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UACE S475 Sub math paper 1 2016

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, $g = 9.8\text{ms}^{-2}$.

SECTION A (40 MARKS)

Answer all the questions in this section

- Given that $(x + 1)$ and $(x - 2)$ the polynomial $ax^3 - 3x^2 - bx + 2$, find the values of a and b. (05 marks)
- The table below shows the oral interview rank (X) and written interview rank (Y) for 12 candidates.

Candidate	A	B	C	D	E	F	G	H	I	J	K	L
Oral interview Rank (X)	8	10	9	4	12	5	11	7	3	6	1	2
Written interview Rank (Y)	11	12	9	7	10	6	8	5	2	4	1	3

Calculate Spearman's rank correlation coefficient and comment on your results. (05marks)

- The sum to infinity of a Geometric Progression (GP) is $\frac{25}{4}$ and the first term is 5. Find the
 - common ratio of the GP. (03marks)
 - sum of the first ten terms of the GP. (02marks)
- The table below shows the number of crates of soda at by a certain shop in 2010

MONTH	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEP	OCT	NOV	DEC
Oral interview Rank (X)	175	783	351	228	378	297	823	338	230	391	410	742

Calculate the four – month moving average for the data. (05marks)

5. Determine the coordinates of the stationary point of the curve

$$y = \frac{1}{4}x^2 - 2x - 5 \quad (05\text{marks})$$

6. Two independent events A and B are such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{3}{5}$.

Find $P(A \cup B)$ (05marks)

7. Given the matrices $A = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix}$, find

(a) matrix C such that $3A - 2C + B = I$, where I is a 2 x 2 identity matrix.(03marks)

(b) the determinant of C. (02 marks)

8. A car of mass 2000kg ascends on an incline of $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal.

The resistance force to motion of the car is 1000N. The power of the car engine is 59,200W.
Calculate the maximum speed of the car. (05 marks)

SECTION B (60 MARKS)

Answer any four questions

All questions carry equal marks

9. The table below shows a frequency distribution of marks scored by 55 students in a test.

Marks	10 -	20 -	30 -	40 -	50 -	60 -	70 -	80 - ≤90
Number of students	2	6	12	15	10	6	3	1

(a) Draw a histogram for the data and use it to estimate the modal mark. (05marks)

(b) Calculate the

(i) mean mark

(ii) standard deviation (10marks)

10. Chemical A is converted into another chemical by a chemical reaction. The rate at which a chemical A is being converted is directly proportional to the amount present at any time. Initially 100g of chemical A was present. After 5 minutes, 90g of A is present.

(a) Form a differential equation for chemical reaction. (03marks)

(b) By solving the differential equation formed in (a), determine the

(i) amount of chemical A present after 20 minutes.

(ii) time taken for the amount of chemical A to be reduced to 20 g

11. The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005.

COMPONENT	A	B	C	D	E
PRICE (\$) 1998	35	70	43	180	480
PRICE (\$) 2005	60	135	105	290	800
WEIGHT	6	5	3	2	1

Taking 1998 as the base year

- (a) Calculate for 2005 the:
- simple aggregate price index. (03marks)
 - price relative of each component. (03marks)
 - weighted aggregate price index. (06marks)
- (b) Estimate the cost of an engine in 1998 given that its cost in 2005 was 1600 US dollars
12. (a) Solve the equation $1 + \cos\theta = 2\sin^2\theta$ for values of θ between 0° and 360° . (09marks)
- (b) By eliminating θ from the equation $x = a\sec\theta$ and $y = b + c\cos\theta$, show that $x(y - b) = ca$. (06marks)

13. A random variable X has a probability density function, $f(x)$, defined by

$$f(x) = \begin{cases} kx(x + 2), & 0 \leq x \leq 2, \\ 0, & \text{otherwise} \end{cases}$$

where k is constant

Determine

- value of k (04marks)
 - $P(1 \leq X \leq 1.5)$ (03marks)
 - Expectation, $E(X)$ (04marks)
 - Variance, $\text{Var}(X)$ (04marks)
14. A car initially at rest accelerated uniformly to a speed of 20ms^{-1} in 16 seconds. The car then travelled at the attained speed for 2 minutes. The car then accelerated uniformly at 2.5ms^{-2} for 8 seconds. It finally decelerated at 2.5ms^{-2} to rest.
- (a) Find the
- greatest speed attained by the car.
 - total time taken by the car to come to rest. (06marks)

(b) Sketch velocity – time graph for the motion of the car. (04 marks)

