UACE S475 Sub math paper 1 2020

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, g = 9.8ms^{-2}.

SECTION A (40 MARKS)

Answer all the questions in this section

1. Without using a calculator, evaluate $\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\ x\ \sqrt{20}}$. (05marks)
2. The mean of eight numbers 13, 5, 6, 10, k, 11, 8 and 7 is 9. Find the;
   (a) Value of k. (02 marks)
   (b) Standard deviation (03marks)
3. The sum of the first 16 terms of an arithmetic progression (A. P) is 1088. The 16th term is twice the 8th term. Determine the value of the first term of the A.P. (05marks)
4. A School student's council consists of 7 girls and 5 boys. Two students are selected at random from the council. Find the probability that;
   (a) Both are girls (02 marks)
   (b) The first is a boy and the second is a girl. (03 marks)
5. Determine the equation of the tangent to the curve y = 2x^3 + 3x at a point x = 2. (05 marks)
6. The table below shows the enrollment of students in an institution over a period of 5 years.

Year	2003	2004	2005	2006	2007
Number of students	145	182	170	155	213

   Calculate
   (a) three-year moving averages. (03marks)
   (b) number of students enrolled in 2008, given that the fourth moving average is 2008. (02 marks)

7. Given **a** = $\begin{pmatrix} 5 \\ -15 \end{pmatrix}$ and **b** = $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find the;
   (a) dot product of **a** and **b**. (02 marks)
   (b) angle between the vectors **a** and **b**. (03 marks)
8. A train moving in a straight line passes a point P with velocity of $20ms^{-1}$. It the moves for 5 seconds with acceleration of $2.5ms^{-2}$.

   Determine the;

   (a) velocity of the train after 5 seconds. (03marks)
   (b) distance of the train from P after 5 seconds. (02 marks)

SECTION B (60 MARKS)

Answer any **four** questions from this section

All questions carry equal marks

9. The table below shows marks obtained in sub-Math and Physics by nine students.

Sub-Math (x)	51	62	64	47	54	44	68	61	56
Physics (y)	45	54	58	56	49	43	59	56	53

   (a) (i) Draw a scatter diagram for the data
   (ii) On your scatter diagram, draw a line of best fit.
   (iii) Use the line of the best fit to estimate the value of x when y 55. (09 marks)
   (b) Calculate the spearman's rank correlation coefficient and comment on the result. (09 marks)
10. (a) Given that α and β are the root of the quadratic equation $x^2 - 3mx + n^2 = 0$, show that $\alpha + \beta = 3m$ and $\alpha\beta = n^2$. (06 marks)
   (b) If α and β are the root of the equation $x^2 - 9x + 4 = 0$, find the;
   (i) value of $\alpha^2 + \beta^2$. (03marks)
   (ii) value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (03marks)
   (iii) quadratic equation whose roots are $\frac{1}{\alpha^2}$ *and* $\frac{1}{\beta^2}$. (03marks)
11. (a) Find the number of all the possible arrangements of all the letters in the word DISAPPEAR. (05 marks)
   (b) In a school, there are nine A-level teachers. In the Science department, there is a teacher for each of the following subjects: Mathematics, Physics, Chemistry and Biology. In the Arts department, there is a teacher for each of the following subjects: Economic, Geography, History, Literature and Fine Art. Three teachers are to be sent for a workshop
   (i) Find the number of all possible combination of teachers that may be sent for the workshop. (02marks)
   (ii) What is probability that at least two teachers from the Science department are sent for the workshop? (06marks)
   (iii) If a Mathematics teacher must attend the workshop, determine the number of possible combination of teachers to be sent. (02marks)

12. (a) Given that M = $\begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix}$, N = $\begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$. K = $\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix}$ and K = M + N,

find the value of x and y. (07marks)

(b) In a football tournament, three teams Arsenal, Chelsea and Liverpool gad the following results

- Arsenal won two matches, drew once and lost one match
- Chelsea won two matches and lost two matches
- Liverpool won 1 match, drew twice and lost one match.

The teams are awarded 3 points of a win, 1 point for a draw and no point for a loss.

