UACE S475 Sub math paper 1 2013

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, g = 9.8ms^{-2}.

SECTION A (40 MARKS)

Answer all the questions in this section

1. Given that p = $\log_a(a^3y^{-2})$ and $\log_a ay^2$, find the value of p + q. (05 marks)

2. The table below shows the age in years of mothers at the time they had their first child.

Age in years	15 -	20 -	25 -	30 -	35 -	40 - 45
Number of mothers	2	14	29	43	33	9

Calculate the modal age of the mothers. (05 marks)

3. Find the sum of the first ten terms of the geometric progression (G.P)

8 + 4 + 2 + ……… (05marks)

4. The table below shows the prices of items and their corresponding weights in the years 2000 and 2004

Item	**Price (U Shs)**		**Weight**
	2000	**2004**	
Food	55,000	60,000	4
Housing	48,000	52,000	2
Transport	16,000	20,000	1

Using 2000as the base year calculate the weighted price index for the items in 2004. (05 marks)

5. Solve the differential equation $8y\frac{dy}{dx} = 9x^2$ (05 marks)

Hence find the solution given that y = 2 when x = 1

6. Solve the equation $\sec^2\theta - \tan\theta = 1$ for $0^0 \leq \theta \leq 90^0$. (05 marks)

7. A bag contains 5 black pens (B) and 4 red pens ®. Two pens are picked at random, one after the other without replacement. Find the probability that both pens are of the same color.

8. A powered trolley in a factory is moving in a straight line with constant acceleration. It passes point A with a velocity of U ms^{-1}. It takes 8 seconds to travel 60m from point A to point B. Finally it takes 4 second to travel from point B to point C 60m apart. Find the value of U. (05 marks)

SECTION B (60 MARKS)

Answer any **four** questions from this section

All questions carry the same marks

9. Eight candidates seeking admission to a University course sat for written and oral test. The scores were as shown in the table below:

Written (X)	55	54	35	62	87	53	71	50
Oral (y)	57	60	47	65	83	56	74	63

(a) (i) draw a scatter diagram for the data.

(ii) Draw a line of best fit on your scatter diagram

(iii) Use the line of the best fit to find the value of Y when X = 70 (08marks)

(b) Calculate spearman's rank coefficient. Comment on your results. (07 marks)

10. (a) Sketch the curve $y = 5 + 4x - x^2$.(10 marks)

(b) Find the area enclosed between the curve and the x-axis from x = -1 to x = 5. (05 marks)

11. The table shows the number of bags of sugar sold by a certain wholesale shop from the year 2009 to 2020

YEAR	QUARTER			
	1^{ST}	2^{nd}	3^{rd}	4^{th}
2009	192	280	320	260
2010	300	360	380	270
2011	342	420	430	320
2012	424	480	510	412

(a) Calculate the four-point averages for the data (06marks)

(b) (i) On the same axes, plot the original data and the four-point moving averages. (05 marks)

(ii) Comment on the trends of the number of bags of sugar sold over the four year period. (01 mark)

(iii) Use your graph to estimate the number of bags to be sold in the first quarter of 2013. (03marks)

12. The points P and Q have position vectors **OP= –2i – 5j** and **OQ = i – 2j** respectively. R is appoint such that **OR = OP + λPQ**.

(a) Find the:

(i) value of **OP.OQ**

(ii) angle between the two vectors **OP** and **OQ.** (07 marks)

(b) Determine the

(i) vector **PQ**

(ii) vector **OR** in terms of λ

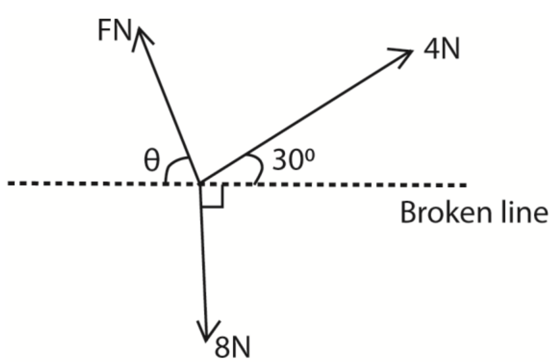
(iii) the value of λ for which OR is perpendicular to **PQ**. (08 marks)

