UACE S475 Sub math paper 1 2019

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, g = 9.8ms^{-2}.

SECTION A (40 MARKS)

Answer all the questions in this section

1. Show that $\sqrt{\dfrac{25^3 + 5^6}{5^7 - 5^6}} = \dfrac{\sqrt{2}}{2}$ (05marks)

2. Two events A and B are such that P(A) = $\dfrac{19}{30}$, P(B) = $\dfrac{2}{5}$ and P(A∩B') = $\dfrac{2}{5}$

   Find:

   (i) P(A∩B) (03marks)

   (ii) P(AUB) (02 marks)

3. Determine the possible values of α for which the equation

   2x2 + (α + 2)x + (a + 2) = 0 has equal roots (05marks)

4. A random variable X has the probability distribution shown in the table below

x	0	1	2	3	4	5
P(X = x)	0.01	0.15	2b	0.2	b	0.10

   Calculate the:

   (a)   value of b (02 marks)

(b)   expectation of X, E(X) (03marks)

5.   Evaluate $\int_1^2 \frac{x^4-1}{x^2}\,dx$ (05 marks)

6.   The ages of eight students in a class are: 12, 13, 14, 15, 12, 17, 13, 16.

Find the;

(a)   mean age   (02 marks)

(b)   variance (03 marks)

7.   Solve the equation cosθ = sin2θ for values of θ from 0^0 to 360^0. (05 marks)

8.   A particle of mass 5kg rests in limiting equilibrium on a rough plane inclines at 20^0 to the horizontal.

Calculate the

(a) Normal reaction
(b) Coefficient of friction between the particle and the plane. (02 marks)

SECTION B (60 MARKS)

Answer any **four** questions

All questions **carry** equal marks

9.   The table below shows the tax collection of a town council in millions of shillings for six consecutive months.

Month	Jan	Feb	Mar	Apr	May	June
Tax (in millions)	21.5	24.3	21.8	26.2	22.7	28.9

(a)   Construct the 3-month moving average for the given data. (06marks)

(b)   Plot the 3-month moving averages and the original data on the same axes. (06 marks)

(c)   Use your graph to estimate the town council's tax collection for the month of July. (03marks)

10.   The equation of a curve is y = 3 +2x − x^2.

(a)   Determine the;

(i)   coordinates and nature of the turning points of the curve. (06 marks)

(ii) y – and x – intercept of the curve (04marks)

(b)   (i)   sketch the curve (02marks)

(ii) find the area enclosed by the curve and the x – axis. (03marks)

11. The marks scored by candidates in an examination are normally distributed with a mean score of μ and standard deviation of δ. Given that 37.5% of the candidates scored below 40 and that 12.5% scored above 60, find the;

(i) values of μ and δ. (09marks)

(ii) probability that a candidate score between 46 and 55. (06 marks)

12. If OA = $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$, OB = $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$ and OC = $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$

(a) find the vectors;

(i) BC

(ii) AB (06 marks)

(b) Show that vectors AB and BC are perpendicular. (03 marks)
(c) Determine the magnitude of the vector 2BC – 3AB. (06 marks)

13. The table below shows the heights to nearest cm and the masses to the nearest kg of 10 students, A to J.

Student	A	B	C	D	E	F	G	H	I	J
Mass (kg)	53	68	57	52	66	64	63	58	57	68
Height (cm)	148	172	156	139	163	158	168	151	144	170

(a) (i) Plot the given data on a scatter diagram

(ii) Draw a line of best fit on the scatter diagram

(iii) Estimate the height of a student whose mass is 60 kg. (08marks)

(b) Calculate the rank correlation coefficient for the data.

Comment on your answer   (07marks)

14. A car of mass 1200kg has a maximum speed of 180kmh^{-1} on level road when the power of the engine is 50kW. When the car ascends an incline of 1 in 5 with the same engine power, the resulting force is 1648N.