(i) Write a 3 x 3 matrix for the results and a column matrix for points. (04marks)

(ii) By matric multiplication, determine the winner of the tournament. (04 marks)

13. A discrete random variable W has a probability distribution shown below

w	-3	-2	-1	0	1
P(W =w)	0.1	0.25	0.3	0.15	d

Find

(a) The value of d (02 marks)
(b) P(-3 ≤ W ≤ -1) (03marks)
(c) P(W > -1) (02 marks)
(d) (i) the mode (01 mark)
(ii) the mean (01mark)
(iii) the variance of the distribution (05marks)

14. A mass of 6kg is lying on a smooth horizontal table. The mass is connected by two light inextensible strings passing over smooth pulleys at the edges of the table, two masses of 5kg and 9kg on opposite sides of the table. With the two strings taut and the masses hanging vertically, the system is released from rest.

Calculate the:

(i) Acceleration of the masses. (10marks)
(ii) Tension in the strings. (05marks)

Suggested answers

1. Without using a calculator, evaluate $\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\,x\,\sqrt{20}}$. (05marks)

   Solution

   $\frac{6\sqrt{10}+2\sqrt{40}}{\sqrt{2}\,x\sqrt{20}} = \frac{6\sqrt{10}+2\sqrt{4x10}}{\sqrt{2}\,x\sqrt{2x1=0}}$

   $= \frac{6\sqrt{10}+2\sqrt{4}\,x\sqrt{10}}{\sqrt{2}\,x\sqrt{2}x\sqrt{10}}$

   $= \frac{6\sqrt{10}+4\,x\sqrt{10}}{2x\sqrt{10}}$

   $= \frac{10\sqrt{10}}{2x\sqrt{10}}$

   = 5

2. The mean of eight numbers 13, 5, 6, 10, k, 11, 8 and 7 is 9. Find the;

   (a) Value of k. (02 marks)

   Solution

   $\frac{13+5+6+10+k+11+8+7}{8} = 9$

   $60 + k = 72$

   k = 72 -60 = 12

   (b) Standard deviation (03marks)

   Solution

   S.D = $\sqrt{\frac{\sum fx^2}{n} - \bar{x}^2}$

   $\sum fx^2$= 13^2+ 5^2 + 6^2+10^2+12^2+11^2+8^2+7^2

   = 169+25+36+100+144+121+64+49

   = 708

   S.D = $\sqrt{\frac{708}{8} - 9^2}$

   = $\sqrt{88.5 - 81}$

   = $\sqrt{7.5}$

   = 2.74

3. The sum of the first 16 terms of an arithmetic progression (A. P) is 1088. The 16th term is twice the 8th term. Determine the value of the first term of the A.P. (05marks)

   Solution

   Let

   The first term = a

   And common difference = d

   nth term in A.P = a + (n – 1)d

   8th term = a + 7d

   16th term = a + 15d

   But 2(a + 7d) = a + 15d

   2a + 14d = a + 15d

   ⇨ a = d

   Sum (S_n) of the first n terms is given by

   $S_n = \frac{n}{2}(2a + (n-1)d)$

   ⇨ $1088 = \frac{16}{2}(2a + (16-1)d)$

   $1088 = 8(2a + 15d)$

   But a = d

   ⇨ 1088 = 8 x 17a

   a = 8

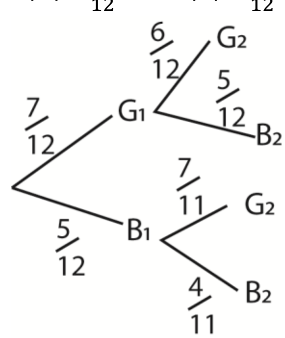
Hence the first term is 8

4. A School student's council consists of 7 girls and 5 boys. Two students are selected at random from the council. Find the probability that;

   Solution

   Total number of students = 7 + 5 = 12

   P(G) = $\frac{7}{12}$ and P(B) = $\frac{5}{12}$



   (a) Both are girls (02 marks)

   P(Both girls) = $P(G_1 \cap G_2)$

   $= \frac{7}{12} x \frac{6}{11} = \frac{7}{22}$

(b) The first is a boy and the second is a girl. (03 marks)

