UACE S475 Sub math paper 1 2017

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, g = 9.8ms^{-2}.

SECTION A (40 MARKS)

Answer all the questions in this section

1. Given that $\log_3 x = 2\log_3 4 - \log_3 5 + \log_3 9$, find the value of x. (05marks)

2. A father and a mother with their five children are to sit on a bench. What is the probability that the father and mother will sit next to each other? (05marks)

3. The vector a = 3i + 2j and b = 4i – 5j

   Determine

   (a) $|b|$
   (b) A.b (05marks)

4. The table below shows the expenditure in shillings of University student for the years 2005 and 2006.

ITEM	EXPENDITURE(Shs)		WEIGHT
	2005	2005	
Text books	100,000	120,000	3
Pocket money	50,000	70,000	2
Research	40,000	50,000	1

Using the year 2005 as the base, calculate the weighted aggregate price index. (05marks)

5. Solve the differential equation

   $3y\frac{dy}{dx} = \frac{1}{x^2}$ given that y = 2 when x = 1 (05 marks)

6. It was observed that 3 seeds in every four seeds planted germinate. If 16 seeds were planted, calculate

   (i) expected number of seeds that will germinate (03marks)

   (ii) probability that exactly 14 seeds will germinate (02marks)

7. Using the matric method, solve the simultaneous equations.

   3x – y = 16

   x + 2y = 3 (05marks)

8. A man of mass 9kg carries a 50kg bag of cement for a distance of 7m up a slope. The slope is inclined an angle 30^0 to the horizontal.

   (a) Find the work done against gravity. (03marks)

   (b) The man took 42 second to do the work. Calculate the power developed. (02mark)

SECTION B (60 MARKS)

Answer any **four** questions from this section

All questions carry equal marks

9. The table below shows the weights in kg of 50 cattle on a farm

60	81	76	68	84	112	76	102	86	67
65	98	107	110	72	99	87	92	76	77
94	102	87	86	73	118	98	120	62	87
65	92	104	116	91	93	78	122	102	92
80	111	73	120	106	123	94	109	80	96

   (a) Form a grouped frequency table for the data with classes of equal intervals, starting with the class 60 – 69. (06 marks)

   (b) Draw a cumulative frequency curve (Ogive) for the given data. (04 marks)

   (c) Use your Ogive to estimate the;

      (i) lower and upper quartile

      (ii) median weight

      (iii) number of cattle which weigh 118kg and above. (05 marks)

10. A particle moves with velocity $V = 2t^2 - 9t + 10$ where t is time.

    The particle is at the origin when t = 0. Determine the

    (a) expression for distance and acceleration in terms of t (07marks)

    (b) distance of the particle from the origin when the particle is at rest. (08 marks)

11. The table below shows quarterly sales of cars for the year 2000, 2001 and 2002 by a company.

YEAR	QUARTER			
	1st	2nd	3rd	4th
2000	390	310	280	355
2001	420	320	305	410
2002	400	350	315	425

    (a) Calculate a four point moving average for the data. (06marks)

    (b) (i) Plot a four – point moving average for the data on the same axes. (06marks)

    (ii) Comment on the trend of the sales of the cars. (01 mark)

    (iii) Use your graph to estimate the number of cars sold in the first quarter of 2003. (02marks)

12. (a) Triangle OAB is such that Angle AOB =900, angle ABO = θ, $\overline{OB}$ = 14.4cm and $\overline{OA}$ = 6cm.

    Find sinθ + cotθ (07marks)

    (b) Solve $2\cos^2 x = \sin x + 1$ for $0^0 \leq x \leq 360^0$. (08marks)

13. A cumulative random variable X, has a probability density function (pdf) given by

$$f(x) = \begin{cases} k(x^2 + 6), & 0 \leq x \leq 3 \\ 0, & otherwise \end{cases}$$

    Where k is a constant

    Determine the:

    (i) value of k (04 marks)

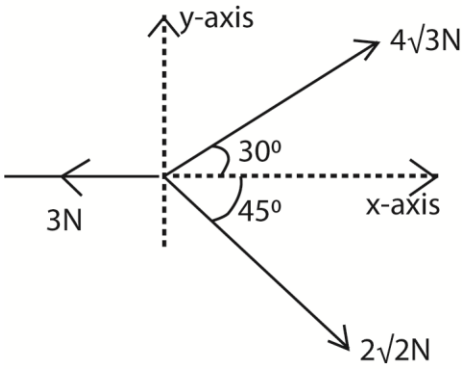
    (ii) P(X> 1). (04 marks)

    (iii) expectation, E(X) (03marks)

    (iv) variance, Var (X)(04marks)

14. (a) Forces P = 10N and Q = 4N act away from point A. The magnitude of their resultant is 8N. Find the angle between P and Q. (05marks)

(b) The diagram below shows three forces 3N, $4\sqrt{3}$N and $2\sqrt{2}$N acting on a particle at the origin.