13. A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

14. The diagram below shows three forces FN, 4N and 8N acting on a particle



If the forces are in equilibrium, find the values of

(i) θ

(ii) F (06marks)

(b) In a rectangle ABCD, AB = 4 and BC = 3m. Forces of magnitude 3N, 10N, 4N, 6N and 5N act in the direction of the letters AB, BC, CD, DA and AC respectively. Taking AB as horizontal, find the magnitude of the resultant force. (09 marks)

Proposed answers

1. Given that p = $\log_a(a^3y^{-2})$ and $\log_a ay^2$, find the value of p + q. (05 marks)

   Solution

   P + Q = $\log_a(a^3y^{-2}) + \log_a ay^2$

   $=\log_a(a^3y^{-2}x\ ay^2)$

   $=\log_a(a^4)$

   = $4\log_a a$

   = 4 x 1 = 4

2. The table below shows the age in years of mothers at the time they had their first child.

Age in years	15 -	20 -	25 -	30 -	35 -	40 - 45
Number of mothers	2	14	29	43	33	9

Calculate the modal age of the mothers. (05 marks)

Using the formula

Mode = Li + $\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) c$

Modal class (30 – 35)

Δ1 = 43 - 29 = 14

Δ2 = 43 – 33 = 10

Li = 30, Cc = 5

Mode = 30 + 5 $\left(\frac{14}{14+10}\right)$ = 32.92 (2D)

3. Find the sum of the first ten terms of the geometric progression (G.P)

   8 + 4 + 2 + ……… (05marks)

$S_n = \frac{a(1-r^n)}{1-r}$

a = 8, r = $\frac{4}{8} = \frac{1}{2}$, n = 10

$S_{10} = \frac{8(1-(0.5)^{10})}{1-(0.5)} = 16$

4. The table below shows the prices of items and their corresponding weights in the years 2000 and 2004

Item	**Price (U Shs)**		**Weight**
	2000	**2004**	
Food	55,000	60,000	4
Housing	48,000	52,000	2
Transport	16,000	20,000	1

Using 2000as the base year calculate the weighted price index for the items in 2004. (05 marks)

W.P.I = $\frac{\sum wP_{2004}}{\sum wP_{2000}} x100 = \frac{60,000\ x\ 4+52,000x2+20000x\ 1}{55,000x\ 4+48,000\ x\ 2+16,000\ x1} x100 = 111.15(2DP)$

5. Solve the differential equation $8y\frac{dy}{dx} = 9x^2$ (05 marks)

$8ydy = 9x^2dx$

$\int 8ydy = \int 9x^2dx$

$4y^2 = 3x^3 + c$

Hence find the solution given that y = 2 when x = 1

Substituting for y = 2 and x = 1

$4 \times 2^2 = 3 \times 1^3 + c$

c = 16 – 3 = 13

Substituting c in the equation

=> $4y^2 = 3x^3 + 13$

6. Solve the equation $\sec^2\theta - \tan\theta = 1$ for $0^0 \le \theta \le 90^0$. (05 marks)

$\sec^2\theta - \tan\theta = 1$

$1 + \tan^2\theta - \tan\theta = 1$

$\tan^2\theta - \tan\theta = 0$

$\tan\theta(\tan\theta - 1) = 0$

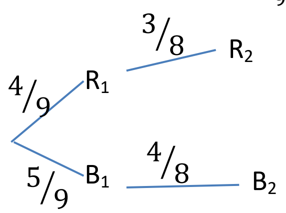
Either $\tan\theta = 0$; $\theta = \tan^{-1} 0 = 0^0$

Or $\tan\theta$-1, $\theta = \tan^{-1} 1 = 45^0$

Hence $\theta = (0^0, 45^0)$

7. A bag contains 5 black pens (B) and 4 red pens (R). Two pens are picked at random, one after the other without replacement. Find the probability that both pens are of the same color.

