20 + \frac{1}{2}x\ 20\ x\ 20$ = 1600m

Method II (area of a trapezium)

A = $\frac{1}{2}h(a+b) = \frac{1}{2}x\ 20(50+110)$ = 1600m

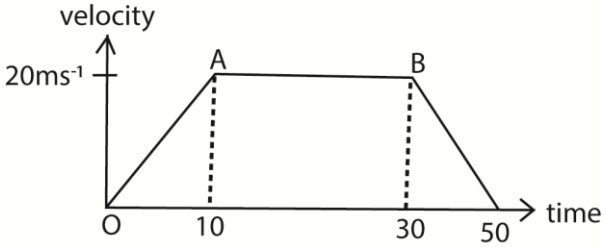
(iii) what is the acceleration of the car?

a= $\frac{v-u}{t} = \frac{20-0}{40} = 0.5ms^{-2}$

Example 11

A car from rest accelerates steadily to 10s up to a velocity f 20ms. It continues with uniform velocity for further 20s and then decelerates so that it stops in 20s.

(a) Draw a velocity-time graph to represent the motion



(b) Calculate

(i) acceleration

a= $\frac{v-u}{t} = \frac{20-0}{10} = 2ms^{-2}$

(ii) deceleration

a= $\frac{v-u}{t} = \frac{0-20}{20} = -1ms^{-2}$

(i) Distance = area under the graph

A = $\frac{1}{2}x\ 10\ x\ 20 + 20x\ 20 + \frac{1}{2}x\ 20\ x\ 20$

= 700m

Method II (area of a trapezium)

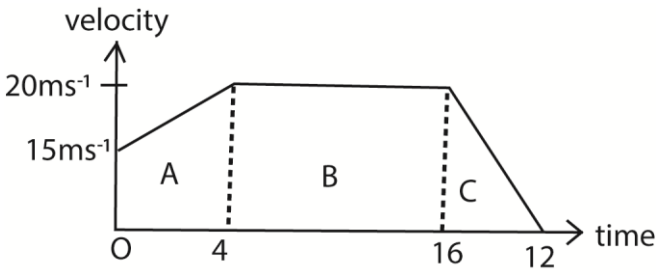
A = $\frac{1}{2}\ x\ 20(50+20)$= 700m

Average speed = $\frac{distance}{time}$

= $\frac{700}{50} = 14m/s$

Example 12

The graph below shows the motion in the body.



(a) Describe the motion of the body
A body with initial velocity of 15m/s accelerates steadily to a velocity of 20m/s in 4s, it then continues with a uniform velocity for 6s and brought to rest in 2s.

(b) Calculate the total distance travelled
Distance = 4 x 15 + $\frac{1}{2}x\ 4\ \ x\ 5 + 20\ x\ 6 + \frac{1}{2}x\ 20\ \ x\ 2$ = 210m

Example 13

A cyclist rider along a straight road from a shop P to shop Q. He passes shop P with a velocity of 3ms^{-1} and accelerates uniformly at 1.25ms^{-2} until he attains a velocity of 12 ms^{-1} at shop Q. Find the

(a) time taken by the cyclist to reach Q. (03marks)

From v = u + at

12 = 3 + 1.25t

t = $\frac{\mathbf{9}}{\mathbf{1.25}}$ = 7.2s

(b) distance PQ. (02 marks)

s = ut + $\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{at^2}$ = 3 x 7.2 + $\frac{\mathbf{1}}{\mathbf{2}}\ \boldsymbol{x\ 1.25\ x\ 7.2^2}$ =54

Example 14

Two points A and B are 800metres apart. A particles moving in a straight line with a constant acceleration passes point A with a velocity of 10m/s. it then passes the point B with a velocity of 40m/s. calculate the time taken by the particle to move from A to B. (05 marks)
Solution
Let a be the acceleration taken
From $v^2 = u^2 + 2as$

a = $\left(\frac{v^2 - u^2}{2s}\right) = \left(\frac{1600 - 100}{2x800}\right) = \frac{1500}{1600} = \frac{15}{16}$

form v = u +at

40 = 10 + $\frac{15}{16}$t

$\frac{15}{16}$t = 30

$t = \frac{30\ x\ 16}{15}$ = 32s

Example 15

A train moving in a straight line passes a point P with velocity of $20ms^{-1}$. It the moves for 5 seconds with acceleration of $2.5ms^{-2}$.

