UACE S475 Sub math paper 1 2015

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, g = $9.8ms^{-2}$.

SECTION A (40 MARKS)

Answer all the questions in this section

1. Evaluate $\frac{\log_6 216+\log_2 64}{\log_3 243-\log_{10} 0.1}$ (05 marks)

2. The table below shows the rank of marks awarded by Judge 1 (Rx) and Judge 2 (Ry) to 7 choir groups A to G.

Choir	A	B	C	D	E	F	G
Rank Judge (Rx)	2	4	6	1	5	3	7
Rank Judge (Ry)	2	3	5	1	6	4	7

Calculate Spearman's rank correlation coefficient between the marks awarded by the two judges.

Comment on your results. (05marks)

3. Solve the equation $3\sin^2\theta + \cos\theta + 1 = 0$ for values of θ from 0^0 to 180^0. (05marks)

4. A committee of 5 people is to be selected from a group of 6men and 7 women.

   (a) Find the number of possible committees. (02 marks)

   (b) What is the probability that there are only 2 women on the committee? (03marks)

5. Find the gradient of the curve $y = 4x^2(3x + 2)$ at the point (1, 20) (05marks)

6. Three A, B and C are such that P(A) =06, P(B) = 0.8, P(B/A) = 0.45 and P(B∩C) = 0.28. Find

(a) $P(A \cap B)$ (03 marks)

(b) $P(C/B)$ (02 marks)

7. The matrix A = $\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$ and **I** is a 2 x 2 Identity matrix.

   Determine the matrix B such $A^2 + \frac{1}{2}B = \mathbf{I}$ (05marks)

8. A bullet of mass 50g is fired toward a stationary wooden block and enters the block when travelling horizontally with a speed of $500ms^{-1}$. The wooden block provides a constant resistance of 36,000N. Find how far into the block the bullet will penetrate. (05marks)

SECTION B (60 MARKS)

Answer any **four** questions

All questions carry equal marks

9. The table below shows the number of students and the mark scored in a test.

MARKS	NUMBER OF STUDENTS
0 – 4	10
5 – 9	7
10 – 14	5
15 – 19	3
20 – 24	7
25 – 29	11
30 – 34	37
35 – 39	20

(a) (i) Draw a cumulative frequency curve (Ogive) for data

(ii) Use the Ogive to estimate the median mark (06marks)

(b) Calculate the

(i) Mean mark

(ii) Standard deviation (09 marks)

10. The rate of decay of a radioactive material is proportional to the amount x grams of the material present at any time t. Initially there was 60 grams of the material. After 8 years the material reduced to 15 grams.

    (a) Form a differential equation for the rat of decay of the material. (03marks)

    (b) Solve the differential equation formed in (a) above. (10 marks)

    (c) Find the time taken for the material to reduce to 10 grams. (02 marks)

11 The table below shows the price (in Ug Shs) of some food Items in January, June and December together with the corresponding weights.

Item	**Price (in Ug Shs)**			**Weight**
	January	June	December	
Matooke (1 bunch)	15,000	13,000	18,000	4
Meat (1kg)	6,500	6000	7,150	1
Posho(1kg)	2,000	1,800	1,600	3
Beans (1kg)	2,200	2,000	2,860	2

(a) Simple aggregate price index for June

Comment on your results (05 marks)

(b) Weighted aggregate price index for December

Comment on your results (10 marks)

12. The roots of the equation $2x^2 - 6x + 7 = 0$ are α and β. Determine the

(a) values of $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ (12 marks)

(b) quadratic equation with integral coefficients whose roots $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$(03marks)

13. A continuous random variable X has a probability density function given by,

$$f(x) = \begin{cases} \frac{kx}{6}, & 1 \le x \le 2, \\ 0 & Otherwise \end{cases}$$

Where k is a constant

(a) Find
   (i) The value of k (04marks)
   (ii) P(X ≥ 1.5) (04marks)
   (iii) The mean of X, E(X) (03 marks)
(b) Sketch the graph of f(x) (04 marks)

