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UACE S475 Sub math paper 1 2018

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, $g = 9.8\text{ms}^{-2}$.

SECTION A (40 MARKS)

Answer all the questions in this section

- The root of the equation $4x^2 + 9x - k = 0$ are α and 2. Find the values of α and k. (05marks)
- A random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} \frac{x}{10}, & x = 1, 2, 3, 4 \\ 0 & \text{elsewher} \end{cases}$$

Calculate:

- $P(1 \leq X < 3)$. (03marks)
 - the mean of X, $E(X)$ (02 marks)
- Show that $\frac{1 - \cos^2 \theta}{\sec^2 - 1} = \cos^2 \theta$

Hence, solve the equation $\frac{1 - \cos^2 \theta}{\sec^2 - 1} = \frac{3}{4}$ for $0^\circ \leq \theta \leq 90^\circ$. (05 marks)

- Events A and B are such that $P(A) = \frac{6}{13}$, $P(B) = \frac{2}{5}$ and $P(A/B) = \frac{1}{4}$

Find:

- $P(A \cap B)$ (02marks)

- (b) $P(A \cup B)$ (03marks)
5. Express $\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}}$ in form $b\sqrt{c}$ where b and c are integers. (05 marks)
6. The marks scored in at test by 8 students are: 5, 9, 11, 15, 19, 15, 10, 14.
Determine the;
- (a) mean mark. (02 marks)
- (b) variance. (03 marks)
7. Evaluate $\int_{-1}^2 \frac{2x^4 - x^5}{x^2} dx$ (05 marks)
8. A force of 65N is inclined at an angle of θ to horizontal. The horizontal component of the force is 25N.
Calculate the
- (a) angle θ (03marks)
- (b) vertical component of the force. (02marks)

SECTION B (60MARKS)

Answer any **four** questions from this section

All questions carry the same marks

9. The table shows scores by 10 students (A to J) in Physics and Mathematics tests

Students	A	B	C	D	E	F	G	H	I	J
Mathematics (x)	28	20	40	28	21	31	36	29	33	24
Physics (y)	30	20	40	28	22	35	35	27	31	23

- (a) (i) Plot a scatter diaram for the given data.
- (ii) Draw a line of the best fit on the scatter diagram
- (iii) Estimate the score in Mathematics for a student who scores 37 in physics. (08marks)
- (b) Calculate the rank correlation coefficient for the data and comment on your result. (07marks)
10. Point A, Band C have position vectors, $2j$, $4i$, and $2i - 2j$ respectively in the $x - y$ plane.
- (a) Find $2OA + 3OB - 4OC$. (04 marks)
- (b) Determine;
- (i) AB and AC (04marks)

(ii) $AB + AC$ (03mark)

(iii) angle BAC (05 marks)

11. A factory sells animals food in bags. The weights of the bags are normally distributed with mean weight 50kg and standard deviation 2.8kg.

(a) Find the probability that the weight of any bag selected at random;

(i) is more than 53kg (04marks)

(ii) lies between 46 and 55kg (05marks)

(b) Determine the percentage of bags whose weights are less than 54kg. (06marks)

12. The equation of a curve is $Y = 3x^2 + 2$

(a) (i) Determine the turning point of the curve

(ii) Find the nature of the turning point

(iii) Sketch the graph of the curve. (07marks)

(b) The curve and the line $y = 14$ intercepts at the point $(-2, 14)$ and $(2, 14)$. Calculate the area of the region enclosed between the line and the curve. (08 marks)

13. The table shows the sales in thousands of copies by a local Newspaper over a period of 12 weeks.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Number of copies sold	135	378	490	430	510	580	565	595	640	660	628	670

(a) Calculate the 3-weeks moving averages for the copies sold. (06marks)

(b) (i) On the same axes, plot the original data and the 3-weeks moving averages. (06marks)

(ii) Use your graphs to estimate the number of copies sold in the 13th week. (03marks)

14. A body of mass 4kg is initially at a point P whose position vector is $(3i + 4j)m$. A constant Force $F = (8i + 4j)N$ acts on the body causing it to move. The body passes through another point Q after 4 seconds.