Determine the;

(a) resistance force along the level road (05marks)

(b) maximum speed of the car up the incline. (06marks)

(c) acceleration of the car up the plane when its speed is 8ms^{-1}. (04 mark)

Proposed answers

1. Show that $\sqrt{\dfrac{25^3+5^6}{5^7-5^6}} = \dfrac{\sqrt{2}}{2}$ (05marks)

$$\sqrt{\dfrac{25^3+5^6}{5^7-5^6}} = \sqrt{\dfrac{(5^2)^3+5^6}{5^7-5^6}} = \sqrt{\dfrac{5^6+5^6}{5 \times 5^6-5^6}} = \sqrt{\dfrac{2 \times 5^6}{4 \, x 5^6}} = \sqrt{\dfrac{2}{4}} = \dfrac{\sqrt{2}}{2}$$

2. Two events A and B are such that P(A) $=\dfrac{19}{30}$, P(B) $= \dfrac{2}{5}$ and P(A∩B') $= \dfrac{2}{5}$

   Find:

   (i) P(A∩B) (03marks)

   P(A∩B) = P(A) - P(A∩B')

   $$= \dfrac{19}{30} - \dfrac{2}{5} = \dfrac{7}{30}$$

   (ii) P(AUB) (02 marks)

   P(AUB) = P(A) + P(B) - P(A∩B)

   $$= \dfrac{19}{30} + \dfrac{2}{5} - \dfrac{7}{30}$$

   $$= \dfrac{24}{30} = \dfrac{4}{5}$$

3. Determine the possible values of α for which the equation

   $2x^2 + (α + 2)x + (α + 2) = 0$ has equal roots (05marks)

   Solution

   For equal roots $b^2 = 4ac$

   $(α + 2)^2 = 4(2) (α + 2)$

   $α^2 + 4 α + 4 = 8 α + 16$

   $α^2 - 4 α - 12 = 0$

   $(α -6)(α +2) = 0$

   Either $α - 6 = 0$, $α = 6$

   Or $α + 2 = 0$, $α = -2$

4.   A random variable X has the probability distribution shown in the table below

x	0	1	2	3	4	5
P(X = x)	0.01	0.15	2b	0.2	b	0.10

Calculate the:

(a)   value of b (02 marks)

$\sum P(X=x) = 1$

=> 0.01 +0.15 + 2b + 0.2 + b + 0.1 = 1

3b = 0.54

b = 0.18

(b)   expectation of X, E(X) (03marks)

E(X) = $\sum xP(X=x)$

x	0	1	2	3	4	5
P(X = x)	0.01	0.15	0.36	0.2	0.18	0.10
xP	0	0.15	0.72	0.6	0.72	0.5

E(X) = 0.15 + 0.72 + 0.6 + 0.72 + 0.5

= 2,69

5.   Evaluate $\int_1^2 \frac{x^4-1}{x^2} dx$ (05 marks)

Solution

$$\int_1^2 \frac{x^4-1}{x^2} dx = \int_1^2 \left(x^2 - \frac{1}{x^2}\right) dx$$

$$= \int_1^2 (x^2 - x^{-2})$$

$$= \left[\frac{x^3}{3} + \frac{1}{x}\right]_1^2$$

$$= \left(\frac{8}{3} + \frac{1}{2}\right) - \left(\frac{1}{3} + 1\right) = \left(\frac{19}{6} - \frac{4}{3}\right)$$

$$= \frac{11}{6}$$

6. The ages of eight students in a class are: 12, 13, 14, 15, 12, 17, 13, 16.

Find the;

Arranging the data

	x	f	fx	x^2	fx^2
12		2	24	144	288
13		2	26	169	338
14		1	14	196	196
15		1	15	225	225
16		1	16	256	256
17		1	17	280	280
		$\sum$f =8	$\sum$fx = 112		$\sum fx^2$=1592

(a) mean age   (02 marks)

$$\text{mean} = \frac{\sum fx}{\sum f} = \frac{112}{8} = 14$$

(b) variance (03 marks)

$$\text{Var (X)} = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2$$

$$= \frac{1592}{8} - 14^2$$

$$= 3$$

7. Solve the equation cosθ = sin2θ for values of θ from 0^0 to 360^0. (05 marks)

cosθ = 2cosθsinθ

cos(1-2sinθ) = 0

Either cosθ = 0 or θ= 90^0, 270^0

Or

1-2sinθ = 0

sinθ = ½ Or θ = 30^0, 150^0

hence θ = 30^0, 90^0, 150^0, 270^0

8. A particle of mass 5kg rests in limiting equilibrium on a rough plane inclines at 20^0 to the horizontal.

Calculate the

(a) Normal reaction

R = mgcosθ = 5 x 9.8 cos20 = 4.7N

(b) Coefficient of friction between the particle and the plane. (02 marks)

