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UACE S475 Sub math paper 1 2022

2hour 40minutes

Instructions to candidates

Answer all the eight questions in section A and only four questions from section B

Where necessary, take the acceleration due to gravity, $g = 9.8\text{ms}^{-2}$.

SECTION A (40 MARKS)

Answer all the questions in this section

1. The determinant of the matrix

$$P = \begin{pmatrix} 3a & -1 \\ 7 & a \end{pmatrix} = 55$$

(03marks)

- (a) Find the value of a. (03 marks)
 (b) Using one of the value of a, determine the inverse of P. (02 marks)

2. The table below shows the 3-month moving averages for the quantity of good (in tonnes) manufactured by a certain company from January to August of 2019.

Month	February	March	April	May	June	July
3-month Moving Average (tonnes)	15	17.5	19	20	21.5	22.5

- (a) Find the moving totals. (03marks)
 (b) If 20 tonnes and 10 tonnes of goods were manufactured in February and March respectively, calculate the quantity that was manufactured in January. (02 marks)
3. A man's monthly salary in his first year of work was 220,000. He got an increment of 5% every year. Calculate;
 (a) the man's total earnings at the end of the year. (02 marks)
 (b) his total earnings after 5years. (03marks)
4. Three letters are chosen at random from the word CLOTHINGS. Determine the probability that two of the three letters chosen are consonants. (05markks)
5. The polynomial $f(x) = x^4 + 2x^2 - 3$ has factors $x - 1$ and $x + 1$. Find the other factors of $f(x)$. (05marks)

6. Given that $P(A) = \frac{3}{4}$ and $P(B/A) = \frac{8}{15}$, find $P(A \cap B')$. (05 marks)
7. Differentiate the following with respect to x .
- (a) $(5x - 2)^2$. (02 marks)
- (b) $\frac{3x^4 + 4x^2 - 1}{2x^2}$ (03 marks)
8. Two points A and B are 800metres apart. A particles moving in a straight line with a constant acceleration passes point A with a velocity of 10m/s. it then passes the point B with a velocity of 40m/s. calculate the time taken by the particle to move from A to B. (05 marks)

SECTION B (60 MARKS)

Answer only **four** questions from this section

All questions carry equal marks

9. The frequency distribution table below shows the marks of 50 students score in a test

Marks	Number of Students
50 – 52	3
53 – 55	16
56 – 58	14
59 – 61	13
62 – 64	2
65 – 67	2

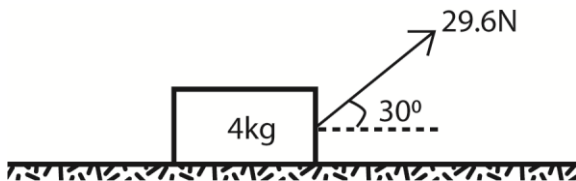
- (a) Calculate the:
- (i) Mean mark (04 marks)
- (ii) Student deviation. (05 marks)
- (b) (i) Plot a cumulative frequency curve (Ogive) for the given data. (04 marks)
- (ii) Use the Orgive to estimate the median mark. (02 marks)
10. The rate of cooling of a body is proportional to the difference in temperature T of the body at any time t and that of the surroundings. If the temperature of the surroundings is 25°C and the body cools from 85°C to 70°C in 15 minutes;
- (a) (i) form a differential equation for the cooling of the body.
- (ii) Solve the differential equation formed in (i) (12marks)
- (b) determine the temperature of the body after 30 minutes
11. The time taken for a bus to make a journey is normally distributed with mean $3\frac{1}{2}$ hours and standard deviation $\frac{3}{4}$ hours.
- (a) Determine the probability that the bus makes a journey:
- (i) in less than 2 hours (05marks)
- (ii) between $3\frac{1}{4}$ and $3\frac{3}{4}$ hours (07 marks)
- (b) If the bus made two hundred journeys, how many of these journeys did it take less than 2 hours? (03 marks)
12. (a) Given that $\tan\theta = \frac{1}{2}$, evaluate $\operatorname{cosec}^2\theta - \sec^2\theta$ without using mathematical tables or calculator. (05marks)
- (b) Prove that $\frac{1+\cos 2x}{2\sin 2x} = \frac{1}{2}\cot x$