P(1st boy, 2nd girl) = $P(B_1 \cap G_2)$

$= \frac{5}{12} x \frac{7}{11} = \frac{35}{132}$

5. Determine the equation of the tangent to the curve y = $2x^3 + 3x$ at a point x = 2. (05 marks)

Gradient = $\frac{dy}{dx}(2x^3 + 3x)$

= $6x^2 + 3$

Substitution for x = 2

Gradient = $6 \times 2^2 + 3$

= 6 x 4 +3

= 24 +3

= 27

6. The table below shows the enrollment of students in an institution over a period of 5 years.

Year	2003	2004	2005	2006	2007
Number of students	145	182	170	155	213

Calculate

(a) three-year moving averages. (03marks)

Solution

Year	Tax	Moving totals	
2003	145		
2004	182	497	165.7
2005	170	507	169.0
2006	155	538	179.3
2007	213		

(b) number of students enrolled in 2008, given that the fourth moving average is 2008. (02 marks)

Solution

Year	Tax	Moving totals	
2003	145		
2004	182	497	165.7
2005	170	507	169.0
2006	155	538	179.3
2007	213	368 + x	$\frac{368+x}{3}$ = 203
2008	x		

$\frac{368+x}{3}$ = 203

$368 + x$ = 203 x 3

$x = 609 - 368 = 241$

7. Given **a** = $\begin{pmatrix} 5 \\ -12 \end{pmatrix}$ and **b** = $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find the;

(a) dot product of **a** and **b**. (02 marks)

(5 x -3) + (-12x 4) = -93

(b) angle between the vectors **a** and **b**. (03 marks)

Solution

Let the angle be θ

$\sqrt{5^2 + (-12)^2}.\sqrt{(-3)^2 + 4^2}\cos\theta = -93$

13 x 5 cosθ = 93

$cos\theta = \frac{-93}{65}$

$\theta = 0$

8. A train moving in a straight line passes a point P with velocity of $20ms^{-1}$. It the moves for 5 seconds with acceleration of $2.5ms^{-2}$.

Determine the;

(a) velocity of the train after 5 seconds. (03marks)

Solution

V = u + at = 20 + 2.5 x 5 = $32.5ms^{-1}$

(b) distance of the train from P after 5 seconds. (02 marks)

S = ut + ½ at^2 = 20 x 5 + ½ x 2.5 x 5^2

= 100 + 31.25

= 131.25m

SECTION B (60 MARKS)

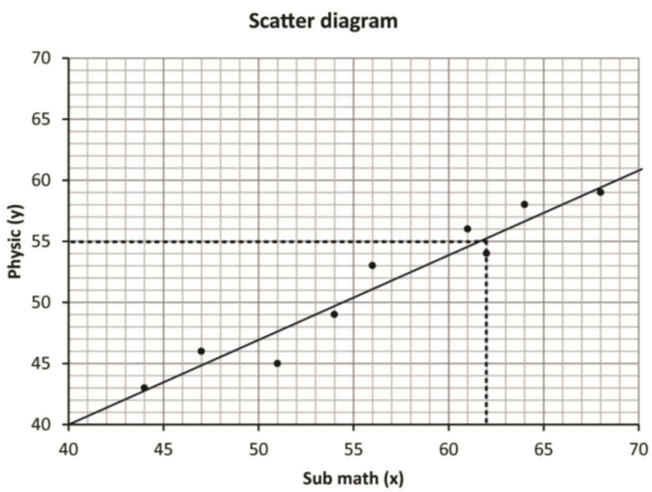
Answer any **four** questions from this section

All questions carry equal marks

9. The table below shows marks obtained in sub-Math and Physics by nine students.

Sub-Math (x)	51	62	64	47	54	44	68	61	56
Physics (y)	45	54	58	56	49	43	59	56	53

(a) (i) Draw a scatter diagram for the data



(ii) On your scatter diagram, draw a line of best fit.

(iii) Use the line of the best fit to estimate the value of x when y 55. (09 marks)

62

(b) Calculate the spearman's rank correlation coefficient and comment on the result. (09 marks)

Sub math(x)	Physics (y)	Rx	Ry	d	d^2
51	45	7	8	-1	1
62	54	3	4	-1	1
64	58	2	2	0	0
47	46	8	7	1	1
54	49	6	6	0	0
44	43	9	9	0	0
68	59	1	1	0	0
61	56	4	3	-1	1
56	53	5	5	0	0
		SUM			4

$\rho = 1- \left[\frac{6\sum d^2}{n(n^2-1)}\right] = 1- \left[\frac{6(4)}{9(9^2-1)}\right] = 1 - \frac{24}{720} = \frac{696}{720} = 0.967$

Comment: There is a high positive correlation between marks of sub math and physics.