Mgsin20 = μR

$$\mu = \frac{5\,x\,9.8\,sin20}{5x\,9.8cos20} = tan20 = 0.364$$

SECTION B (60 MARKS)

Answer any **four** questions

All questions **carry** equal marks

9. The table below shows the tax collection of a town council in millions of shillings for six consecutive months.

Month	Jan	Feb	Mar	Apr	May	June
Tax (in millions)	21.5	24.3	21.8	26.2	22.7	28.9

(a) Construct the 3-month moving average for the given data. (06marks)

Solution

Month	Tax	Moving totals	Moving average
Jan	21.5		
Feb	24.3	67.6	22.5
Mar	21.8	72.3	24.1
Apr	26.2	70.7	23.6
May	22.7	77.8	25.9
June	28.9		
x			

(b) Plot the 3-month moving averages and the original data on the same axes. (06 marks)

A graph showing aoriginal data and movinga average
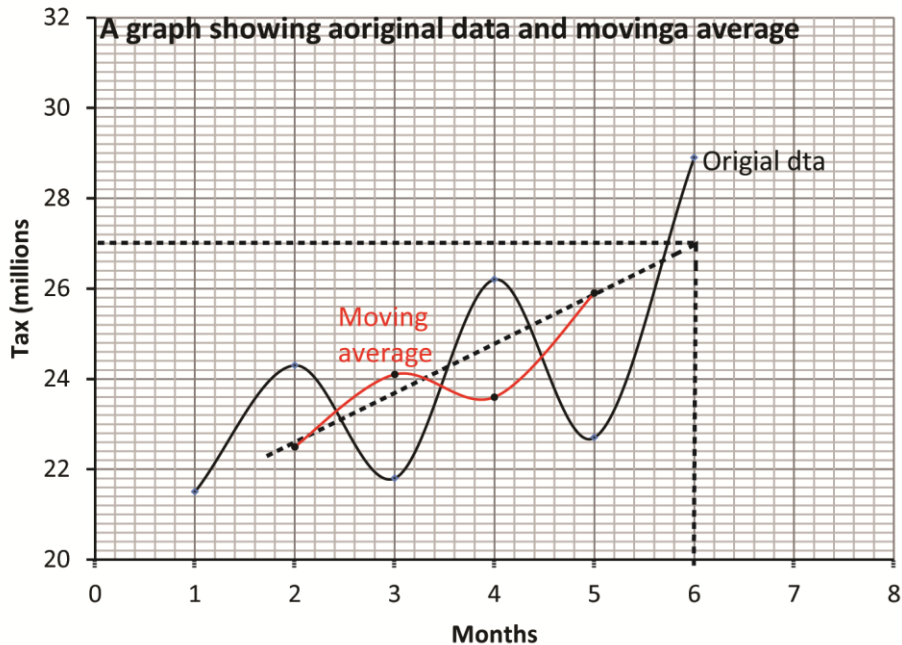
(c) Use your graph to estimate the town council's tax collection for the month of July. (03marks)

From the graph the moving average for June = 27 millions

If x are the sales for July

Then $\frac{x+22.7+28.9}{3}$ = 27

x + 51 = 81

x = 30

10. The equation of a curve is y = 3 +2x − x².

(a) Determine the;

(i) coordinates and nature of the turning points of the curve. (06 marks)

Turning points when $\frac{dy}{dx}(3 + 2x - x^2) = 0$

2 − 2x = 0

x = 1

when x = 1, y = 3 + 2 -1 = 4

turning point is (1, 4)

Nature of turning point

$\frac{d^2y}{dx^2} = -2$, since $\frac{d^2y}{dx^2} < 0$ the turning point is a maxima

(ii) y – and x – intercept of the curve (04marks)

y intercept when x = 0, i.e. y = 3 or (0, 3)
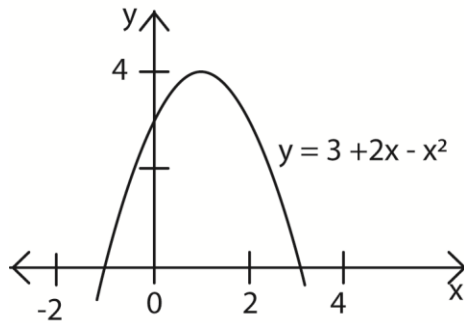
x intercept when y = 0