(c) Use your graph to find the total distance travelled by the car. (05 marks)

Proposed answers

1. Given that $(x + 1)$ and $(x - 2)$ the polynomial $ax^3 - 3x^2 - bx + 2$, find the values of a and b. (05 marks)

For $(x + 1) = 0$; $x = -1$

Substituting $x = -1$ in the polynomial

$$a(-1)^3 - 3(-1)^2 - b(-1) + 2 = 0$$

$$-a - 3 + b + 2 = 0$$

$$b = (1 + a) \dots\dots\dots(i)$$

For $(x - 2) = 0$; $x = 2$

Substituting $x = 2$ in the polynomial

$$a(2)^3 - 3(2)^2 - b(2) + 2 = 0$$

$$8a - 12 - 2b + 2 = 0$$

$$8a - 2b = 10 \dots\dots\dots(ii)$$

Substituting (i) into (ii)

$$8a - 2(1 + a) = 10$$

$$6a = 12$$

$$a = 2$$

substituting a into eqn. (i)

$$b = 1 + 2 = 3$$

hence $a = 2$ and $b = 3$

2. The table below shows the oral interview rank (X) and written interview rank (Y) for 12 candidates.

Candidate	A	B	C	D	E	F	G	H	I	J	K	L
Oral interview Rank (X)	8	10	9	4	12	5	11	7	3	6	1	2
Written interview Rank (Y)	11	12	9	7	10	6	8	5	2	4	1	3

Calculate Spearman's rank correlation coefficient and comment on your results. (05marks)

Solution

Rearranging data

Candidate	Rx	Ry	d	d ²
A	8	11	-3	9
B	10	12	-2	4
C	9	9	0	0
D	4	7	3	9
E	12	10	2	4
F	5	6	-1	1
G	11	8	3	9
H	7	5	2	4
I	3	2	1	1
J	6	4	2	4
K	1	1	0	0
L	2	3	-1	1
				$\sum d^2 = 46$

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 46}{12(12^2-1)} = 0.892$$

Comment: there is high positive correlation between the oral interview and written interview.

3. The sum to infinity of a Geometric Progression (GP) is $\frac{25}{4}$ and the first term is 5. Find the

(a) common ratio of the GP. (03marks)

$$S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow \frac{5}{1-r} = \frac{25}{4}$$

$$25(1-r) = 20$$

$$25r = 5$$

$$r = \frac{1}{5} = 0.2$$

(b) sum of the first ten terms of the GP. (02marks)

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_{10} = \frac{5(1-(0.2)^{10})}{1-0.2} = 6.25$$

4. The table below shows the number of crates of soda by a certain shop in 2010

MONTH	JAN	FEB	MAR	APR	MAY	JUNE	JULY	AUG	SEP	OCT	NOV	DEC
Oral interview Rank (X)	175	783	351	228	378	297	823	338	230	391	410	742
Moving total			1537	1740	1254	1726	1836	1688	1782	1369	1773	
Moving averages			384.3	435.0	313.5	431.5	459	422	445.5	342.3	443.3	

Calculate the four – month moving average for the data. (05marks) **(in the table)**

5. Determine the coordinates of the stationary point of the curve

$$y = \frac{1}{4}x^2 - 2x - 5 \quad (05\text{marks})$$

$$\text{At the stationary point } \frac{dy}{dx} = \frac{d}{dx} \left(\frac{1}{4}x^2 - 2x - 5 \right) = 0$$

$$\frac{1}{2}x - 2 = 0$$

$$x = 4$$

substituting for $x = 4$ in the equation

$$y = \frac{1}{4}(4)^2 - 2(4) - 5 = -9$$

Hence stationary point is (4, -9)

6. Two independent events A and B are such that $P(A) = \frac{1}{4}$ and $P(B) = \frac{3}{5}$.

Find $P(A \cup B)$ (05marks)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{4} + \frac{3}{5} - \frac{1}{4} \cdot \frac{3}{5}$$

$$= \frac{17}{20} - \frac{3}{20} = \frac{14}{20} = 0.7$$

7. Given the matrices $A = \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix}$, find

(a) matrix C such that $3A - 2C + B = I$, where I is a 2×2 identity matrix. (03marks)

$$3 \begin{pmatrix} 3 & 5 \\ -2 & 4 \end{pmatrix} - 2C + \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$2C = \begin{pmatrix} 9 & 15 \\ -6 & 12 \end{pmatrix} + \begin{pmatrix} 8 & -3 \\ -4 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 16 & 12 \\ -10 & 18 \end{pmatrix}$$

$$C = \frac{1}{2} \begin{pmatrix} 16 & 12 \\ -10 & 18 \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ -5 & 9 \end{pmatrix}$$