10. (a) Given that α and β are the root of the quadratic equation $x^2 - 3mx + n^2 = 0$, show that $\alpha + \beta = 3m$ and $\alpha\beta = n^2$. (06 marks)

Solution

$(x - \alpha)(x - \beta) = 0$

$x^2 - \alpha x - \beta x + \alpha\beta = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

Comparing

$\alpha + \beta = 3m$ and $\alpha\beta = n^2$

(b) If α and β are the root of the equation $x^2 - 9x + 4 = 0$, find the;

(i) value of $\alpha^2 + \beta^2$. (03marks)

Solution

From $(\alpha+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (9)^2 - 2x4$

$= 81 - 8$

$= 73$

(ii) value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$. (03marks)

Solution

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{73}{4^2} = \frac{73}{16} = 4.5625$$

(iii) quadratic equation whose roots are $\frac{1}{\alpha^2}$ *and* $\frac{1}{\beta^2}$. (03marks)

Solution

Sum of the root, $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{73}{16}$

Product of the roots, $\frac{1}{\alpha^2} \cdot \frac{1}{\beta^2} = \frac{1}{(\alpha\beta)^2} = \frac{1}{16}$

Hence quadratic equation

$x^2 - \frac{73}{16}x + \frac{1}{16} = 0$

11. (a) Find the number of all the possible arrangements of all the letters in the word DISAPPEAR. (05 marks)

Solution

In the word 'DISAPPEAR' there are 9 letters, 2 letters of A and 2 letters of P

The number of arrangements = $\frac{9!}{2!2!} = \frac{9x8x7x6x5x4x3x2x1}{2x1x2x1}$ =90,720

(b) In a school, there are nine A-level teachers. In the Science department, there is a teacher for each of the following subjects: Mathematics, Physics, Chemistry and Biology. In the Arts department, there is a teacher for each of the following subjects: Economic, Geography, History, Literature and Fine Art. Three teachers are to be sent for a workshop

(i) Find the number of all possible combination of teachers that may be sent for the workshop. (02marks)

Solution

Possible selection

4 science teachers	5 arts teachers
3	0
2	1
1	2
0	3

Number of selections $= {}^4C_3.{}^5C_0 + {}^4C_2.{}^5C_1 + {}^4C_1.{}^5C_2 + {}^4C_0.{}^5C_3$

$= 4 \times 1 + 6 \times 5 + 4 \times 10 + 1 \times 10$

$= 4 + 30 + 40 + 10$

$= 84$

(ii) What is probability that at least two teachers from the Science department are sent for the workshop? (06marks)

Possible selection

4 science teachers	5 arts teachers
2	1
3	0

Number of selections $= {}^4C_3.{}^5C_0 + {}^4C_2.{}^5C_1$

$= 6 \times 5 + 4 \times 1$

$= 30 + 4$

$= 34$

Selection of 3 teachers from 9 teachers $= {}^9C_3 = 84$

P(at least two science teachers selected) $= \frac{34}{84} = \frac{17}{42}$

(iii) If a Mathematics teacher must attend the workshop, determine the number of possible combination of teachers to be sent. (02marks)

Possible selection

4 science teachers	5 arts teachers
2	0
1	1
0	2

Number of selections $= {}^4C_2.{}^5C_0 + {}^4C_1.{}^5C_1 + {}^4C_0.{}^5C_2$

$= 3 \times 1 + 3 \times 5 + 1 \times 10$

$= 3 + 15 + 10$

$= 28$

12. (a) Given that M = $\begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix}$, N = $\begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$. K = $\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix}$ and K = M + N,

find the value of x and y. (07marks)

Solution

$$\begin{pmatrix} y & 4 \\ -2 & 12 \end{pmatrix} = \begin{pmatrix} 4x & 6 \\ -5 & -2x \end{pmatrix} + \begin{pmatrix} -1 & -2 \\ 3 & 3y \end{pmatrix}$$

y = 4x – 1 ……….(i)

12 = -2x + 3y…….(ii)

Substitution of (i) in (ii)

12 = -2x + 3(4x -1)

12 = -2x + 12x – 3

10x = 15

x = 1.5

Using equation (i)

y = 4 x 1.5 – 1 = 5

therefore x = 1.5 and y = 5

(b) In a football tournament, three teams Arsenal, Chelsea and Liverpool gad the following results

- Arsenal won two matches, drew once and lost one match
- Chelsea won two matches and lost two matches
- Liverpool won 1 match, drew twice and lost one match.