Calculate the,

(i) magnitude of the resultant force

(ii) angle the resultant force makes with x-axis. (10 marks)

Proposed answers

1. Given that $\log_3 x = 2\log_3 4 - \log_3 5 + \log_3 9$, find the value of x. (05marks)

   $\log_3 x = \log_3 \frac{4^2 x 9}{5} = \log_3 \frac{16x9}{5} = \log_3 28.8$

   Comparing both sides

   x = 28

2. A father and a mother with their five children are to sit on a bench. What is the probability that the father and mother will sit next to each other? (05marks)

   Solution

   Total member in a family = 7

   The number of ways of sitting on the bench = 7! = 5040

   If the father and mother are seated next to each other, they can be treated as couple.

   Number of ways = 6! = 720

   But the father and mother can sit in two ways i.e. Father-mother or Mother – father

   Therefore the total number of ways in which mother and father can sit next to another

   = 6! X 2 = 1440

   P(mother and father and mother seated next to each other) = $\frac{1440}{5040} = \frac{2}{7}$

   If the father and mother are seated next to each other, they can sit as father – mother or

3. The vector a = 3i + 2j and b = 4i – 5j

   Determine

   (a) $|b| = \sqrt{4^2 + 5^2} = 6.403$

   (b) A.b (05marks)

   (3 x 4)+ (2 x -5) = 12 – 10 = 2

4. The table below shows the expenditure in shillings of University student for the years 2005 and 2006.

ITEM	EXPENDITURE(Shs)		WEIGHT
	2005	2006	
Text books	100,000	120,000	3
Pocket money	50,000	70,000	2
Research	40,000	50,000	1

Using the year 2005 as the base, calculate the weighted aggregate price index. (05marks)

Weighted aggregate index $= \frac{\sum wP_{2006}}{\sum wP_{2005}} \ x\ 100$

$= \frac{3\ x\ 120{,}000+2\ x\ 70{,}000+1\ x\ 50{,}000}{3\ x\ 100{,}000+2\ x\ 50{,}000+1\ x\ 40{,}000} x100$

$= \frac{550{,}000}{440{,}000} x100$

= 125

5. Solve the differential equation

$3y\frac{dy}{dx} = \frac{1}{x^2}$ given that y = 2 when x = 1 (05 marks)

$3ydy = x^{-2}dx$

$\int 3ydy = \int x^{-2}dx$

$\frac{3y^2}{2} = -\frac{1}{x} + c$.....(i)

Substituting for y = 2 and x = 1 in equation (i)

$\frac{3(2)^2}{2} = -\frac{1}{1} + c$ => =7

Substituting for c = 7 in equation (i)

$\frac{3y^2}{2} = -\frac{1}{x} + 7$

6. It was observed that 3 seeds in every four seeds planted germinate. If 16 seeds were planted, calculate

(i) expected number of seeds that will germinate (03marks)

Given p = $\frac{3}{4}$, q = $\frac{1}{4}$, n = 16

Let X be the random variable that the seed germinates

X~B(16, $\frac{3}{4}$)

E(x) = np = 16 x $\frac{3}{4}$ = 12 seeds

(ii) probability that exactly 14 seeds will germinate (02marks)

P(X = 14) = 16C14$\left(\frac{3}{4}\right)^{14}\left(\frac{1}{4}\right)^{2}$ = 0.1336

7. Using the matric method, solve the simultaneous equations.

3x – y = 16

x + 2y = 3 (05marks)

Solution

$\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}16\\ 3\end{pmatrix}$

Let A = $\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}$

Det(A) = (3 x 2) – (-1 x 1) = 6 + 1 = 7

A-1 =$\frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}$

Pre-multiply both side with the inverse

$\frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}3 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{7}\begin{pmatrix}2 & 1\\ -1 & 3\end{pmatrix}\begin{pmatrix}16\\ 3\end{pmatrix}$

$\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{7}\begin{pmatrix}2\,x16 & 1x3\\ -1\,x16 & 3x3\end{pmatrix}$

$\begin{pmatrix}x\\ y\end{pmatrix}=\frac{1}{7}\begin{pmatrix}35\\ -7\end{pmatrix}$

$\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}5\\ -1\end{pmatrix}$

Hence x = 5 and y = -1

8. A man of mass 9kg carries a 50kg bag of cement for a distance of 7m up a slope. The slope is inclined an angle 30^0 to the horizontal.