(b) the determinant of C. (02 marks)

$$\text{Det}(C) = 8 \times 9 - (-5 \times 5) = 72 + 30 = 102$$

8. A car of mass 2000kg ascends on an incline of $\sin^{-1}\left(\frac{1}{10}\right)$ to the horizontal.

The resistance force to motion of the car is 1000N. The power of the car engine is 59,200W.
Calculate the maximum speed of the car. (05 marks)

$$\text{Power} = (R + mg\sin\theta)v$$

$$59,200 = (1000 + 2000 \times 9.8 \times \frac{1}{10})v$$

$$v = \frac{59200}{2960} = 20\text{ms}^{-1}$$

SECTION B (60 MARKS)

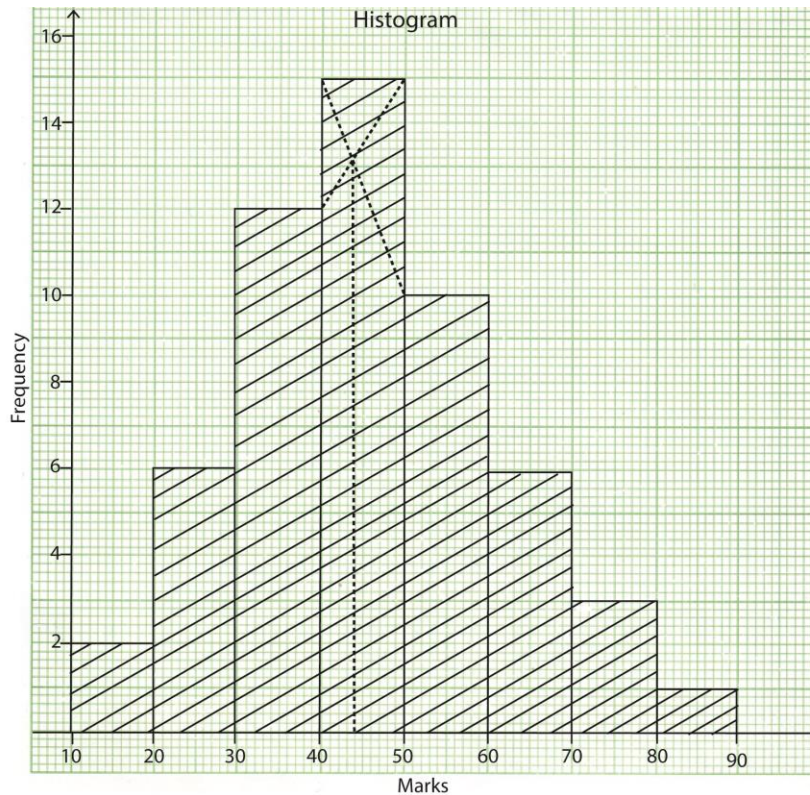
Answer any four questions

All questions carry equal marks

9. The table below shows a frequency distribution of marks scored by 55 students in a test.

Marks	10 -	20 -	30 -	40 -	50 -	60 -	70 -	80 - ≤90
Number of students	2	6	12	15	10	6	3	1

(a) Draw a histogram for the data and use it to estimate the modal mark. (05marks)



(b) Calculate the

Marks	x	f	fx	fx ²
10 - 20	15	2	30	450
20 - 30	25	6	150	3750
30 - 40	35	12	420	14700
40 - 50	45	15	675	30375
50 - 60	55	10	550	30250
60 - 70	65	6	390	25350
70 - 80	75	3	225	16875
80 - 90	85	1	85	7225
		$\Sigma f = 55$	$\Sigma fx = 2525$	$\Sigma fx^2 = 128975$

(i) mean mark

$$\text{Means, } \bar{x} = \frac{\sum fx}{\sum f} = \frac{2525}{55} = 45.91$$

(ii) standard deviation (10marks)

$$\text{S.D} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2} = \sqrt{\frac{128975}{55} - (45.91)^2} = 15.4$$

10. Chemical A is converted into another chemical by a chemical reaction. The rate at which a chemical A is being converted is directly proportional to the amount present at any time. Initially 100g of chemical A was present. After 5 minutes, 90g of A is present.