Hence solve the equation $\frac{1+\cos 2x}{2\sin 2x} = 1$ for $0^\circ \leq x \leq 180^\circ$. (10marks)

13. The table below shows the expenditure of a family for the months of January and July in a certain year.

Item	Expenditure (Shs)		Weight
	January	July	
Food	150,000	174,000	8
Rent	50,000	60,000	2
Clothing	100,000	125,000	6
Power	20,000	25,000	1
Water	60,000	90,000	4

- (a) Calculate the:
- Price relative for each item (03 marks)
 - Simple aggregate index (04 marks)
- (b) (i) find the weighted aggregate price index. (06 marks)
- comment on your result in (b) (i) (02marks)
14. (a) The diagram below shows a block of mass 4kg in limiting equilibrium on a rough horizontal table under the action of a force of 29.6N. The force is inclined at an angle 30° to the horizontal.



Calculate the:

- normal reaction exerted by the table on the block. (03 marks)
 - coefficient of friction between the block and the table (03marks)
- (b) Forces of magnitude 3N, 2N, 6N and 5N act from a point in the direction 090° , 180° , 330° and 060° respectively
- magnitude of the resultant force. (07marks)
 - inclination of the resultant force to the 3N force (02 marks)

Proposed answers

1. The determinant of the matrix

$$P = \begin{pmatrix} 3a & -1 \\ 7 & a \end{pmatrix} = 55$$

(03marks)

- (a) Find the value of a. (03 marks)

$$[P] = 3a \times a - (7 \times -1) = 55$$

$$3a^2 + 7 = 55$$

$$3a^2 = 48$$

$$a^2 = 16$$

$$a = 4$$

Using one of the value of a, determine the inverse of P. (02 marks)

$$\text{Adjunct of } P = \begin{pmatrix} 4 & 1 \\ -7 & 14 \end{pmatrix}$$

$$[P] = 55$$

$$\text{Inverse of } P^{-1} = \frac{1}{55} \begin{pmatrix} 4 & 1 \\ -7 & 14 \end{pmatrix} = \begin{pmatrix} \frac{4}{55} & \frac{1}{55} \\ \frac{-7}{55} & \frac{14}{55} \end{pmatrix}$$

2. The table below shows the 3-month moving averages for the quantity of good (in tonnes) manufactured by a certain company from January to August of 2019.

Month	February	March	April	May	June	July
3-month Moving Average (tonnes)	15	17.5	19	20	21.5	22.5

- (a) Find the moving totals. (03marks)

Solution

Month	Tax	Moving totals	
Jan			
February		45	15
March		52.5	17.5
April		57	19
May		60	20
June		64.5	21.5
July		67.5	22.5

- (b) If 20 tonnes and 10 tonnes of goods were manufactured in February and March respectively, calculate the quantity that was manufactured in January. (02 marks)

Solution

Let the quantity manufactured in January be x

$$x + 20 + 10 = 45$$

$$x + 30 = 45$$

$$x = 15$$

3. A man's monthly salary in his first year of work was 220,000. He got an increment of 5% every year. Calculate;

(a) the man's total earnings at the end of the year. (02 marks)

$$= 220,000 \times 12 = 2,640,000$$

(b) his total earnings after 5 years. (03 marks)

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$a = 2,640,000 \text{ and } r = \frac{105}{100} = 1.05$$

substituting

$$= S_5 = \frac{2,640,000(1.05^5 - 1)}{1.05 - 1} = \frac{2,640,000(1.05^5 - 1)}{0.05} = 14,587,666.5$$

4. Three letters are chosen at random from the word CLOTHINGS. Determine the probability that two of the three letters chosen are consonants. (05 marks)