14. A motorist moving at $90ms^{-1}$ decelerates uniformly to a velocity V ms^{-1} in 10 seconds. He maintains his speed for 30 seconds and then decelerates uniformly to rest in 20 seconds

(a) Sketch a velocity – time graph for the motion of the motorist. (06marks)

(b) Given that the total distance travelled is 800m, use your graph to calculate the value of V. (05marks)

(c) Determine the two decelerations. (04 marks)

Proposed answers

Evaluate $\frac{\log_6 216+\log_2 64}{\log_3 243-\log_{10} 0.1}$ (05 marks)

$$\frac{\log_6 216+\log_2 64}{\log_3 243-\log_{10} 0.1} = \frac{\log_6 6^3+\log_2 2^6}{\log_3 3^5-\log_{10} 10^{-1}} = \frac{3\log_6 6+6\log_2 2}{5\log_3 3- -1\log_{10} 10} = \frac{3x1+6x1}{5x1+1x1} = \frac{9}{6} = 1.5$$

2. The table below shows the rank of marks awarded by Judge 1 (Rx) and Judge 2 (Ry) to 7 choir groups A to G.

Choir	A	B	C	D	E	F	G	sum
Rank Judge (Rx)	2	4	6	1	5	3	7	
Rank Judge (Ry)	2	3	5	1	6	4	7	
d	0	1	1	0	-1	-1	0	
d^2	0	1	1	0	1	1	0	4

Calculate Spearman's rank correlation coefficient between the marks awarded by the two judges.

$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6x\ 4}{7(49-1)}$ = 0.9286

Comment on your results. (05marks)

There is a high positive correlation between the two Judges' marks

3. Solve the equation $3\sin^2\theta + \cos\theta + 1 = 0$ for values of θ from 0^0 to 180^0. (05marks)

$3(1- \cos^2\theta) + \cos\theta+ 1 = 0$

$3 \cos^2\theta – \cos\theta – 4 = 0$

$(\cos\theta + 1)(3\cos \theta -4) = 0$

Either $\theta= \cos^{-1} -1 = 180^0$

Or $\theta= \cos^{-1}\frac{4}{3}$ = no value

Hence = 180^0

4. A committee of 5 people is to be selected from a group of 6men and 7 women.

(a) Find the number of possible committees. (02 marks)

Total number of members = 6+7 = 13

Number of committees = $^{13}C_5$ = 1287

(b) What is the probability that there are only 2 women on the committee? (03marks)

Possible selection 2 of 7 and 3 men out of 6 = ${}^{7}C_{2}.\, {}^{6}C_{3}$ = 21 x 20 = 420

P(2 women on a committee) = $\frac{420}{1287} = 0.3263$

5. Find the gradient of the curve y = $4x^2(3x + 2)$ at the point (1, 20) (05marks)

y = $4x^2(3x + 2) = 12x^3 + 8x^2$

$\frac{dy}{dx} = 36x^2 + 16x$

At x = 1

$\frac{dy}{dx} = 36 + 16 = 52$

Hence gradient = 52

6. Three A, B and C are such that P(A) =0.6, P(B) = 0.8, P(B/A) = 0.45 and P(B∩C) = 0.28. Find

(a) P(A∩B) (03 marks)

P(A∩B) = P(B/A). P(A) = 0.45 x 0.6 = 0.27

(b) P(C/B) (02 marks)

$P(C/B) = \frac{P(B\cap C)}{P(B)} = \frac{0.28}{0.8} = 0.35$

7. The matrix A = $\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}$ and **I** is a 2 x 2 Identity matrix.

Determine the matrix B such A^2 + ½ B = **I** (05marks)

$\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}^2 + \frac{1}{2}B = I$

$\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ -3 & 0 \end{pmatrix} + \frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 \\ -6 & -3 \end{pmatrix} + \frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\frac{1}{2}B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 2 \\ -6 & -3 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 6 & 4 \end{pmatrix}$