Total pens 5 + 4 = 9; P(B) = $\frac{5}{9}$; P(R) = $\frac{4}{9}$;



P(both are same color) = P($R_1 \cap R_2$) + P($B_1 \cap B_2$)

$$= \frac{4}{9}.\frac{3}{8} + \frac{5}{9}.\frac{4}{8} = \frac{20}{72} + \frac{12}{72} = \frac{32}{72} = \frac{4}{9}$$

$$= 0.4444$$

8. A powered trolley in a factory is moving in a straight line with constant acceleration. It passes point A with a velocity of U ms^{-1}. It takes 8 seconds to travel 60m from point A to point B. Finally it takes 4 second to travel from point B to point C 60m apart. Find the value of U. (05 marks)

s = Ut + $\frac{1}{2}at^2$

60 = 8U + 32a ……………(i)

120 = 12U + 72a ………..(ii)

(ii) – 1.5(i)

30 = 24a

a = 1.25

Substituting a in eqn. (i)

8U = 60 – 32 x 1.25 = 20

U = 2.5m^{-1}

SECTION B (60 MARKS)

Answer any **four** questions from this section

All questions carry the same marks

9. Eight candidates seeking admission to a University course sat for written and oral test. The scores were as shown in the table below:

Written (X)	55	54	35	62	87	53	71	50
Oral (y)	57	60	47	65	83	56	74	63

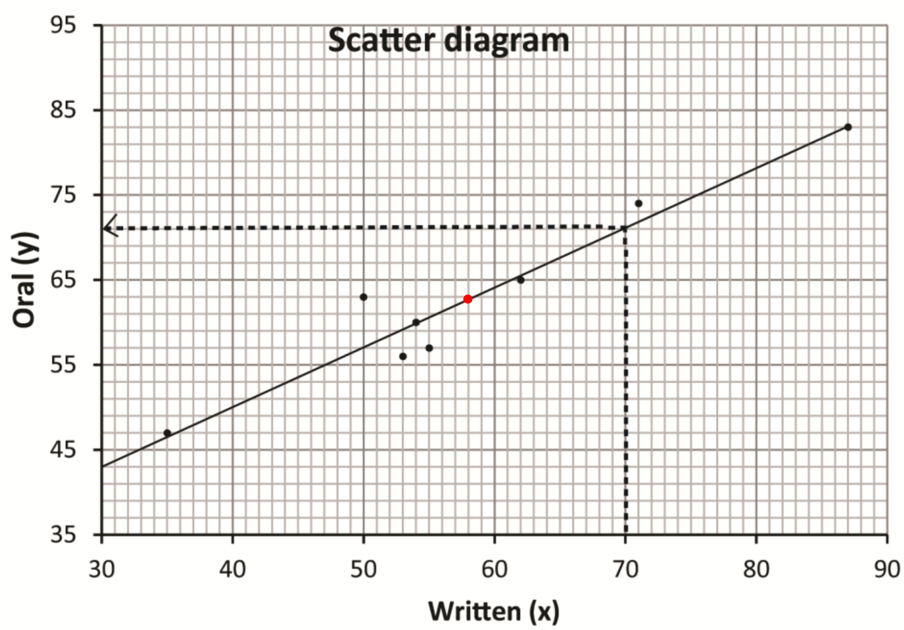
(a) (i) draw a scatter diagram for the data.

(ii) Draw a line of best fit on your scatter diagram

Note: The line of the best fit must pass through mean of y and mean and mean of x, here shown in red below although it should not be plotted.

$\bar{x} = \frac{55+54+35+62+87+53+71+50}{8} = 58$

$\bar{y} = \frac{57+60+47+65+83+56+74+63}{8} = 63$



(iii) Use the line of the best fit to find the value of Y when X = 70 (08marks)

71

(b) Calculate spearman's rank coefficient. Comment on your results. (07 marks)

X	Y	Rx	Ry	d	d^2
55	57	4	6	-2	4
54	60	5	5	0	0
35	47	8	8	0	0
62	65	3	3	0	0
87	83	1	1	0	0
53	56	6	7	-1	1
71	74	2	2	0	0
50	63	7	4	3	9
					$\sum d^2 = 14$

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6 x 14}{8 x 63}$ = 0.8333

Comment: there is a **high positive** correlation between marks of oral and written interview

10. (a) Sketch the curve $y = 5 + 4x - x^2$.(10 marks)

Finding x - intercepts when y = 0

$x^2 - 4x - 5 = 0$

$(x - 5)(x+1) = 0$

Either $(x - 5) = 0$; $x = 5$

Or $x + 1 = 0$; $x = -1$

Hence x – intercept are (0, -1) and (0, 5)

y – intercept, when x = 0, y = 5

Hence y-intercept is (0, 5)