Solution

Total number of letters in CLOTHINGS = 9 letters

Selection of 3 letters from 9 letters ${}^9C_3 = 84$

Possible selection

7 consonants	2 Vowels
2	1

$$\begin{aligned} \text{Number of selections} &= {}^7C_2 \cdot {}^2C_1 \\ &= 21 \times 2 \\ &= 42 \end{aligned}$$

$$\text{Probability} = \frac{42}{84} = \frac{1}{2}$$

5. The polynomial $f(x) = x^4 + 2x^2 - 3$ has factors $x - 1$ and $x + 1$. Find the other factors of $f(x)$. (05 marks)

$$(x - 1)(x + 1) = x^2 - 1$$

$$\begin{array}{r} x^2 + 3 \\ x^2 - 1 \overline{) x^4 + 2x^2 - 3} \\ \underline{-x^4 - x^2} \\ 3x^2 - 3 \\ \underline{-3x^2 - 3} \\ 0 \end{array}$$

Hence the other factor is $x^2 + 3$

6. Given that $P(A) = \frac{3}{4}$ and $P(B/A) = \frac{8}{15}$, find $P(A \cap B')$. (05 marks)

Solution

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B/A) \cdot P(A)$$

$$= \frac{8}{15} \times \frac{3}{4} = \frac{2}{5}$$

$$P(A \cap B) = P(A) - P(A \cap B')$$

$$\Rightarrow P(A \cap B') = P(A) - P(A \cap B)$$

$$= \frac{3}{4} - \frac{2}{5}$$

$$= \frac{3}{20}$$

7. Differentiate the following with respect to x.

(a) $(5x - 2)^2$. (02 marks)

Solution

$$(5x - 2)^2 = (5x - 2)(5x - 2) = 25x^2 - 20x + 4$$

$$\frac{dy}{dx} (5x - 2)^2 = \frac{dy}{dx} (25x^2 - 20x + 4)$$

$$= 50x - 20$$

$$= 10(5x - 2)$$

(b) $\frac{3x^4 + 4x^2 - 1}{2x^2}$ (03 marks)

Solution

$$\text{Let } u = 3x^4 + 4x^2 - 1$$

$$\frac{du}{dx} = 12x^3 + 8x$$

$$\text{Let } v = 2x^2$$

$$\frac{dv}{dx} = 4x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{2x^2(12x^3 + 8x) - 4x(3x^4 + 4x^2 - 1)}{(2x^2)^2}$$

$$= \frac{24x^5 + 16x^3 - 12x^5 - 16x^3 + 4x}{4x^4}$$

$$= \frac{12x^5 + 4x}{4x^4}$$

$$= \frac{3x^4 + 1}{x^3}$$

8. Two points A and B are 800metres apart. A particles moving in a straight line with a constant acceleration passes point A with a velocity of 10m/s. it then passes the point B with a velocity of 40m/s. calculate the time taken by the particle to move from A to B. (05 marks)

Solution

Let a be the acceleration taken

$$\text{From } v^2 = u^2 + 2as$$

$$a = \left(\frac{v^2 - u^2}{2s} \right) = \left(\frac{1600 - 100}{2 \times 800} \right) = \frac{1500}{1600} = \frac{15}{16}$$

$$\text{form } v = u + at$$

$$40 = 10 + \frac{15}{16}t$$

$$\frac{15}{16}t = 30$$

$$t = \frac{30 \times 16}{15} = 32s$$

SECTION B (60 MARKS)

Answer only **four** questions from this section

All questions carry equal marks

9. The frequency distribution table below shows the marks of 50 students score in a test

Marks	Number of Students
50 – 52	3
53 – 55	16
56 – 58	14
59 – 61	13
62 – 64	2
65 – 67	2

- (a) Calculate the:

Solution

Marks	Class boundaries	Number of Students (f)	fx	Fx ²	CF
50 – 52	49.5 – 52.5	3	153	7803	3
53 – 55	52.5 – 55.5	16	864	46656	19
56 – 58	55.5 – 58.5	14	798	45486	33
59 – 61	58.5 – 61.5	13	780	46800	46
62 – 64	62.5 – 64.5	2	126	7938	48
65 – 67	64.5 - 67.5	2	132	712	50
		50	2853	163395	

- (i) Mean mark (04 marks)