$B = 2\begin{pmatrix} 0 & -2 \\ 6 & 4 \end{pmatrix} = \begin{pmatrix} 0 & -4 \\ 12 & 8 \end{pmatrix}$

Hence matrix of B = $\begin{pmatrix} 0 & -4 \\ 12 & 8 \end{pmatrix}$

8. A bullet of mass 50g is fired toward a stationary wooden block and enters the block when travelling horizontally with a speed of 500ms^{-1}. The wooden block provides a constant resistance of 36,000N. Find how far into the block the bullet will penetrate. (05marks)

F = ma

36000 = $\frac{50}{1000} xa$

a = $\frac{36000x\ 1000}{50}$ 720,000

From $v^2 = u^2 - 2as$, v = 0

s = $\frac{500^2}{2x720,00}$ =0.1736m

SECTION B (60 MARKS)

Answer any **four** questions

All questions carry equal marks

9. The table below shows the number of students and the mark scored in a test.

MARKS	NUMBER OF STUDENTS
0 – 4	10
5 – 9	7
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25 – 29	11
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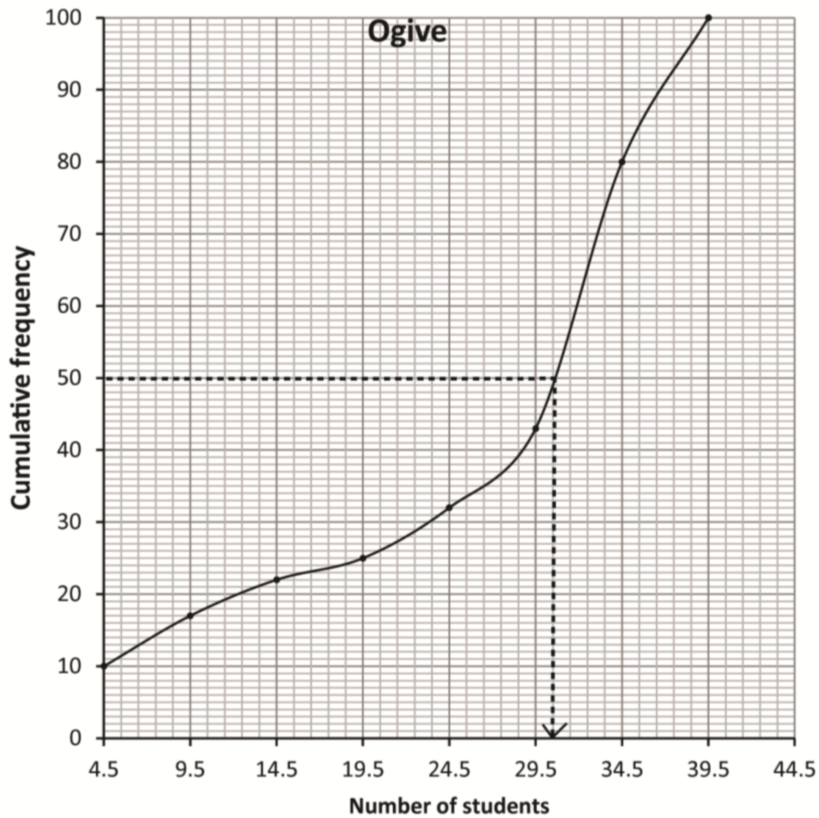
35 – 39	20

(c) (i) Draw a cumulative frequency curve (Ogive) for data

MARKS	Class boundaries	NUMBER OF STUDENTS (f)	Cf	x	fx	fx^2
0– 4	0 – 4.5	10	10	2	20	40
5 – 9	4.5 – 9.5	7	17	7	49	343
10 – 14	9.5 – 14.5	5	22	12	60	720
15 – 19	14.5 – 19.5	3	25	17	51	867
20 – 24	19.5 – 24.5	7	32	22	154	3388
25 – 29	24.5 – 29.5	11	43	27	297	019
30 – 34	29.5 – 34.5	37	80	32	1184	37888
35 – 39	34.5 – 39.5	20	100	23	740	27380
		$\sum f = 100$			∑fx =2555	∑fx^2=78645