(a) Form a differential equation for chemical reaction. (03marks)

Let the amount present at time $t = x$

$$\frac{dx}{dt} = -kx$$

(b) By solving the differential equation formed in (a), determine the

(i) amount of chemical A present after 20 minutes.

$$\int \frac{dx}{x} = -k \int dt$$

$$\ln x = -kt + c$$

$$\text{At } t=0, x = 100 \Rightarrow c = \ln 100$$

$$\text{At } t = 5$$

$$\ln 90 = -k \times 5 + \ln 100$$

$$k = \frac{1}{5} \ln \frac{100}{90} = 0.021 \text{min}^{-1}$$

$$\Rightarrow \ln \frac{100}{x} = 0.021t$$

$$\text{At } t = 20$$

$$\ln \frac{100}{x} = 0.021 \times 20 = 0.42$$

$$\frac{100}{x} = 1.522 ; = 65.7$$

Hence the amount present after 20 minutes = 65.7g

(ii) time taken for the amount of chemical A to be reduced to 20 g (12 marks)

$$\begin{aligned} \text{From } \ln \frac{100}{x} &= 0.021t \\ \Rightarrow \ln \frac{100}{20} &= 0.021t \\ t &= 76.64 \text{ minutes} \end{aligned}$$

11. The table below shows the prices in US dollars and weights of five components of an engine, in 1998 and 2005.

COMPONENT	A	B	C	D	E
PRICE (\$) 1998	35	70	43	180	480
PRICE (\$) 2005	60	135	105	290	800
WEIGHT	6	5	3	2	1

Taking 1998 as the base year

(a) Calculate for 2005 the:

(i) simple aggregate price index. (03marks)

$$\text{S.A.P.I} = \frac{\sum P_{2005}}{\sum P_{1998}} \times 100 = \frac{60+135+105+290+800}{35+70+43+180+480} \times 100 = \frac{1390}{808} \times 100 = 172$$

(ii) price relative of each component. (03marks)

$$\text{Price relative} = \frac{P_{2005}}{P_{1998}} \times 100$$

$$\text{Price relative for A} = \frac{60}{35} \times 100 = 171.4$$

$$\text{Price relative for B} = \frac{135}{70} \times 100 = 192.9$$

$$\text{Price relative for C} = \frac{105}{43} \times 100 = 244.2$$

$$\text{Price relative for D} = \frac{290}{180} \times 100 = 161.1$$

$$\text{Price relative for E} = \frac{800}{480} \times 100 = 166.7$$

(iii) weighted aggregate price index. (06marks)

$$\text{W.A.P.I} = \frac{\sum wP_{2005}}{\sum wP_{1998}} \times 100 = \frac{60 \times 6 + 135 \times 5 + 105 \times 3 + 290 \times 2 + 800 \times 1}{35 \times 6 + 70 \times 5 + 43 \times 3 + 180 \times 2 + 480 \times 1} \times 100 = \frac{2730}{1529} \times 100 = 178.5$$

(b) Estimate the cost of an engine in 1998 given that its cost in 2005 was 1600 US dollars

Method 1: using S.A.P.I

Let x be the cost

$$\frac{1600}{x} \times 100 = 172; x = 930.2 \text{ US dollars}$$

Method 2 Using W.A.P.I

$$\frac{1600}{x} \times 100 = 178.5; x = 896.4 \text{ US dollars}$$

12. (a) Solve the equation $1 + \cos\theta = 2\sin^2\theta$ for values of θ between 0° and 360° . (09marks)

$$1 + \cos\theta = 2(1 - \cos^2\theta)$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\text{Either } \theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ, 300^\circ$$

$$\text{Or } \theta = \cos^{-1}(-1) = 180^\circ$$

$$\text{Hence } \theta = 60^\circ, 180^\circ, 300^\circ$$

- (b) By eliminating θ from the equation $x = a\sec\theta$ and $y = b + c\cos\theta$,

show that $x(y - b) = ca$. (06marks)

$$\sec\theta = \frac{x}{a} \Rightarrow \cos\theta = \frac{a}{x} = \frac{y-b}{c}$$

by cross multiplication

$$x(y - b) = ca$$

13. A random variable X has a probability density function, $f(x)$, defined by

$$f(x) = \begin{cases} kx(x + 2), & 0 \leq x \leq 2, \\ 0, & \text{otherwise} \end{cases}$$

where k is constant

Determine

- (a) value of k (04marks)

$$\int f(x) dx = 1$$

$$\Rightarrow k \int_0^2 (x^2 + 2x) dx = 1$$

$$k \left[\frac{x^3}{3} + x^2 \right]_0^2 = 1$$

$$k \left(\frac{8}{3} + 4 \right) = 1 \Rightarrow k = \frac{3}{20}$$

(b) $P(1 \leq X \leq 1.5)$ (03marks)

$$P(1 \leq X \leq 1.5) = \frac{3}{20} \left[\frac{x^3}{3} + x^2 \right]_1^2 = \frac{3}{20} \left[\left(\frac{8}{3} + 4 \right) - \left(\frac{1}{3} + 1 \right) \right] = 0.306$$