(a) Find the work done against gravity. (03marks)

w = F x d = 50 x 9.8 x 7 = 3,430J

(b) The man took 42 second to do the work. Calculate the power developed. (02mark)

P = $\frac{W}{t}=\frac{3,430}{42}$ = 81.6W

SECTION B (60 MARKS)

Answer any **four** questions from this section

All questions carry equal marks

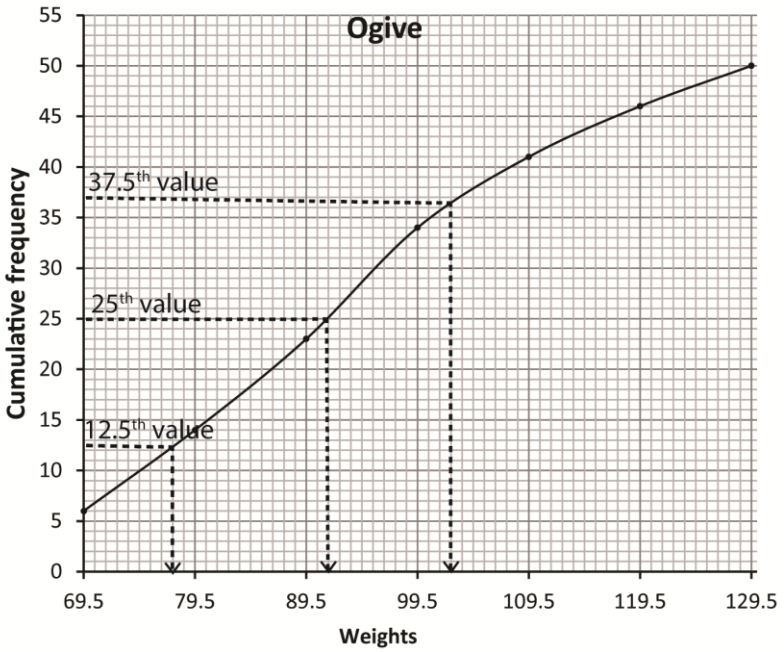
9. The table below shows the weights in kg of 50 cattle on a farm

60	81	76	68	84	112	76	102	86	67
65	98	107	110	72	99	87	92	76	77
94	102	87	86	73	118	98	120	62	87
65	92	104	116	91	93	78	122	102	92
80	111	73	120	106	123	94	109	80	96

(a) Form a grouped frequency table for the data with classes of equal intervals, starting with the class 60 – 69. (06 marks)

Classes	Class boundaries	Frequency, f	Cumulative frequency, CF
60 – 69	59.5 – 69.5	6	6
70 – 79	69.5 – 79.5	8	14
80 – 89	79.5 – 89.5	9	23
90 – 99	89.5 – 99.5	11	34
100 – 109	99.5 – 109.5	7	41
110 – 119	109.5 – 119.5	5	46
120 – 129	119.5 – 129.5	4	50

(b) Draw a cumulative frequency curve (Ogive) for the given data. (04 marks)



Note that CF is plotted against the upper limit value of the class

(c) Use your Ogive to estimate the;

(i) lower and upper quartile

Lower quartile, $q_1 = \left(\frac{N}{4}\right)^{th} = \left(\frac{50}{4}\right)^{th} = 12.5^{th} value = 69.5 + 8 = 77.5$

(ii) median weight

Median = $\left(\frac{N}{2}\right)^{th} = \left(\frac{50}{2}\right)^{th} = 25^{th} value = 89.5 + 2 = 91.5$

(iii) number of cattle which weigh 118kg and above. (05 marks)

Upper quartile, $q_3 = \left(\frac{3N}{4}\right)^{th} = \left(\frac{3x50}{4}\right)^{th} = 37.5^{th} value = 99.5 + 3 = 102.5$

10. A particle moves with velocity $V = 2t^2 - 9t + 10$ where t is time.

The particle is at the origin when t = 0. Determine the

(a) expression for distance and acceleration in terms of t (07marks)

Distance = $\int v dt = \int (2t^2 - 9t + 10)\, dt = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 10t + c$

Acceleration a = $\frac{dv}{dt} = \frac{d}{dt}(2t^2 - 9t + 10) = 4t - 9$

(b) distance of the particle from the origin when the particle is at rest. (08 marks)