(iii) Use the Ogive to estimate the median mark (06marks)



Note that Cf is plotted against the upper limit of the class

Median $=\left(\frac{N}{2}\right)^{th} = \left(\frac{100}{5}\right)^{th} = 50^{th}$ value= 30.5

(b) Calculate the

(i) Mean mark

Mean $= \frac{\sum fx}{\sum f} = \frac{2555}{100} = 25.55$

(ii) Standard deviation (09 marks)

S.d $= \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{78645}{100} - (25.55)^2} = 11.56$

10. The rate of decay of a radioactive material is proportional to the amount x grams of the material present at any time t. Initially there was 60 grams of the material. After 8 years the material reduced to 15 grams.

(a) Form a differential equation for the rat of decay of the material. (03marks)

$\frac{dx}{dt} = -kx$

(b) Solve the differential equation formed in (a) above. (10 marks)

$\int \frac{dx}{x} = -k \int dt$

Inx = -kt + c

At t = 0, c = In60

t = 8, x = 15

In$\frac{60}{15}$ = 8k

k $= \frac{1}{8} In \frac{60}{15}$ = 0.173 year^{-1}

Thus, $In \frac{60}{x} = 0.173t$

(c) Find the time taken for the material to reduce to 10 grams. (02 marks)

$In \frac{60}{10} = 0.173t$

t = 10.36 years

11 The table below shows the price (in Ug Shs) of some food Items in January, June and December together with the corresponding weights.

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Matooke (1 bunch)	15,000	13,000	18,000	4
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Beans (1kg)	2,200	2,000	2,860	2

(a) Simple aggregate price index for June

S.A.P.I $= \frac{\sum P_{June}}{\sum P_{Jan}} x100 = \frac{13{,}000+6000+1800+2000}{15000+6500+2000+2200} x100 = 88.71$

Comment on your results (05 marks)

There was a general decrease in the prices of items in the month of June by 11.29%

(b) Weighted aggregate price index for December

W.A.P.I $= \frac{\sum wP_{Dec}}{\sum wP_{June}} x100 = \frac{18{,}000x4+7150x1+1600x3+2860x2}{13000x4+6000x1+1800x2+2000x2} x100 = 116.6$

Comment on your results (10 marks)

There was a general increase in the prices of items in the month of December by 11.29%

12. The roots of the equation $2x^2 - 6x + 7 = 0$ are α and β. Determine the

(a) values of $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$ (12 marks)

Solution

$\alpha+\beta = 3$

$\alpha\beta = \frac{7}{2}$

$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

But $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

=> $(\alpha - \beta)^2 = (\alpha+\beta)^2 - 2\alpha\beta - 2\alpha\beta$

$= (\alpha+\beta)^2 - 4\alpha\beta$

$= 3^2 - 4 \text{ x } \frac{7}{2}$

$= -5$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2} = \frac{\alpha+\beta}{\alpha^2\beta^2} = \frac{\alpha+\beta}{(\alpha\beta)^2} = \frac{2}{\left(\frac{7}{2}\right)^2} = \frac{3x4}{49} = \frac{12}{49}$$

(b) Quadratic equation with integral coefficients whose roots $(\alpha - \beta)^2$ and $\frac{1}{\alpha^2\beta} + \frac{1}{\alpha\beta^2}$(03marks)

Sum of root = $-5 + \frac{12}{49} = -\frac{233}{49}$

Product of roots $= -5 \text{ x } \frac{12}{49} = -\frac{60}{49}$

Equation

$$x^2 - \left(-\frac{233}{49}\right)x - \frac{60}{49} = 0$$

Hence equation is $x^2 - \frac{233}{49}x - \frac{60}{49} = 0$

13. A continuous random variable X has a probability density function given by,

$$f(x) = \begin{cases} \frac{kx}{6}, & 1 \leq x \leq 2, \\ 0 & Otherwise \end{cases}$$