Find the;

(a) acceleration of the body (02marks)

(b) velocity of the body as it passes through Q. (03marks)

(c) Kinetic energy of the body after the 4 seconds. (04marks)

(d) Distance between the points P and Q (06marks)

Proposed answers

1. The root of the equation $4x^2 + 9x - k = 0$ are α and 2. Find the values of α and k . (05marks)

$$2 + \alpha = -\frac{9}{4}$$

$$\alpha = -\frac{9}{4} - 2 = -\frac{17}{4} = -4\frac{1}{4}$$

$$2\alpha = -\frac{k}{2}$$

$$2x - \frac{17}{4} = -\frac{k}{2}$$

$$-\frac{k}{2} = -\frac{34}{4}$$

$$k = 34$$

2. A random variable X has a probability distribution given by

$$P(X = x) = \begin{cases} \frac{x}{10}, & x = 1, 2, 3, 4 \\ 0 & \text{elsewhere} \end{cases}$$

Calculate:

- (a) $P(1 \leq X < 3)$. (03marks)

$$P(1 \leq X < 3) = P(X=1) + P(X=2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10} = 0.3$$

- (b) the mean of X , $E(X)$ (02 marks)

X	1	2	3	4
$X=x$	0.1	0.2	0.3	0.4
$x(P(X=x))$	0.1	0.4	0.9	1.6

$$E(X) = 0.1 + 0.4 + 0.9 + 1.6 = 3$$

3. Show that $\frac{1-\cos^2\theta}{\sec^2-1} = \cos^2\theta$

$$\frac{1-\cos^2\theta}{\sec^2-1} = \frac{\sin^2\theta}{\frac{1}{\tan^2\theta}} = \sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} = \cos^2\theta \text{ as required}$$

Hence, solve the equation $\frac{1-\cos^2\theta}{\sec^2-1} = \frac{3}{4}$ for $0^\circ \leq \theta \leq 90^\circ$. (05 marks)

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \sqrt{\frac{3}{4}} = \pm 0.866$$

For $\cos\theta = 0.866$; $\cos^{-1}(0.866) = 30^\circ, 330^\circ$

For $\cos\theta = -0.866$; $\cos^{-1}(-0.866) = 150^\circ, 210^\circ$

Hence $\theta = 30^\circ$ for $0^\circ \leq \theta \leq 90^\circ$

4. Events A and B are such that $P(A) = \frac{6}{13}$, $P(B) = \frac{2}{5}$ and $P(A/B) = \frac{1}{4}$

Find:

- (a) $P(A \cap B)$ (02 marks)

$$P(A \cap B) = P(B) \cdot P(A/B) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

- (b) $P(A \cup B)$ (03 marks)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{6}{13} + \frac{2}{5} - \frac{1}{10} = \frac{99}{130}$$

5. Express $\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}}$ in form $b\sqrt{c}$ where b and c are integers. (05 marks)

$$\frac{4}{\sqrt{3}+\sqrt{2}} + \frac{4}{\sqrt{3}-\sqrt{2}} = \frac{4(\sqrt{3}-\sqrt{2})+4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{8\sqrt{3}}{1}$$

Hence $b = 8$ and $c = 3$

6. The marks scored in at test by 8 students are: 5, 9, 11, 15, 19, 15, 10, 14.

Determine the;

Use of a table

x	f	fx	x^2	fx^2
5	1	5	25	25
9	1	9	81	81
10	1	10	100	100
11	1	11	121	121
14	1	14	196	196
15	2	30	225	450
19	1	19	361	361
	$\sum f = 8$	$\sum fx = 98$		$\sum fx^2 = 1,334$

- (a) mean mark. (02 marks)

$$\text{Mean} = \frac{\sum fx}{\sum f} = \frac{98}{8} = 12.25$$

- (b) variance. (03 marks)

$$\text{Var}(X) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2 = \frac{1334}{8} - (12.25)^2 = 16.69$$

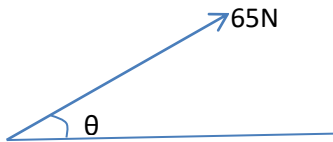
7. Evaluate $\int_{-1}^2 \frac{2x^4 - x^5}{x^5} dx$ (05 marks)

$$\begin{aligned} \int_{-1}^2 \frac{2x^4 - x^5}{x^5} dx &= \int_{-1}^2 (2x^{-1} - x) dx \\ &= \left[2x^0 - \frac{x^2}{2} \right]_{-1}^2 \\ &= \left(\frac{16}{2} - 4 \right) - \left(-\frac{2}{2} - \frac{1}{2} \right) \\ &= \frac{4}{2} + \frac{11}{2} = \frac{27}{2} = 13.5 \end{aligned}$$

8. A force of 65N is inclined at an angle of θ to horizontal. The horizontal component of the force is 25N.

Calculate the

Let the angle be θ



- (a) angle θ (03marks)

$$65 \cos \theta = 25$$

$$\theta = \cos^{-1} \frac{25}{65} = 67.38$$

- (b) vertical component of the force. (02marks)

$$\text{Vertical component of force} = 65 \sin 67.38 = 60 \text{N}$$

SECTION B (60MARKS)

Answer any **four** questions from this section

All questions carry the same marks

9. The table shows scores by 10 students (A to J) in Physics and Mathematics tests

Students	A	B	C	D	E	F	G	H	I	J
Mathematics (x)	28	20	40	28	21	31	36	29	33	24
Physics (y)	30	20	40	28	22	35	35	27	31	23

- (a) (i) Plot a scatter diagram for the given data.

