UACE S475 Sub math paper 1 2014

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, $g = 9.8ms^{-2}$.

SECTION A (40 MARKS)

Answer all the questions in this section

1. The roots of the equation $2x^2 + 4x - 1 = 0$ are α and β. Find the value of $\alpha^2 + \beta^2$. (05marks)

   $2x^2 + 4x - 1 = 0$

   $x^2 + 2x - \frac{1}{2} = 0$

   $\alpha + \beta = -2$

   $\alpha\beta = -\frac{1}{2}$

   $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

   $= 4 - (-1) = 5$

2. The ninth term of an Arithmetic Progression (A.P) is greater than the fifth term by 6. The sum of the first twelve terms is 123. Find the

   nth term = a+ (n-1)d

   (a) common difference of the A.P (03 marks)

   9^{th} term = a + 8d

5^{th} term = a + 4d

=> a + 8d = a + 4d + 6

4d = 6

d = 1.5

(b) first term of the A.P. (02mark)

$s_\infty = \frac{n}{2}[2a + (n-1)d]$

123 = $\frac{12}{2}(2a + 11x1.5)$

6(2a + 16.5) = 123

12a =24

a = 2

3. (a) How many arrangements can be made using the letters in the word "TROTTING"? (03marks)

TROTTING has 8 letters and 3T'2

Number of arrangements = $\frac{8!}{3!}$ = 6720

(b) In how many of these arrangements are the letters N and G next to each other. (02marks)

Since N and G are always together, they are counted as one letter

=> TROTTING has 7 letters and 3T'2

Number of arrangements = $\frac{7!}{3!}$ = 840

Number of arrangements of N and G = 2! =2

Total number of arrangements = 840 x 2 = 1680

4. Solve the differential equation $\frac{dy}{dx}$ = 2x + 5, given that y = -1 when x = 3 (05marks)

dy = (2x +5)dx

$\int dy = \int(2x + 5)dx$

y = x^2 + 5x + c

substituting for y = -1 and x =3

$-1 = 9 + 15 + c$

$c = -25$

Hence equation: $y = x^2 + 5x - 25$

5. A class of n students sat for a Mathematics test. Given that $\sum fx = 400$, $\sum fx^2 = 6500$ and the mean $\bar{x} = 16$, where x is the mark and f the frequency; determine the value of

   (a) n (02 marks)

   Mean = $\frac{\sum fx}{n}$

   $16 = \frac{400}{n}$, $n = 25$

   (b) the standard deviation. (03marks)

   S.d = $\sqrt{\frac{\sum fx^2}{n} - (\bar{x})^2} = \sqrt{\frac{6500}{25} - (16)^2}$ =2

6. Show that $\sec^2\theta + \text{cosec}^2\theta = \sec^2\theta\text{cosec}^2\theta$ (05 marks)

   $\sec^2\theta + \text{cosec}^2\theta = \frac{1}{cos^2\theta} + \frac{1}{sin^2\theta} = \frac{cos^2\theta + sin^2\theta}{cos^2\theta sin^2\theta} = \frac{1}{cos^2\theta sin^2\theta} = sec^2\theta cosec^2\theta$

7. In Binomial experiment , the probability of a success for n trials is 0.6. If the mean is 7.2, find the

   (a) value of n. (02 marks)

   Mean = np

   $7.2 = 0.6n$

   $n = 12$

   (b) probability of obtaining 7 success. (03marks)

   $P(X = x) = {}^nC_x p^x q^{(n-x)}$

   $= {}^{12}C_7(0.6)^7(0.4)^5$

   $= 0.227$

8. A cyclist rider along a straight road from a shop P to shop Q. He passes shop P with a velocity of $3ms^{-1}$ and accelerates uniformly at $1.25ms^{-2}$ until he attains a velocity of $12\ ms^{-1}$ at shop Q. Find the

   (a) time taken by the cyclist to reach Q. (03marks)

   From $v = u + at$

12 = 3 + 1.25t

t = $\frac{9}{1.25}$ = 7.2s

(b) distance PQ. (02 marks)

s = ut + $\frac{1}{2}at^2$ = 3 x 7.2 +$\frac{1}{2}$ x 1.25 x 7.2^2 =54

SECTION B (60 MARKS)