Solution

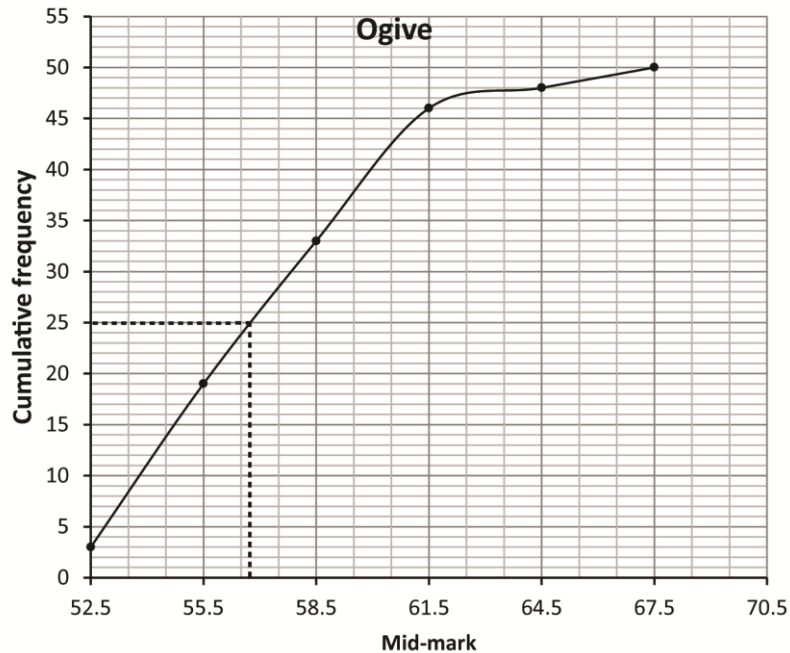
$$\text{Mean, } \bar{x} = \frac{\sum fx}{\sum f} = \frac{2853}{50} = 57.06$$

- (ii) Standard deviation. (05 marks)

$$\begin{aligned} \text{s.d} &= \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} \\ &= \sqrt{\frac{163395}{50} - 57.06^2} \\ &= 12.06 \end{aligned}$$

- (b) (i) Plot a cumulative frequency curve (Ogive) for the given data. (04 marks)

Note that CF is plotted against the upper limit of each class



- (ii) Use the Ogive to estimate the median mark. (02 marks)
56.5

10. The rate of cooling of a body is proportional to the difference in temperature T of the body at any time t and that of the surroundings. If the temperature of the surroundings is 25°C and the body cools from 85°C to 70°C in 15 minutes;

- (a) (i) Form a differential equation for the cooling of the body.

Solution

$$\frac{dT}{dt} = -kT$$

- (ii) Solve the differential equation formed in (i) (12marks)

Solution

$$\int \frac{dT}{T} = - \int kt$$

$$\ln T = -kt + C$$

$$\text{At } t = 0, T_0 = (85 - 25) = 60^{\circ}$$

$$\ln T = -kt + \ln 60$$

$$\text{At } t = 15, T = (70 - 25) = 45^{\circ}$$

$$\Rightarrow \ln 45 = -15k + \ln 60$$

$$15k = \ln 60 - \ln 45$$

$$15k = \ln \frac{60}{45}$$

$$k = \frac{1}{15} \ln \frac{60}{45} = 0.02 \text{min}^{-1}$$

therefore the equation $\ln T = -0.02t + \ln 60$ or $\ln \frac{60}{T} = 0.02t$

(b) determine the temperature of the body after 30 minutes

Solution

after 30 minutes

$$\ln \frac{60}{T} = 0.02 \times 30 = 0.6$$

$$\frac{60}{T} = 1.82$$

$$T = \frac{60}{1.82} = 33^{\circ}$$

$$\text{The temperature} = 85 - 33 = 52^{\circ}$$

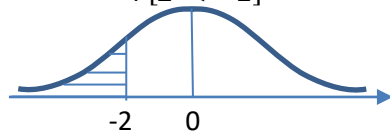
11. The time taken for a bus to make a journey is normally distributed with mean $3\frac{1}{2}$ hours and standard deviation $\frac{3}{4}$ hours.