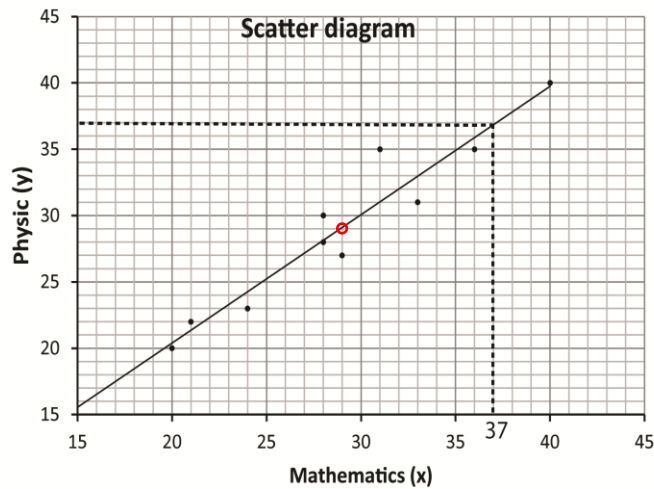
- (ii) Draw a line of the best fit on the scatter diagram

Note that the line of the best fit passes through the mean of x and the mean of y; these should be calculated to guide you when drawing a line of the best fit but should not be plotted

$$\bar{x} = \frac{28+20+40+28+21+31+36+29+33+24}{10} = 29$$

$$\bar{y} = \frac{30+20+40+28+22+35+35+27+31+23}{10} = 29.1$$

On the graph below the point (\bar{x}, \bar{y}) are encircled by red



(iii) Estimate the score in Mathematics for a student who scores 37 in physics. (08marks)

37

(b) Calculate the rank correlation coefficient for the data and comment on your result. (07marks)

Using spearman's rank correlation coefficient

Student	R _x	R _y	d	d ²
A	6.5	5	2.5	2.25
B	10	10	0	0
C	1	1	0	0
D	6.5	6	0.5	0.25
E	9	9	0	0
F	4	2.5	1.5	2.25
G	2	2.5	-0.5	0.25
H	5	7	-2	4
I	3	4	-1	1
J	8	8	0	0
SUM				10

$$\rho = 1 - \frac{6 \sum d^2}{n(n^2-1)} = 1 - \frac{6 \times 10}{10 \times 99} = 0.939$$

Comment: there is a high positive correlation between the marks of physics and mathematics

10. Point A, B and C have position vectors, $2j$, $4i$, and $2i - 2j$ respectively in the $x - y$ plane.

(a) Find $2OA + 3OB - 4OC$. (04 marks)

$$2 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + 3 \begin{pmatrix} 4 \\ 0 \end{pmatrix} - 4 \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 + 12 - 8 \\ 4 + 0 + 8 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \end{pmatrix}$$

(b) Determine;

(i) AB and AC (04marks)

$$AB = AO + OB = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$$

$$AC = AO + OC = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

(ii) AB + AC (03mark)

$$\begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix}$$

(iii) angle BAC (05 marks)

$$\text{Dot product of vector } BA = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \text{ and } AC = \begin{pmatrix} 2 \\ -4 \end{pmatrix} = (-4 \times 2) + (2 \times -4) = -16$$

Let the angle be θ

$$|BA||AC|\cos\theta = \text{dot product}$$

$$\sqrt{4^2 + (2)^2} \cdot \sqrt{(2)^2 + (-4)^2} \cos\theta = -16$$

$$20\cos\theta = -16$$

$$\theta = \cos^{-1}\left(\frac{16}{20}\right) = 143.1^\circ$$

11. A factory sells animals food in bags. The weights of the bags are normally distributed with mean weight 50kg and standard deviation 2.8kg.