Determine the;

(a) velocity of the train after 5 seconds. (03marks)
Solution
$V = u + at = 20 + 2.5 \times 5 = 32.5ms^{-1}$

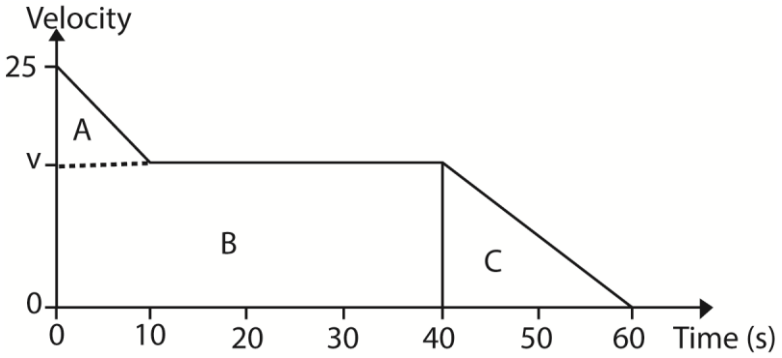
(b) distance of the train from P after 5 seconds. (02 marks)
$S = ut + \frac{1}{2} at^2 = 20 \times 5 + \frac{1}{2} \times 2.5 \times 5^2$
$= 100 + 31.25$
$= 131.25m$

Example 16

A motorist moving at $90ms^{-1}$ decelerates uniformly to a velocity V ms^{-1} in 10 seconds. He maintains his speed for 30 seconds and then decelerates uniformly to rest in 20 seconds

(a) Sketch a velocity – time graph for the motion of the motorist. (06marks)

$90kmh^{-1} = \frac{\mathbf{90x1000}}{\mathbf{3600}} = 25ms^{-1}$



(b) Given that the total distance travelled is 800m, use your graph to calculate the value of V. (05marks)

Total area = A + B + C

$800 = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x10}(\mathbf{25} - \boldsymbol{v}) + \mathbf{40}\boldsymbol{v} + \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{20}\boldsymbol{v})$ = 125 -5v +40v + 10v = 125 +45v

45v = 800-125 = 675

$v = 15ms^{-1}$

(c) Determine the two decelerations. (04 marks)

Let deceleration in region A be a

From v = u –at

15 = 25 – 10a

10a = 10

a = 1ms^{-2}

Deceleration in region C

0 = 14 – 20a

a = $\frac{\mathbf{14}}{\mathbf{20}}$ = 0.7ms^{-2}

Example 17

A car initially at rest accelerated uniformly to a speed of 20ms^{-1} in 16 seconds. The car then travelled at the attained speed for 2 minutes. The car then accelerated uniformly at 2.5ms^{-2} for 8 seconds. It finally decelerated at 2.5ms^{-2} to rest.

(a) Find the

(i) greatest speed attained by the car.

v = u + at = 20 + 2.5 x 8 = 40ms^{-1}

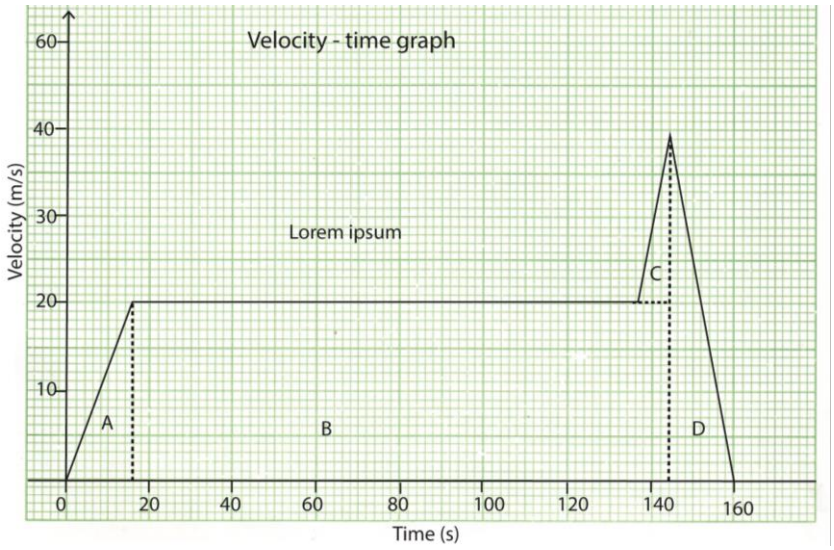
(ii) total time taken by the car to come to rest. (06marks)