Finding turning point when $\frac{dy}{dx} = 0$

$\frac{d}{dx}(5 + 4x - x^2) = 4 - 2x = 0$; $x = 2$

When x = 2

y = 5 +8 – 4 = 9

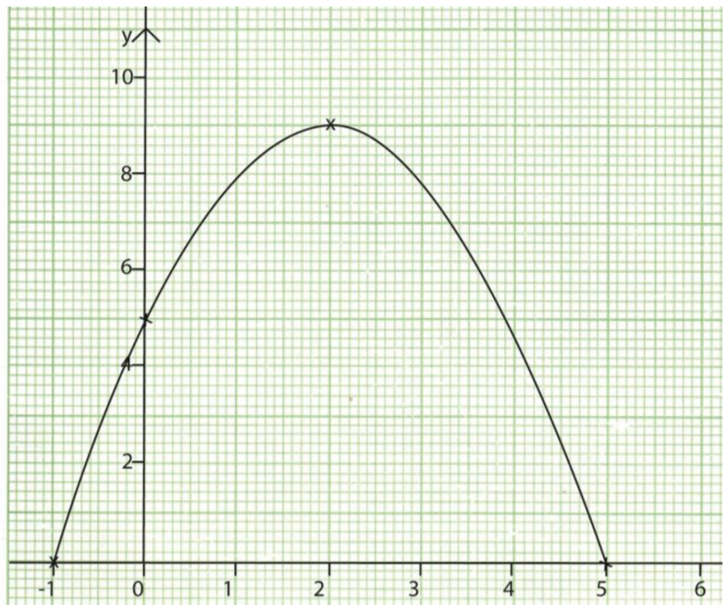
Hence turning point is (2, 9)

Nature of turning point

$\frac{d^2y}{dx^2} = -2$

Since $\frac{d^2y}{dx^2} < 0$ the turning point is a maximum

Sketch



Note that the turning points and intercepts must be plotted clearly.

(b) Find the area enclosed between the curve and the x-axis from x = -1 to x = 5. (05 marks)

A = $\int_{-1}^{5}((5 + 4x - x^2))dx$

= $\left[5x + 2x^2 - \frac{x^3}{3}\right]_{-1}^{5}$

= $\left(5(5) + 2(5)^2 - \frac{(5)^3}{3}\right) - \left(5(1) + 2(1)^2 - \frac{(1)^3}{3}\right)$

= $\left(25 + 50 - \frac{125}{3}\right) - \left(-5 + 2 + \frac{1}{3}\right)$

=$\frac{100}{3} + \frac{8}{3}$

= 36 sq. units

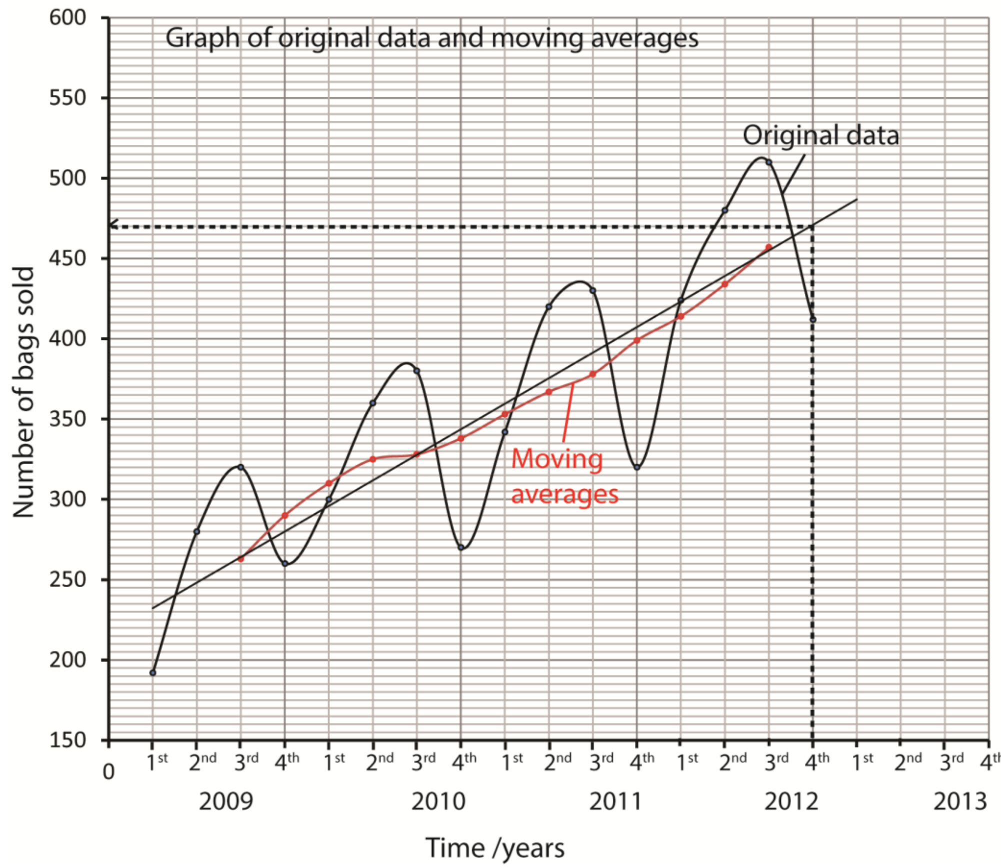
11. The table shows the number of bags of sugar sold by a certain wholesale shop from the year 2009 to 2020