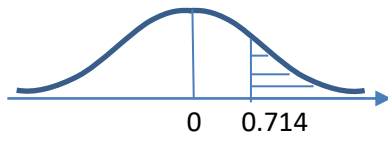
(a) Find the probability that the weight of any bag selected at random;

Let X = random variable for weight of bags

Given $\mu = 50$ and $\sigma = 2.8$ kg

(i) is more than 53kg (04marks)

$$P(X > 52) = P\left(Z > \frac{52-50}{2.8}\right) = P(Z > 0.714)$$



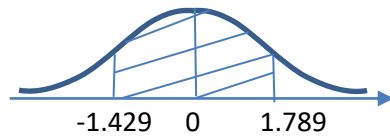
$$P(Z > 0.714) = 0.5 - P(0 < Z < 0.714)$$

$$= 0.5 - 0.2623$$

$$= 0.2377$$

(ii) lies between 46 and 55kg (05marks)

$$P(46 < X < 55) = P\left(\frac{46-50}{2.8} < Z < \frac{55-50}{2.8}\right)$$



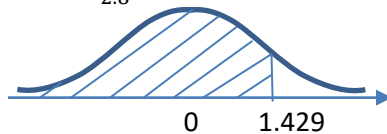
$$P(-1.429 < Z < 1.786) = P(0 < Z < 1.429) + P(0 < Z < 1.789)$$

$$= 0.4235 + 0.4630$$

$$= 0.8865$$

(b) Determine the percentage of bags whose weights are less than 54kg. (06marks)

$$P(X < 54) = P\left(Z < \frac{54-50}{2.8}\right) = P(Z < 1.429)$$



$$P(Z < 1.429) = 0.5 + P(0 < Z < 1.429)$$

$$= 0.5 + 0.4235 = 0.9235$$

12. The equation of a curve is $y = 3x^2 + 2$

(a) (i) Determine the turning point of the curve

$$\text{At turning point } \frac{dy}{dx} (3x^2 - 2) = 6x = 0$$

$$x = 0 \text{ and } y = 3(0)^2 + 2 = 2$$

Hence turning point = (0, 2)

(ii) Find the nature of the turning point

$$\frac{d^2y}{dx^2} = 6$$

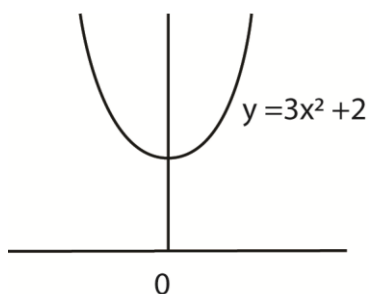
Since $\frac{d^2y}{dx^2} > 0$, the turning point is a minimum

(iii) Sketch the graph of the curve. (07marks)

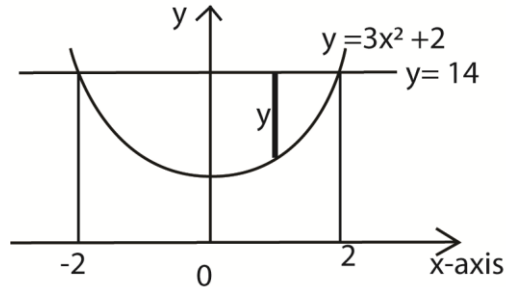
Finding x – intercepts when $y = 0$

$$3x^2 + 2 = 0, x^2 = -\frac{2}{3}$$

x	0	unidentified
y	2	0



(b) The curve and the line $y = 14$ intercepts at the point (-2, 14) and (2, 14). Calculate the area of the region enclosed between the line and the curve. (08 marks)



$$\text{A of the curve and x-axis} = \int_{-2}^2 y dx = \int_{-2}^2 (3x^2 + 2) = [x^3 + 2x]_{-2}^2 = (8 + 4) - (-8 - 4) = 24$$