(a) Determine the probability that the bus makes a journey:

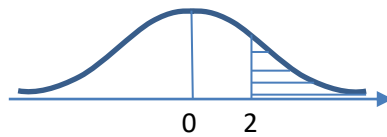
(i) in less than 2 hours (05marks)

Solution

$$\begin{aligned} P(X < 2) &= P\left[Z < \frac{2-3.5}{0.75}\right] \\ &= P[Z < -2] \end{aligned}$$



By symmetry, $P[Z < -2] = P[Z > 2]$



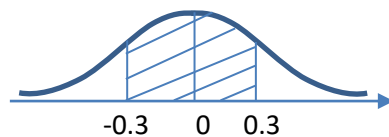
$$\begin{aligned} P[Z > 2] &= 0.5 - (0 < Z < 2) \\ &= 0.5 - 0.4772 \\ &= 0.0228 \end{aligned}$$

Hence the probability = 0.0228

(ii) between $3\frac{1}{4}$ and $3\frac{3}{4}$ hours (07 marks)

Solution

$$\begin{aligned} P(3.25 < X < 3.75) &= P\left[\frac{3.25-3.5}{0.75} < Z < \frac{3.75-3.5}{0.75}\right] \\ &= P[-0.3 < Z < 0.3] \end{aligned}$$



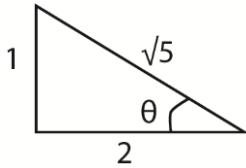
$$\begin{aligned} P[-0.3 < Z < 0.3] &= 2P(0 < Z < 0.3) \\ &= 2 \times 0.1179 \\ &= 0.2358 \end{aligned}$$

(b) If the bus made two hundred journeys, how many of these journeys did it take less than 2 hours? (03 marks)

Number of journeys that take less than 2 hours = $0.0228 \times 200 = 4$

12. (a) Given that $\tan\theta = \frac{1}{2}$, evaluate $\operatorname{cosec}^2\theta - \sec^2\theta$ without using mathematical tables or calculator. (05marks)

Solution



$$\operatorname{cosec}\theta = \frac{\sqrt{5}}{1} = \sqrt{5}; \operatorname{cosec}^2\theta = 5$$

$$\sec\theta = \frac{\sqrt{5}}{2}; \sec^2\theta = \frac{5}{4}$$

$$\Rightarrow \operatorname{cosec}^2\theta - \sec^2\theta = 5 - \frac{5}{4} = \frac{15}{4} = 3\frac{3}{4}$$

- (b) Prove that $\frac{1+\cos 2x}{2\sin 2x} = \frac{1}{2} \cot x$

Solution

$$\frac{1+\cos 2x}{2\sin 2x} = \frac{1+\cos^2 x - \sin^2 x}{4\cos x \sin x} = \frac{2\cos^2 x}{4\cos x \sin x} = \frac{\cos x}{2\sin x} = \frac{1}{2} \cot x$$

Hence solve the equation $\frac{1+\cos 2x}{2\sin 2x} = 1$ for $0^\circ \leq x \leq 180^\circ$. (10marks)

$$\text{Hence } \frac{1}{2} \cot x = 1$$

$$\cot x = 2$$

$$x = 26.6^\circ$$

13. The table below shows the expenditure of a family for the months of January and July in a certain year.

Item	Expenditure (Shs)		Weight
	January	July	
Food	150,000	174,000	8
Rent	50,000	60,000	2
Clothing	100,000	125,000	6
Power	20,000	25,000	1
Water	60,000	90,000	4

(a) Calculate the:

- (i) Price relative for each item (03 marks)

Solution

$$\text{Price relative} = \frac{P_{July}}{P_{June}} \times 100$$

$$\text{Price relative for food} = \frac{174,000}{150,000} \times 100 = 116$$

$$\text{Price relative for rent} = \frac{60,000}{50,000} \times 100 = 120$$

$$\text{Price relative for clothing} = \frac{125,000}{100,000} \times 100 = 125$$

$$\text{Price relative for Power} = \frac{25,000}{20,000} \times 100 = 125$$

$$\text{Price relative for Water} = \frac{90,000}{60,000} \times 100 = 150$$

- (ii) Simple aggregate index (04 marks)