$s = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 10t + c$

At t = 0, s =0, c= 0

$\therefore s = = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 10t$

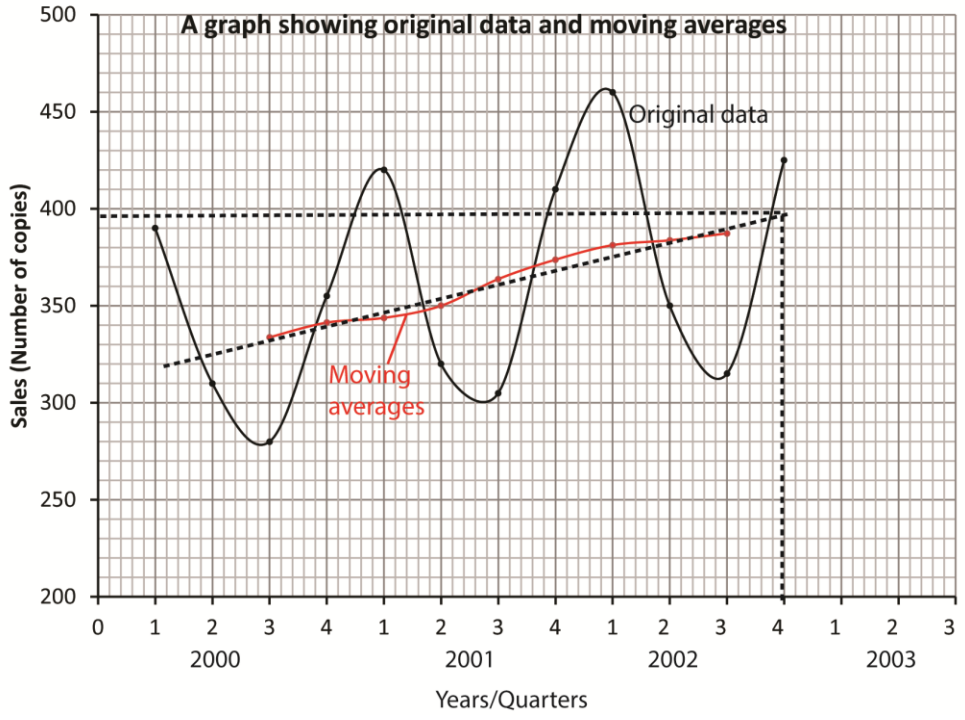
11. The table below shows quarterly sales of cars for the year 2000, 2001 and 2002 by a company.

YEAR	QUARTER			
	1st	2nd	3rd	4th
2000	390	310	280	355
2001	420	320	305	410
2002	400	350	315	425

(a) Calculate a four point moving average for the data. (06marks)

Year	Quarter	Sales	Moving totals	Moving averages
2000	1	390		
	2	310		
	3	280	1335	333.75
	4	355	1365	341.35
2001	1	420	1375	343.75
	2	320	1400	350
	3	305	1455	363.75
	4	410	1495	373.75
2002	1	460	1525	381.25
	2	350	1535	383.75
	3	315	1550	387.3
	4	425		
2003	1	x		

(b) (i) Plot a four – point moving average for the data on the same axes. (06marks)



(ii) Comment on the trend of the sales of the cars. (01 mark)

There is a general increase in the sale of cars with the years

(iii) Use your graph to estimate the number of cars sold in the first quarter of 2003. (02marks)

The estimated 10^{th} moving average = 395 cars

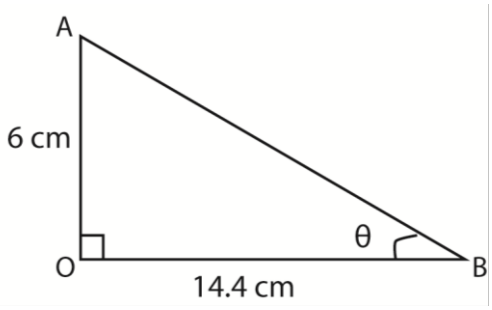
Let the number of cars sold in the 1^{st} quarter of 2003 be x

$$\frac{x+425+315+350}{4} = 395$$

x + 1,115 = 395 x 4 =1580

x = 490

12. (a) Triangle OAB is such that Angle AOB =90^0, angle ABO = θ, $\overline{OB}$ = 14.4cm and $\overline{OA}$ = 6cm.