YEAR	QUARTER			
	1ST	2nd	3rd	4th
2009	192	280	320	260
2010	300	360	380	270
2011	342	420	430	320
2012	424	480	510	412

(a) Calculate the four-point averages for the data (06marks)

Year	Quarter	Bags sold	Moving Totals	Moving averages
2009	1st	192		
	2nd	280		
	3rd	320	1052	263
	4th	260	1160	290
2010	1st	300	1240	310
	2nd	360	1300	325
	3rd	380	1310	328
	4th	270	1352	338
2011	1st	342	1412	353
	2nd	420	1462	367
	3rd	430	1512	378
	4th	320	1594	399
2012	1st	424	1654	414
	2nd	480	1734	434
	3rd	510	1826	457
	4th	412		
2013	1st	x		

(b) (i) On the same axes, plot the original data and the four-point moving averages. (05 marks)



(ii) Comment on the trends of the number of bags of sugar sold over the four year period. (01 mark)

There is a general increase in the number of bags sold with increase in time

(iii) Use your graph to estimate the number of bags to be sold in the first quarter of 2013. (03marks)

The estimates 14th moving average = 460

Let the number of bags sold in the first quarter of 2013 be x

$$\frac{x+480+510+412}{4} = 470$$

x = 1,880 – 1402 = 478

12. The points P and Q have position vectors **OP= –2i – 5j** and **OQ = i – 2j** respectively. R is appoint such that **OR = OP + λPQ**.

(a) Find the:

(i) value of **OP.OQ**

-1 x 2 + -5 x -2 = 8

(ii) angle between the two vectors **OP** and **OQ.** (07 marks)

OP.OQ=$|OP||OQ|cos\theta$

$8 = \sqrt{(-2)^2 + (-5)^2}.\sqrt{(1)^2 + (-2)^2} cos\theta$

$8 = \sqrt{29}.\sqrt{5} cos\theta$

$8 = \sqrt{145} cos\theta$

$\cos\theta = \frac{8}{\sqrt{145}}$

$\theta = \cos^{-1}\frac{8}{\sqrt{145}}$ = 48.37^0(2D)

(b) Determine the

(i) vector **PQ**

PQ = PO + OQ

$= \binom{2}{5} + \binom{1}{-2}$

$= \binom{3}{3}$ Or (3i + 3j)

(ii) vector **OR** in terms of λ

OR = $\binom{-2}{-5} + \lambda\binom{3}{3} = \binom{-2+3\lambda}{-5+3\lambda}$

(iii) the value of λ for which OR is perpendicular to **PQ**. (08 marks)

When OR is perpendicular to PQ

Then OR.PQ = 0

$\binom{-2+3\lambda}{-5+3\lambda}.\binom{3}{3}$

-6 + 9λ- 15 + 9λ =

18λ = 21

$\lambda = \frac{21}{18} = \frac{7}{6}$

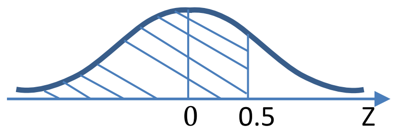
13. A bakery produces loaves of bread whose weight is normally distributed with mean 1,000g and standard deviation 40g.