The teams are awarded 3 points of a win, 1 point for a draw and no point for a loss.

(i) Write a 3 x 3 matrix for the results and a column matrix for points. (04marks)

$$\begin{matrix} & W & D & l \\ A & 2 & 1 & 1 \\ C & 2 & 0 & 2 \\ L & 1 & 2 & 1 \end{matrix} = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \text{ and } \begin{matrix} w \\ D \\ L \end{matrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

(ii) By matric multiplication, determine the winner of the tournament. (04 marks)

$$\begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3\,x3 + 1x1 + 1x0 \\ 2x3 + 0x1 + 2x0 \\ 1x3 + 2\,x1 + 1x0 \end{pmatrix} = \begin{pmatrix} 6+1+0 \\ 6+0+0 \\ 3+2+0 \end{pmatrix} = (5)$$

Asernal won with 7 points

13. A discrete random variable W has a probability distribution shown below

w	-3	-2	-1	0	1
P(W =w)	0.1	0.25	0.3	0.15	d

Find

(a) The value of d (02 marks)

Solution

Total probability = 1 = 0.1 + 0.25 + 0.3 + 0.15 + d

d + 0.8 = 1

d = 0.2

(b) P(-3 ≤ W ≤ -1) (03marks)

P(-3 ≤ W ≤ -1) = P(W = -3) + P (W = -3) + P(W = -1)

= 0.1 + 0.25 + 0.3

= 0.65

(c) P(W > -1) (02 marks)

P(W > -1) = P(W = 0) + P(W = 1)

= 0.15+ 0.2

= 0.35

(d) (i) the mode (01 mark)

-1

(ii) the mean (01mark)

(iii) the variance of the distribution (05marks)

w	-3	-2	-1	0	1
w^2	9	4	1	0	1
P(W =w)	0.1	0.25	0.3	0.15	0.2
w. P(W =w)	-0.3	-0.5	-0.3	0	0.2
w^2. P(W =w)	0.9	1.0	0.3	0	0.2

Mean E(W) = $\sum_{all} wP(W = w)$ = -0.3 – 0.5 – 0.3+ 0.2 = -0.9

Var(W) = $\sum_{all} w^2P(W = w) - (\sum_{all} wP(W = w))^2$

= 0.9 + 0.1 + 0.3 + 0.2 – (-0.9)2

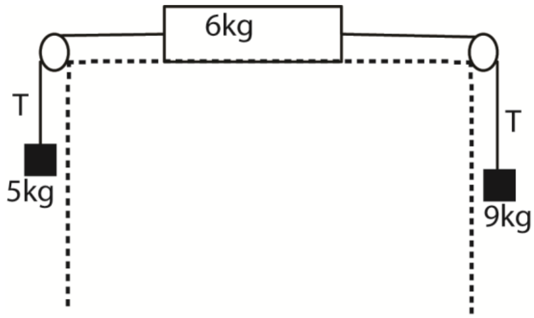
= 1.5 – 0.81

= 0.69

14. A mass of 6kg is lying on a smooth horizontal table. The mass is connected by two light inextensible strings passing over smooth pulleys at the edges of the table, two masses of 5kg and 9kg on opposite sides of the table. With the two strings taut and the masses hanging vertically, the system is released from rest.

Calculate the:

(i) Acceleration of the masses. (10marks)



Let the acceleration be a

T – 5g = 5a

9g – T = 9a

4 x 9.8 = 14a

A = 2.8 ms^{-2}

(ii) Tension in the strings. (05marks)

9 x 9.8 - T = 9 x 2.8

T = 63N

Please obtain free downloadable notes, exams and marking guides of Sub-math, ICT, general paper, biology, economics, geography etc. from digitalteachers.co.ug website

Thanks

Dr. Bbosa Science