Answer any **four** questions form this section

All questions carry equal marks

9. The table below shows the marks of eight students in the mid-term test and end of term test in Economics.

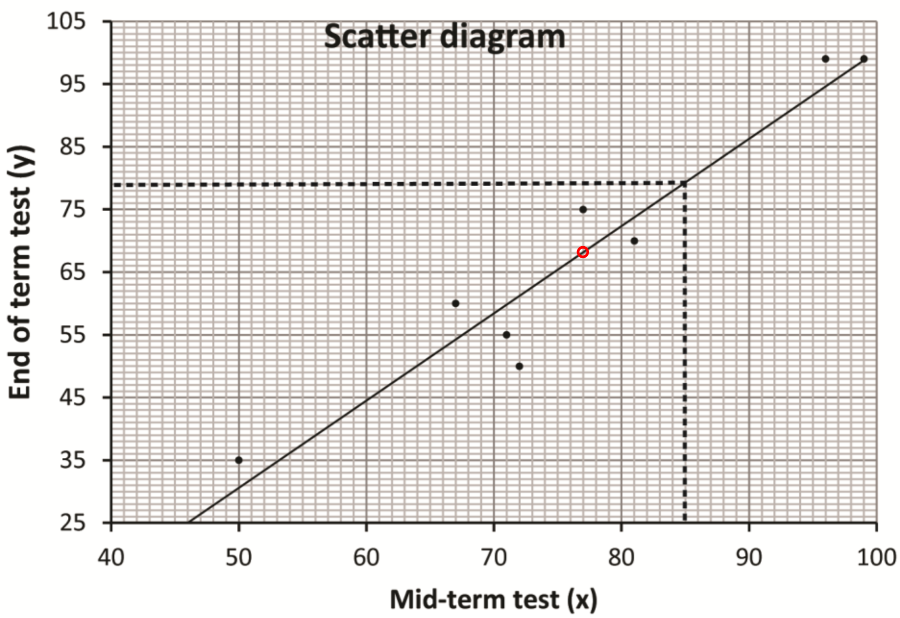
Mid-term tests (x)	99	71	50	67	77	81	96	72
End of term test (y)	99	55	35	60	75	70	99	50

(a) (i) Draw a scatter diagram for the data

(ii) On the same diagram draw a line of the best fit

Note that the line of the best fit passes through the mean of x and mean of y. these should be calculated to guide you as you draw the line of the best fit but should not be plotted. On the plot below $(\bar{x}, \bar{y})$ are encircled in red

$$\bar{x} = \frac{99+71+50+67+77+81+96+72}{8} = 76.6$$
$$\bar{y} = \frac{99+55+35+60+75+70+99+50}{8} = 67.9$$



(iii) Use the line of the best fit to find the value of y when x – 85. (08 marks)

78

(b) Calculate the Spearman's rank correlation coefficient. Comment on your results. (07marks)

Mid-term tests (x)	End of term test (y)	Rx	Ry	d	d^2
99	99	1	1.5	-0.5	0.25
71	55	6	6	0	0
50	35	8	8	0	0
67	60	7	5	2	4
77	75	4	3	1	1
81	70	3	4	-1	1
96	99	2	1.5	0.5	0.25
72	50	5	7	-2	4
					$\sum d^2=10.5$

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\ x\ 10.5}{8x63} = 0.875$$

10. (a) Given that A = $\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}$ and B = $\begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}$.

Find

(i) AB

$$\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 8+0 & 2+6 \\ 4+0 & 1-2 \end{pmatrix} = \begin{pmatrix} 8 & 8 \\ 4 & -1 \end{pmatrix}$$

(ii) BA

$$\begin{pmatrix} 4 & 1 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 8+1 & -12+1 \\ 0-2 & 0-2 \end{pmatrix} = \begin{pmatrix} 9 & -11 \\ -2 & -2 \end{pmatrix}$$

Comment on your result (05marks)

AB $\neq$BA; hence the commutative property does not apply for matrix

(b) A family bought the following items for three successive days. The first day it bought three bunches of matooke, two kilogram of rice, five kilograms meat and two kilogram of sugar. The second day it bought one kilogram of sugar. The third the family bought a bunch of matooke and two kilogram of rice. A bunch of matooke costs shs. 15,000. A kilogram of rice, meat and sugar cost 3,300, shs 8,000 and shs 3,000 respectively.

(i) represent the family's requirements in a 3 x 4 matrix

Purchase

$$\begin{array}{cc} \text{D} & \begin{array}{cccc} \text{B} & \text{R} & \text{M} & \text{S} \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} \end{array} = \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix}$$

(ii) write down the cost of each item as column matrix

Item costs

$$\begin{array}{c} B \\ R \\ M \\ S \end{array} \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix} = \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

(iii) Use the matrices in b(i) and b(ii) to find the family's total expenditure for the three days. (10 marks)

$$\begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} 15000 \\ 3300 \\ 8000 \\ 3000 \end{pmatrix}$$

= 3x 15000+2 x 3300 + 5 x 8000 + 2 x 3000 + 1 x 3000 + 1 x 15000 + 2 x 3300

= 45000 + 6600 + 40,000 +6000 + 3000 + 15000 + 6600

= 122,200

11. (a) The table below shows the price (U shs) of flour and eggs in the years of 2000 and 2010

COMMODITY	PRICE (U shs)	
	2000	2010
Flour	3000	5000
Eggs (1tray)	5000	7000

Taking 2000 as the base year, calculate the:

(i) Price relative of each commodity

Price relative = $\frac{P_{2010}}{P_{2000}} x100$

Price relative for flour = $\frac{5000}{3000} x100 = 166.7$

Price relative for eggs = $\frac{7000}{5000} x100 = 140$

(ii) Simple aggregate price index

S.A.P.I = $\frac{\sum P_{2010}}{\sum P_{2000}} x100 = \frac{5000+7000}{3000+5000} x100 = 150$

Comment on your results (08marks)

The prices of items increased by 50% from 2000 to 2010

(b) The data below shows items with their corresponding prices relatives and weights

ITEM	PRICE RELATIVE	WEIGHT
Food	120	172
Clothing	124	160
Housing	125	170
Transport	135	210
Others	104	140

(i) Find the cost of living index

Cost of living index = weighted price index

$= \frac{\sum Pw}{\sum W}$ where p = price relative, w = weights

ITEM	PRICE RELATIVE(P)	WEIGHT (w)	Pw
Food	120	172	20640
Clothing	124	160	19840
Housing	125	170	21250
Transport	135	210	28350
Others	104	140	14560
SUM		852	104640

Cost of living index $= \frac{104640}{852} = 122.8$

(ii) Comment on your result . (07 marks)

The cost of living increased by 22.8%

12. Given the curve $y = 3x^3 - 4x^2 - x$

(a) find the turning points of the curve (10 mark)

Turning points when $\frac{dy}{dx} = 0$

=> $9x^2 - 8x - 1 = 0$

$(9x + 1)(x - 1) = 0$

Either $9x + 1 = 0$; $x = -\frac{1}{9}$

Or $(x - 1) = 0$; $x = 1$

When $x = -\frac{1}{9}$

$y = 3\left(-\frac{1}{9}\right)^3 - 4\left(-\frac{1}{9}\right)^2 - \left(-\frac{1}{9}\right) = \frac{14}{243}$

Turning point $= \left(-\frac{1}{9}, \frac{14}{243}\right)$

When $x = 1$

y = $3(1)^3 - 4(1)^2 - (-1)$=-2

Turning point (1, -2)

Hence the turning points (x, y) are $\left(-\frac{1}{9}, \frac{14}{243}\right)$ and $(1, -2)$

(b) distinguish between the nature of the turning points. (05 marks)

$\frac{d^2}{dx^2}(9x^2 - 8x - 1)$ = 18x – 8

When x = 1

$\frac{d^2y}{dx^2}$ = 18- 8 = 10

Since $\frac{d^2y}{dx^2} > 0$; the turning point (1, -2) is a minimum

When x = $-\frac{1}{9}$

$\frac{d^2y}{dx^2}$ = $\frac{-18}{9} - 8$ = - 10

Since $\frac{d^2y}{dx^2} < 0$; the turning point $\left(-\frac{1}{9}, \frac{14}{243}\right)$ is a maximum

13. The table below shows the probability distribution of the number of Compact Discs (CDs) sold.

Number of CDs (x)	0	1	2	3	4
Probability, P(X = x)	0.05	0.28	c	0.22	0.09

Determine the:

(a) Value of c (03 marks)

$\sum P(X = x) = 1$

⇨ 0.05 + 0.28 + c + 0.22+ 0,09 = 1

c =0.36

(b) Probability that at least 2 CD's are sold. (03 marks)

P(x ≥2) = P(x =2) + P(x = 3) + P(x = 4) = 0.36 + 0.22 + 0.09 = 0.67

(c) Expectation, E(X) (03 marks)

E(X) = ∑xP(X= x) = 0.05 x 0 + 0.28 x 1 + 0.36 x 2 + 0.22 x 3 + 0.09 x 4 =2.02

(d) Standard deviation. (06marks)

E(X^2) = ∑x^2P(X= x) = 0.05 x 0^2 + 0.28 x 1^2 + 0.36 x 2^2 + 0.22 x 3^2 + 0.09 x 4^2 =5.14

S.D =$\sqrt{E(X^2) - (E(x))^2} = \sqrt{5.14 - (2.02)^2}$ = 1.0294

14. (a) A brick of mass 750 g is dragged by a horizontal force at a uniform speed along a rough horizontal surface, through a distance of 20m. The work done against friction is 49.8J. Calculate the coefficient of friction between the brick and the surface. (06 marks)

Work = force x distance

Force = $\frac{49.8}{20} = 2.49 =$ μR =μmg

750g = $\frac{750}{1000} = 0.75kg$

μ = $\frac{2.49}{0.75x9.8} = 0.339$

(b) A truck of mass 8 tonnes has a maximum speed of 20ms^{-1} up an incline of aresin $\frac{1}{50}$ when the engine is working against resistances of 30,000N. Calculate the maximum power of the engine. (09 marks)

Power = (R + Mgsinθ) v

= (30,000 +8,000 x 9.8 x $\frac{1}{50}$) x 20

= 631,360W