3 +2x – x² = 0

Or

x² – 2x – 3 =0

(x – 3)(x + 1) = 0

Either x – 3 = 0 and x = 3

Or x +1 = 0 and x = -1

Hence x intercepts are (-1, 0)and (3 , 0)

(b)   (i) sketch the curve (02marks)

(ii) find the area enclosed by the curve and the x – axis. (03marks)

Area = $\int_{-1}^{3}(3 + 2x - x^3) = \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3}$

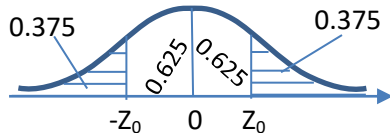= (9 +9-9)-(-3 + 1 + $\frac{1}{3}$)

= $10\frac{2}{3}$ unit²

11. The marks scored by candidates in an examination are normally distributed with a mean score of μ and standard deviation of δ. Given that 37.5% of the candidates scored below 40 and that 12.5% scored above 60, find the;

(i)   values of μ and δ. (09marks)

Let X be marks scored

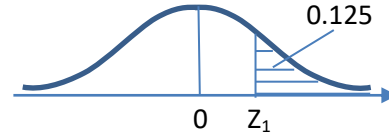$P(X < 40) = \frac{37.5}{100} = 0.375$

$P(x< 40) = P(Z < Z_0) = 0.375$

0.375   0.375

0.625   0.625

-Z_0 0 Z_0

By symmetry, $P(Z < Z_0) = P(Z > Z_0)$

But $Z = \frac{X - \mu}{\sigma}$

$-0.319 = \frac{40 - \mu}{\sigma}$

$\mu - 0.319\sigma = 40$ ……………………(i)

$P(X> 60) = P(Z > Z_1) = \frac{12.5}{100} = 0.125$

0.125

0   Z_1

$P(0 < Z < Z_1) = 0.875$

$Z_1 = 1.15$

⇨ $1.15 = \frac{60 - \mu}{\sigma}$

$\mu - 0.319\sigma = 60$ …………………….(ii)
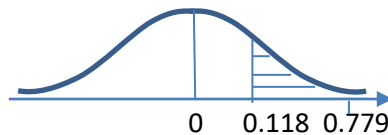
from equation (i) and (ii)

$\sigma = 13.6$ and $\mu = 44.4$

(ii)   probability that a candidate score between 46 and 55. (06 marks)

$P(46< X < 55 = \left[\frac{46 - 44.4}{13.6} < Z < \frac{55 - 44.4}{13.6}\right]$

$= P[0.118 < Z < 0.779\,]$

0   0.118   0.779

$= P(0 < Z < 0.779) - P(0<Z<0.118)$

$= 0.2822 - 0.0470$

$= 0.2352$

12. If OA = $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$, OB = $\begin{pmatrix} 9 \\ 2 \end{pmatrix}$ and OC = $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$

   (a)　find the vectors;

   　　(i)　BC

   　　　Solution

   　　　BC = BO + OC

   　　　$= \begin{pmatrix} -9 \\ -2 \end{pmatrix} + \begin{pmatrix} 7 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

   　　(ii)　AB　(06 marks)

   　　　AB = AO + OB

   　　　$= \begin{pmatrix} -6 \\ -5 \end{pmatrix} + \begin{pmatrix} 9 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

   (b) Show that vectors AB and BC are perpendicular. (03 marks)
   For perpendicular vectors
   AB.BC =0
   AB.BC = $\begin{pmatrix} 3 \\ -3 \end{pmatrix} . \begin{pmatrix} -2 \\ -2 \end{pmatrix}$ = (3 x -2)+(-3 x -2) = -6 + 6 = 0
   Hence AB and BC are perpendicular

   (c) Determine the magnitude of the vector 2BC − 3AB. (06 marks)
   2BC − 3AB = $2\begin{pmatrix} -2 \\ -2 \end{pmatrix} - 3\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ -6 \end{pmatrix} = \begin{pmatrix} -13 \\ 2 \end{pmatrix}$
   $|2BC - 3AB| = \sqrt{(13)^2 + 2^2} = \sqrt{169 + 4} = \sqrt{173}$ = 13.15