Let time taken during deceleration be t

0 = 40 – 2.5 t => t = 16s

Total time for the journey = 16 + 120 + 8 + 16 = 160s

(b) Sketch velocity – time graph for the motion of the car. (04 marks)



(c) Use your graph to find the total distance travelled by the car. (05 marks)

Total distance travelled = sum of areas A+ B +C + D

= $\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x}\mathbf{16}\ \boldsymbol{x}\ \mathbf{20} + \mathbf{128}\ \boldsymbol{x}\ \mathbf{20} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x}\ \mathbf{8}\ \boldsymbol{x}\mathbf{20} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{x}\ \mathbf{16}\ \boldsymbol{x}\mathbf{40}$

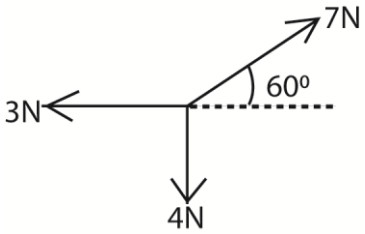
= 160 + 2,560 + 80 + 320

= 3,120m

Example 18

A body of mass 1kg is acted on by the forces shown below. Find

(i) Magnitude of the resultant force
(ii) acceleration of the body
(iii) distance moved in 2s



$F_R = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + \begin{pmatrix} 7\cos 60 \\ 7sin60 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 2.06 \end{pmatrix}$

$F_R = \sqrt{(0.5)^2 + (2.06)^2}$ = 2.12N

but $F_R = ma$

2.12 =1a

a = 2.12ms^{-2}

From s = ut + ½ at^2

u = 0, t = 2s, a =2.12ms^{-2}

s = 0 x 2 + ½ x 2.12 x 2^2 = 4.24m from the origin

Revision exercise

(Answers are given in square brackets at the end of each question)

1. P, Q and R are points on a straight road such that PQ = 20m and QR = 55m. A cyclist moving with uniform acceleration passes O and then notices that it takes him 10s and 15s to travel between P and Q and Q and R respectively. find the acceleration [a = $\frac{2}{15}ms^{-2}$]
2. A car travels from Kampala to Jinja and back. It takes average speed on the return journey is 4km/h greater than that on the outward journey and it takes 12 minutes less. Given that Kampala and Jinja are 80km apart, find the average speed on the outward journey.[30.05kmh]
3. Car A traveling at 35ms^{-1} along a straight horizontal road, accelerates uniformly at 0, 4ms^{-2}. At the same time, another car B moving at 44ms^{-1} and accelerating uniformly at 0.5ms^{-2} is 200m behind A
   (i) Find the time taken before car B over takes car A. [20s]
   (ii) speed with which B over takes A. [55m/s]
4. A car is being driven along a road at 72kmh^{-1} notices a fallen tree on the road 800m ahead and suddenly reduces the speed to 36kh^{-1} by applying brakes. For how long were the brakes applied [53.33s]
5. A train starts from station a with a uniform acceleration of 0.2ms^{-2} for 2 minutes and attains a maximum speed and moves uniformly for 15 minutes. it is then brought to rest at constant retardation of 5/3ms^{-2} at station B. find the distance between A and B. [23212.8m]
6. A motorcycle decelerated uniformly from 20kmh^{-1} to 8kmh^{-1} in travelling 896m. find the rate of deceleration in ms2 [0.0145ms^{-2}]
7. A body moves with a uniform acceleration and covers a distance of 27m in 3s; it then moves with a uniform velocity and covers a distance of 60m in 5s. Find the initial velocity and acceleration of the body. [6ms^{-1}, 2ms^{-2}]
8. A particle is projected away from an origin O with initial velocity of 0.25ms^{-1}. The particle travels in a straight line and accelerates at 1.5ms^{-2}. find
   (i) how far the particle is from O after 4s [7.5m]
   (ii) the distance travelled by the particle during the fourth second after projection. [5.5m]
9. A taxi which is moving with a uniform acceleration is observed to take 20s and 30s to travel successive 400m. find
   (i) initial speed of the taxi. [$\frac{68}{3}ms^{-1}$]
   (ii) the further distance it covers before stopping [163.3m]
10. Two cyclist A and B are 36m apart on a straight road. Cyclist B starts from rest with an acceleration of 6ms^{-2} while A is in pursuit of B with velocity of 20ms^{-1} and acceleration of 4ms^{-1}. Find the time taken when A overtakes B [1.3466s]