Find sinθ + cotθ (07marks)

tan θ = $\frac{6}{14.4}$ => θ = $\tan^{-1}\frac{6}{14.4}$ = 22.62^0

sinθ + cotθ = sin 22.62 + cot 22.62

= 0.3846 + 2.4000

= 3.7846

(c) Solve $2\cos^2 x = \sin x + 1$ for $0^0 \le x \le 360^0$. (08marks)

$2(1- \sin^2 x) = \sin x + 1$

$2\sin^2 x - \sin x - 1 = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

Either $\sin x + 1 = 0$

$\sin x = -1$; $x = 270^0$

Or

$2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$; $x = 30^0, 150^0$

$x = (30^0, 150^0, 270^0)$

13. A cumulative random variable X, has a probability density function (pdf) given by

$$f(x) = \begin{cases} k(x^2+6), & 0 \le x \le 3 \\ 0, & otherwise \end{cases}$$

Where k is a constant

Determine the:

(i) value of k (04 marks)

$\int_0^3 f(x)dx = 1$

$k\int_0^3 (x^2+6)dx = 1$

$k\left[\frac{x^3}{3}+6x\right]_0^3 = 1$

$k\left(\frac{3^3}{3}+6x3\right) = 1$

$27k = 1$

$k = \frac{1}{27}$

(ii) P(X> 1). (04 marks)

$\frac{1}{27}\left[\frac{x^3}{3}+6x\right]_1^3 = \frac{1}{27}\left[27-6\frac{1}{3}\right] = \frac{62}{81} = 0.7654$

(iii) expectation, E(X) (03marks)

$E(X) = \frac{1}{27}\int_0^3 xf(x)dx$

$= \frac{1}{27}\int_0^3 x(x^2+6)dx = \int_0^3 (x^3+6x)$

$= \frac{1}{27}\left[\frac{x^4}{4}+3x^2\right]_0^3 = \frac{1}{27}\left(\frac{81}{4}+27\right)$

$= 1.75$

(iv) variance, Var (X)(04marks)

$E(X^2) = \frac{1}{27}\int_0^3 x^2 f(x)dx$

$= \frac{1}{27}\int_0^3 x^2(x^2+6)dx = \int_0^3 (x^4+6x^2)$

$= \frac{1}{27}\left[\frac{x^5}{5}+2x^3\right]_0^3 = \frac{1}{27}\left(\frac{243}{5}+54\right)$

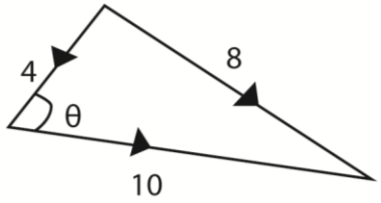
$= 3.8$

$Var(X) = E(X^2) - (E(x))^2$

$= 3.8 - (1.75)^2$

$= 0.7375$

14. (a) Forces P = 10N and Q = 4N act away from point A. The magnitude of their resultant is 8N. Find the angle between P and Q. (05marks)



Using: $\boldsymbol{b}^2 = \boldsymbol{a}^2 + \boldsymbol{c}^2 - 2\boldsymbol{ac}\cos B$

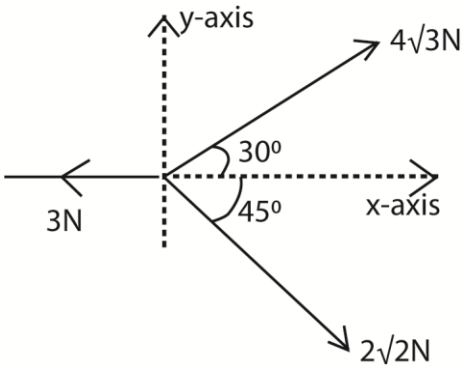
$8^2 = 4^2 + 10^2 - 2 \times 4 \times 10\cos\theta$

$80\cos\theta = 116 - 64 = 52$

$\cos\theta = \frac{52}{80} = 0.56$

$\theta = 49.5^0$

(b) The diagram below shows three forces 3N, $4\sqrt{3}$N and $2\sqrt{2}$N acting on a particle at the origin.



Calculate the,

(i) magnitude of the resultant force

Horizontal component of the forces = $4\sqrt{3}\cos 30 - 3 + 2\sqrt{2}\cos 45 = 5$

Vertical component of the forces $= 4\sqrt{3}\sin 30 - 2\sqrt{2}\sin 45 = 4.93$

Resultant force $= \sqrt{5^2 + (4.93)^2} = 7.0$N

(ii) angle the resultant force makes with x-axis. (10 marks)

Let the angle be x

$$\tan x = \frac{4.93}{5}$$

$$x = \tan^{-1}\frac{4.93}{5} = 44.6^0$$

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Thanks

Dr. Bbosa Science