13. The table below shows the heights to nearest cm and the masses to the nearest kg of 10 students, A to J.

Student	A	B	C	D	E	F	G	H	I	J
Mass (kg) (x)	53	68	57	52	66	64	63	58	57	68
Height (cm)(y)	148	172	156	139	163	158	168	151	144	170

   (a)　(i)　Plot the given data on a scatter diagram

   　　(ii)　Draw a line of best fit on the scatter diagram

   　　Note that the line of the best fit passes through the mean mass ($\bar{x}$) and mean of height ($\bar{y}$) and these should be calculated to guide you when drawing the line of the best fit

   　　$\bar{x} = \frac{53+68+57+52+66+64+63+58+57+68}{10} = 60.6$

   　　$\bar{y} = \frac{148+172+156+139+163+158+168+151+144+170}{10} = 156.9$

The point $(\bar{x}, \bar{y})$ is shown on the graph below by a red circle but should not be plotted
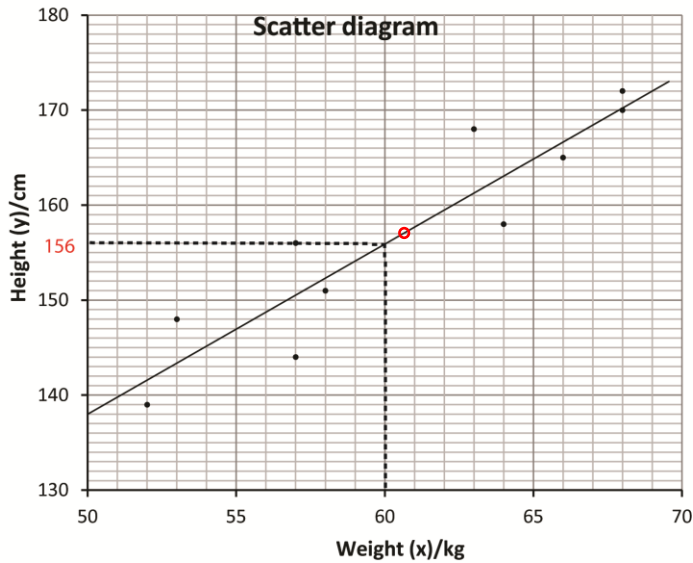
(iii) Estimate the height of a student whose mass is 60 kg. (08marks)

156 (cm)

(c) Calculate the rank correlation coefficient for the data.

Student	Rx	Ry	d	d^2
A	10	8	2	4
B	1.5	1	0.5	0.25
C	7.5	6	1.5	1.25
D	9	10	0.5	0.25
E	3	4	-1	1
F	4	5	-1	1
G	5	3	2	4
H	6	7	-1	1
I	7.5	9	-1.5	2.25
J	1.5	2	-0.5	0.25
sum				17

$$\rho = 1 - \frac{6\sum d^2}{m(n^2-1)}$$

$$= 1 - \frac{6 \times 17}{10(10^2-1)}$$

$$= 0.897$$

Comment on your answer (07marks)

There is high positive correlation between height and weight of student.

14. A car of mass 1200kg has a maximum speed of 180kmh^{-1} on level road when the power of the engine is 50kW. When the car ascends an incline of 1 in 5 with the same engine power, the resulting force is 1648N.

Determine the;

(a) resistance force along the level road (05marks)
at constant speed, the engine power is balanced by resistance force,
Power = force x velocity
180kmh^{-1} = $\frac{180 \, x \, 1000}{3600}$ = 50ms^{-1}
Thus 50,000 = R x 50
F = 1000N

(b) maximum speed of the car up the incline. (06marks)
$\frac{Power}{V}$ = (R + mgsinθ + ma)
50000= (1000 + 1200 x 9.8 x $\frac{1}{5}$ +1648)V
V = 10ms^{-1}

(c) acceleration of the car up the plane when its speed is 8ms^{-1}. (04 mark)
$\frac{Power}{V}$ = R + mgsinθ + ma
$\frac{50000}{8}$ = (1000 + 1200 x 9.8 x $\frac{1}{5}$ +1200a) x 8
6250 = 3,352 + 1200a

1200a = 2,898

a = 2.415ms^{-2}

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Thanks

Dr. Bbosa Science