Where k is a constant

(c) Find

(i) The value of k (04marks)

$\int f(x)dx = 1$

$\frac{k}{6}\int_1^2 xdx = \frac{k}{6}\left[\frac{x^2}{2}\right]_1^2 = 1$

$\frac{k}{6}\left(2-\frac{1}{2}\right) = \frac{k}{6}\left(\frac{3}{2}\right) = 1$

$k = 4$

(ii) P(X ≥ 1.5) (04marks)

P(X ≥ 1.5)= $\frac{4}{6}\left[\frac{x^2}{2}\right]_{1.5}^2 = \frac{2}{3}\left(2-\frac{2.25}{2}\right) = 0.58$

(iii) The mean of X, E(X) (03 marks)

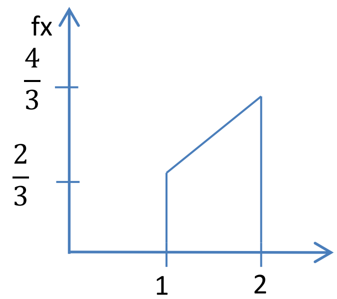
E(X) = $\int xf(x)dx = \frac{4}{6}\int_1^2 x^2dx = \frac{4}{6}\left[\frac{x^3}{3}\right]_1^2 = \frac{4}{6}\left(\frac{8}{3}-\frac{1}{3}\right) = \frac{2}{3}.\frac{7}{3} = \frac{14}{9}$

(d) Sketch the graph of f(x) (04 marks)

f(x) = $\frac{2}{3}x$

At x = 1, fx = $\frac{2}{3}$

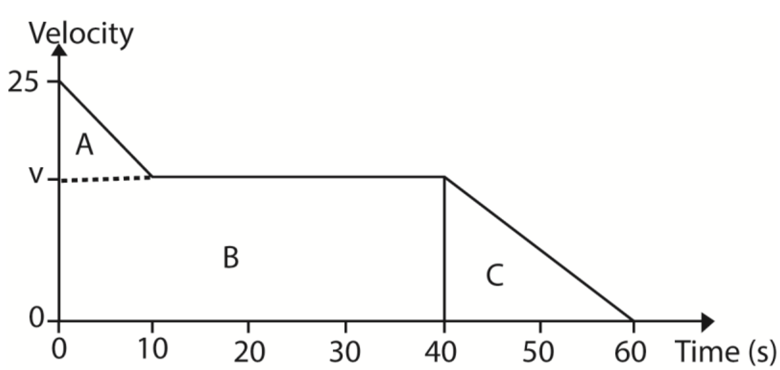
At x = 2, fx = $\frac{4}{3}$



14. A motorist moving at 90ms^{-1} decelerates uniformly to a velocity V ms^{-1} in 10 seconds. He maintains his speed for 30 seconds and then decelerates uniformly to rest in 20 seconds

(a) Sketch a velocity – time graph for the motion of the motorist. (06marks)

90kmh^{-1} = $\frac{90x1000}{3600}$ = 25ms^{-1}



(b) Given that the total distance travelled is 800m, use your graph to calculate the value of V. (05marks)

Total area = A + B + C

800 = $\frac{1}{2}x10(25 - v) + 40v + \frac{1}{2}(20v)$ = 125 -5v +40v + 10v = 125 +45v

45v = 800-125 = 675

v = 15ms^{-1}

(c) Determine the two decelerations. (04 marks)

Let deceleration in region A be a

From v = u –at

15 = 25 – 10a

10a = 10

a = 1ms^{-2}

Deceleration in region C

0 = 14 – 20a

a = $\frac{14}{20}$ = 0.7ms^{-2}