$$\text{Total area of the rectangle} = 4 \times 14 = 56$$

$$\text{Area of the curve and the line } (y = 14)$$

$$56 - 24 = 36 \text{unit}^2$$

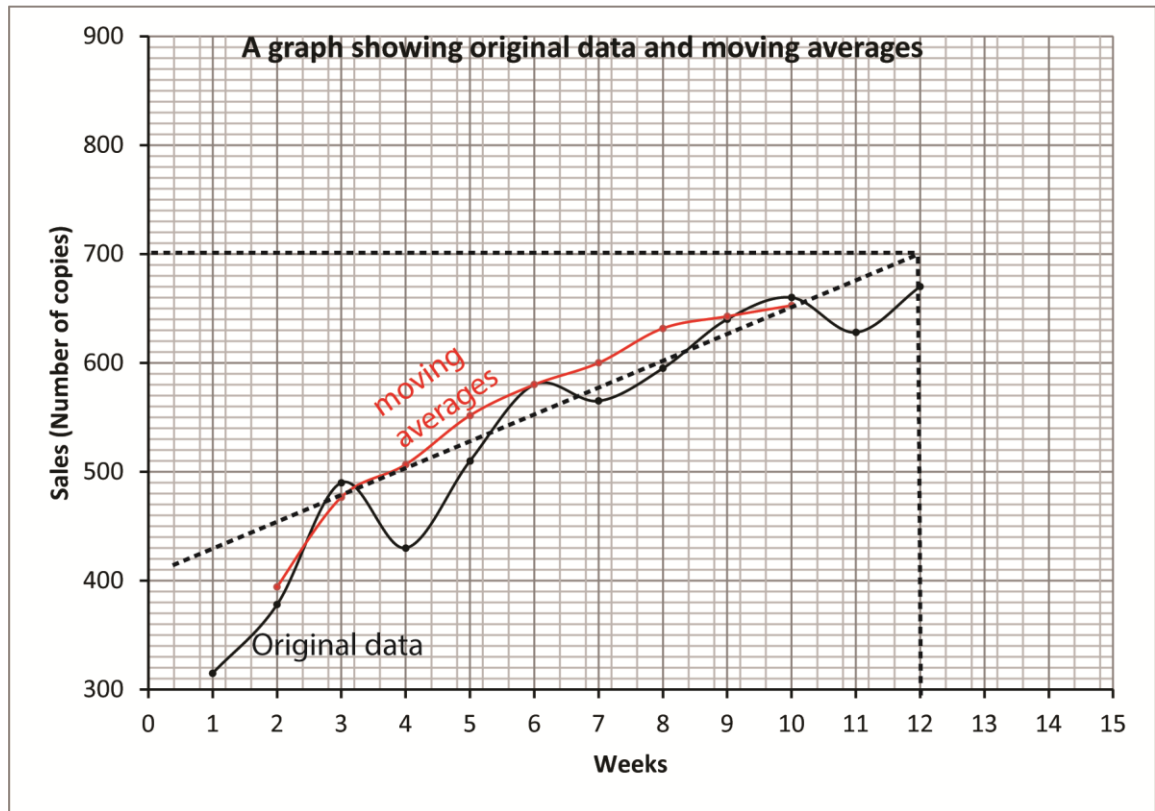
13. The table shows the sales in thousands of copies by a local Newspaper over a period of 12 weeks.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Number of copies sold	135	378	490	430	510	580	565	595	640	660	628	670

- (a) Calculate the 3-weeks moving averages for the copies sold. (06marks)

week	Number of copies	Moving totals	Moving averages
1	315		
2	378	1183	394.3
3	490	1298	476.7
3	430	1430	476.7
5	510	1520	506.7
6	580	1655	551.7
7	565	1740	580
8	595	1800	600
9	640	1895	631.7
10	660	1928	642.7
11	628	1958	652.7
12	670		
13	x		

- (b) (i) On the same axes, plot the original data and the 3-weeks moving averages. (06marks)



- (ii) Use your graphs to estimate the number of copies sold in the 13th week. (03marks)

From the graph the 11th moving average = 700

Let x be the number of copies sold in the 13th week

$$\frac{x+628+670}{3} = 720$$

$$x = (700 \times 3) - (628 + 670) = 802$$

14. A body of mass 4kg is initially at a point P whose position vector is $(3i + 4j)m$. A constant Force $F = (8i + 4j)N$ acts on the body causing it to move. The body passes through another point Q after 4 seconds.

Find the;

- (a) acceleration of the body (02marks)

$$\text{Magnitude of force} = \sqrt{8^2 + 4^2} = 8.94$$

$$F = ma$$

$$8.94 = 4a$$

$$a = 0.45\text{ms}^{-2}$$

- (b) velocity of the body as it passes through Q. (03marks)

$$v = u + at = 0 + 4 \times 0.45 = 1.8\text{ms}^{-1}$$

(c) Kinetic energy of the body after the 4 seconds. (04marks)

$$\text{K.E} = \frac{1}{2}mv^2 = \frac{1}{2} \times 4 \times 1.8^2 = 6.48\text{J}$$

(d) Distance between the points P and Q (06marks)

$$\text{From } s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2} \times 0.45 \times 4^2 = 3.6\text{m}$$

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Thanks

Dr. Bbosa Science