Solution

$$\begin{aligned} \text{Simple aggregate index} &= \frac{\sum P_{July}}{\sum P_{June}} \\ &= \frac{174000+60000+125000+25000+900}{150000+50000+100000+20000+60000} \\ &= \frac{474,000}{380,000} \\ &= 1.25 \end{aligned}$$

- (b) (i) find the weighted aggregate price index. (06 marks)

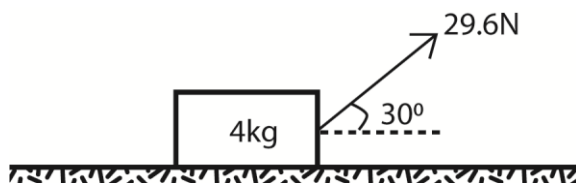
Solution

$$\begin{aligned} \text{Weighed aggregate price index} &= \frac{\sum P_{July} \times w}{\sum P_{June} \times w} \\ &= \frac{174000 \times 8 + 60000 \times 2 + 125000 \times 6 + 25000 \times 1 + 900 \times 4}{150000 \times 8 + 50000 \times 2 + 100000 \times 6 + 20000 \times 1 + 60000 \times 4} \\ &= \frac{2,647,000}{1,160,000} \\ &= 1.23 \end{aligned}$$

- (ii) comment on your result in (b) (i) (02marks)

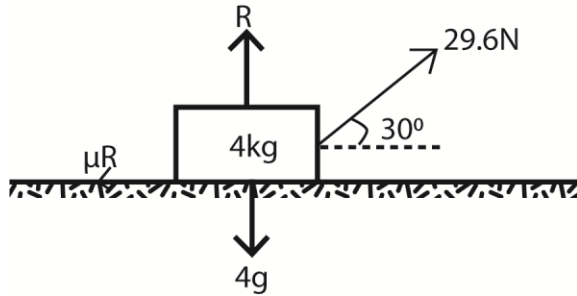
Weighed aggregate price index is less than simple aggregate index

14. (a) The diagram below shows a block of mass 4kg in limiting equilibrium on a rough horizontal table under the action of a force of 29.6N. The force is inclined at an angle 30° to the horizontal.



Calculate the:

- (i) normal reaction exerted by the table on the block. (03 marks)
 Normal reaction $R = mg = 4 \times 9.8 - 29.6 \sin 30^\circ = 24.4\text{N}$
- (ii) coefficient of friction between the block and the table (03marks)

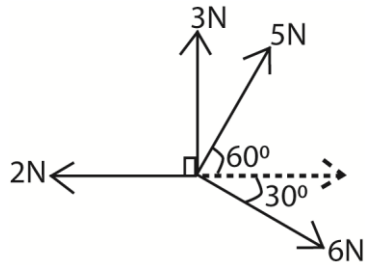


$$29.6\cos 30^{\circ} = \mu R = 24.4\mu$$

$$\mu = 1.05$$

(b) Forces of magnitude 3N, 2N, 6N and 5N act from a point in the direction 090° , 180° , 330° and 060° respectively

(i) magnitude of the resultant force. (07marks)



$$\begin{aligned} \text{Forces acting in x direction} &= 5\cos 60 + 0 - 2 + 6\cos 30 \\ &= 2.5 - 2 + 5.2 \\ &= 5.7\text{N} \end{aligned}$$

$$\begin{aligned} \text{Forces acting in y direction} &= 5\sin 60 + 3 + 0 - 6\sin 30 \\ &= 4.33 + 3 - 3 \\ &= 4.33\text{M} \end{aligned}$$

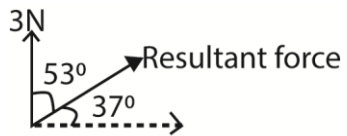
$$\text{Resultant force} = \sqrt{5.7^2 + 4.33^2} = 7.158\text{N}$$

(ii) inclination of the resultant force to the 3N force (02 marks)

Let the angle the resultant makes with horizontal axis = x

$$\tan x = \frac{4.33}{5.7}$$

$$x = \tan^{-1} \frac{4.33}{5.7} = 37^{\circ}$$



Hence the angle the resultant makes with 3N (or vertical axis) = $90 - 37 = 53^\circ$

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Thanks

Dr. Bbosa Science