(c) Expectation, $E(X)$ (04marks)

$$\begin{aligned} E(X) &= \int x f(x) dx = \frac{3}{20} \int_0^2 (x^3 + 2x^2) dx \\ &= \frac{3}{20} \left[\frac{x^4}{4} + \frac{2x^3}{3} \right]_0^2 \\ &= \frac{3}{20} \left(\frac{16}{4} + \frac{16}{3} \right) \\ &= 1.4 \end{aligned}$$

(d) Variance, $\text{Var}(X)$ (04marks)

$$\begin{aligned} E(X^2) &= \int x^2 f(x) dx = \frac{3}{20} \int_0^2 (x^4 + 2x^3) dx \\ &= \frac{3}{20} \left[\frac{x^5}{5} + \frac{2x^4}{4} \right]_0^2 \\ &= \frac{3}{20} \left(\frac{32}{5} + \frac{32}{4} \right) \\ &= 2.16 \end{aligned}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - (E(X))^2 \\ &= 2.16 - (1.4)^2 = 0.2 \end{aligned}$$

14. A car initially at rest accelerated uniformly to a speed of 20ms^{-1} in 16 seconds. The car then travelled at the attained speed for 2 minutes. The car then accelerated uniformly at 2.5ms^{-2} for 8 seconds. It finally decelerated at 2.5ms^{-2} to rest.

(a) Find the

(i) greatest speed attained by the car.

$$v = u + at = 20 + 2.5 \times 8 = 40\text{ms}^{-1}$$

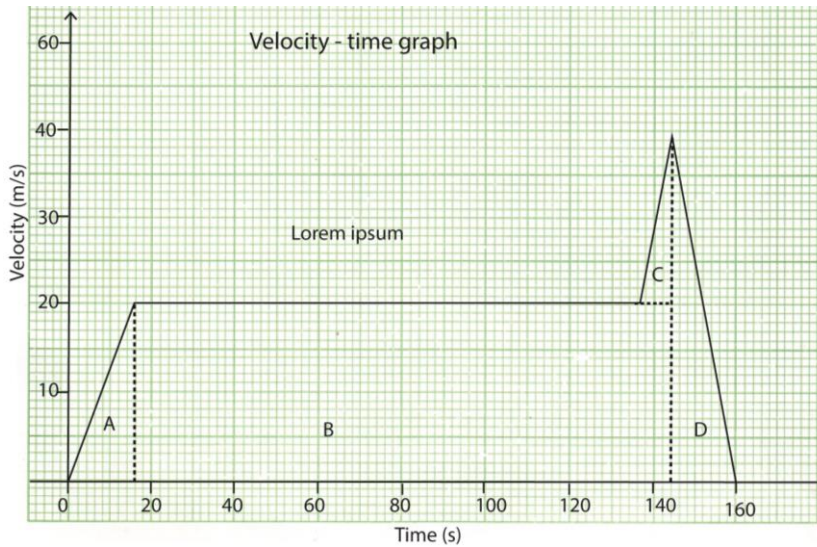
(ii) total time taken by the car to come to rest. (06marks)

Let time taken during deceleration be t

$$0 = 40 - 2.5 t \Rightarrow t = 16\text{s}$$

$$\text{Total time for the journey} = 16 + 120 + 8 + 16 = 160\text{s}$$

(b) Sketch velocity – time graph for the motion of the car. (04 marks)



(c) Use your graph to find the total distance travelled by the car. (05 marks)

Total distance travelled = sum of areas A+ B +C + D

$$\begin{aligned}
 &= \frac{1}{2} \times 16 \times 20 + 128 \times 20 + \frac{1}{2} \times 8 \times 20 + \frac{1}{2} \times 16 \times 40 \\
 &= 160 + 2,560 + 80 + 320 \\
 &= 3,120\text{m}
 \end{aligned}$$

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Thanks

Dr. Bbosa Science