(a) Find the probability that a randomly selected loaf has a weight of utmost 1,020g.

μ = 1000, σ = 40

Let x be the weight of bread

P(x≤ 1020) =P(z ≤ $\frac{1020-1000}{40}$ = P(z ≤ 0.5)



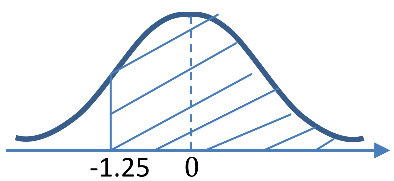
P(z ≤ 0.5)= 0.5 + P(0≤ x≤ 0.5)

= 0.5+0.1915

= 0.6915

(b) Assuming that the bakery makes 10,500 loaves, find the approximate number of loaves with weight greater than 950g.

P(x≤ 950) =P(z ≥ $\frac{950-1000}{40}$ = P(z ≥ -1.25)



P(z ≥ -1.25)= P(-1.25≤ z≤ 0) + 0.5

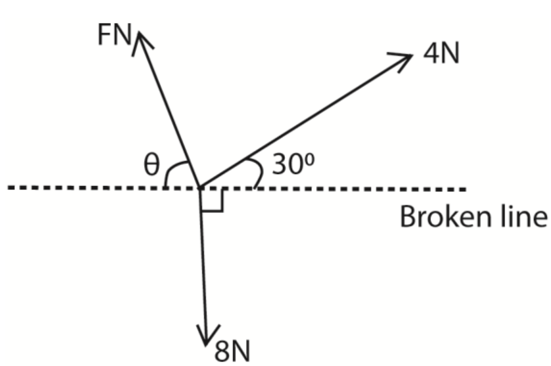
= (0 ≤ z ≤ 1.25) + 0.5

= 0.5 +0.3944

= 0.8944

Number of loaves = 0.8944 x 10,500 = 9391

14. The diagram below shows three forces FN, 4N and 8N acting on a particle



If the forces are in equilibrium, find the values of

(i) θ

Both horizontal and vertical component of force = 0

$F\cos\theta = 4\cos 30$ ……………….(i)

$F\sin\theta = 8 - 4\sin 30$ …………….. (ii)

(ii)÷ (i)

$\tan\theta = \frac{8-4sin30}{4cos30} = \frac{6}{3.464} = 1.732$

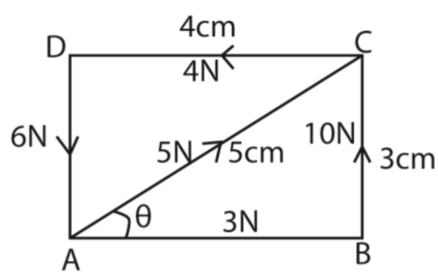
$\theta = \tan^{-1} 1.732 = 60^0$

(ii) F (06marks)

From eqn. (i)

$F = \frac{4cos30}{cos60} = 6.93N$

(b) In a rectangle ABCD, AB = 4 and BC = 3m. Forces of magnitude 3N, 10N, 4N, 6N and 5N act in the direction of the letters AB, BC, CD, DA and AC respectively. Taking AB as horizontal, find the magnitude of the resultant force. (09 marks)



$AC^2 = AB^2 + BC^2 = 4^2 + 3^2 = 16 + 9 = 25$

$AC = \sqrt{25} = 5$

$R = \binom{3}{0} + \binom{0}{10} + \binom{-4}{0} + \binom{0}{-6} + \binom{5cos\theta}{5sin\theta}$

$= \binom{3}{0} + \binom{0}{10} + \binom{-4}{0} + \binom{0}{-6} + \begin{pmatrix} 5x\frac{4}{5} \\ 5x\frac{3}{5} \end{pmatrix}$

$= \binom{3}{0} + \binom{0}{10} + \binom{-4}{0} + \binom{0}{-6} + \binom{4}{3}$

$= \binom{3}{7}$

$|R| = \sqrt{3^2 + 7^2} = \sqrt{58}$ = 7.6158N (4D)

Hence resultant force = 7.6158N

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Thanks

Dr. Bbosa Science