SENIOR FIVE TERM 2

TOPIC 2/6: Dynamics 1

Competency: The learner determines the resultant force and analyses the effect of forces on bodies by applying Newton's laws to solve problems in real world phenomena.

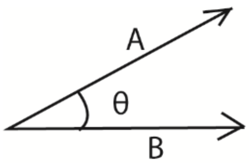
Contents

Resultant of forces

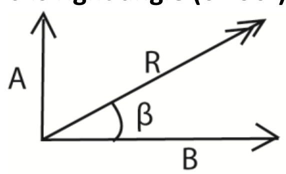
A force is anything which changes a body's state of rest or uniform motion in a straight line. Examples of forces are weight, tension, reaction, friction, resistance force.

Resultant of two forces

Consider two forces A and B inclined to each other at an angle θ



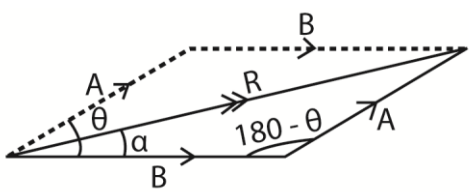
(i) θ is right angle (θ =90^0)



Resultant, R = $\sqrt{A^2 + B^2}$

Direction of resultant, β =$\tan^{-1}\left(\frac{A}{B}\right)$

(ii) **θ is acute (0^0 ≤ θ ≤ 90^0)**

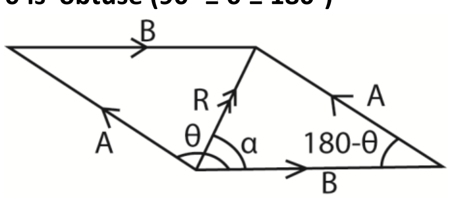


Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

$\alpha = \sin^{-1}\left(\frac{A\sin(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\ (180-\theta)]}$

(iii) **θ is obtuse (90^0 ≤ θ ≤ 180^0)**



Direction of resultant, $\frac{\sin\alpha}{A} = \frac{\sin(180-\theta)}{R}$

$\alpha = \sin^{-1}\left(\frac{A\sin(180-\theta)}{R}\right)$

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\ (180-\theta)]}$

Example 1

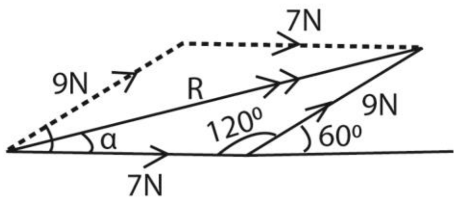
Two forces of magnitude 5N and 12N act on a particle with their direction inclined at 90^0. Find the magnitude and direction of the resultant

$R = \sqrt{5^2 + 12^2}$ = 13N α =$\tan^{-1}\left(\frac{5}{12}\right)$ = 22.6^0

The resultant = 13N at 22.6^0 to 12N force

Example 2

Forces of magnitude 7N and 9N act on a particle at an angle of 60^0 between them. Find the magnitude and direction of the resultant.



Direction of resultant, $\frac{\sin\alpha}{9} = \frac{\sin(180-\theta)}{13.89}$

$\alpha = \sin^{-1}\left(\frac{9\sin(180-60)}{13.89}\right)$ = 34.13^0

Resultant, R =$\sqrt{[A^2 + B^2 - 2ABcos\ (180-\theta)]}$

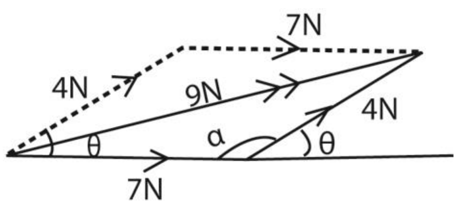
=$\sqrt{[7^2 + 9^2 - 2x7\ x\ 9cos\ (180-60)]}$

= 13.89N

Example 3

Find the angle between a force of 7N and 4N their resultant has a magnitude of 9N

Solution



$9^2 = 7^2 + 4^2 - 2 \times 7 \times 4 \times \cos\alpha$

Cosα =$\frac{-2}{7}$

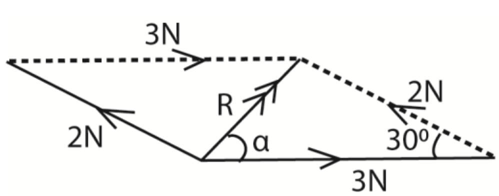
$\alpha = \cos^{-1}\left(-\frac{2}{7}\right)$ = 106.6^0

the angle θ between the forces = 180 – 106.6

= 73.4^0

Example 4

Forces of 3N and 2N act on a particle at an angle of 150^0 between them. Find the magnitude and direction of the resultant.



Direction of resultant, $\frac{\sin\alpha}{2} = \frac{\sin(30)}{1.61}$

$\alpha = \sin^{-1}\left(\frac{2\sin(180-60)}{1.61}\right)$ = 38.3^0

$R^2 = 2^2 + 3^2 - 2 \times 2 \times 2 \times \cos(30)$

R = 1.61N

Revision exercise resultant force of two forces

1. Two forces of magnitude 7N and 24N act on a particle with their direction at 90^0. Find the magnitude and direction of the resultant. [25N, 16.26^0 with 24N force]
2. Forces of 5N and 8N act on a particle at an angle of 50^0 between them. Find the magnitude and direction of the resultant. [11.9N at 19^0 with 8N force]
3. Forces of 4N and 6N act on a particle at angle 60^0 between them. Find the magnitude and the direction of the resultant. [5.29N, at 40.9^0 with 6N force]
4. Forces of 9N and 10N act on a particle at angle 40^0 between them. Find the magnitude and the direction of the resultant. [17.9N, at 18.9^0 with 10N force]
5. Forces of 12N and 10N act on a particle at angle 105^0 between them. Find the magnitude and the direction of the resultant. [13.5N, at 45.7^0 with 12N force]
6. Forces of 8N and 3N act on a particle at angle 160^0 between them. Find the magnitude and the direction of the resultant. [5.28N, at 11.2^0 with 8N force]
7. Find the angle between a force of 10N and 4N their resultant has a magnitude of 8N. [130.5^0]
8. The angle between a force αN and a force of 3N is 120^0. If the resultant of the two forces has magnitude 7N, find the value of α. [8N]

The angle between a force βN and a force of 8N is 45^0. If the resultant of the two forces has a magnitude 15N, find the value of β. [8.24N]

Parallel forces in equilibrium

Conditions for a body to be in equilibrium

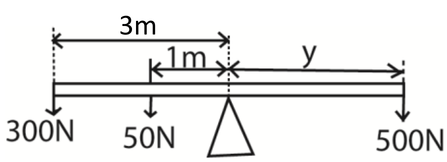
When a system of parallel forces act on a body then it will be in equilibrium when;

(i) the sum of forces acting in one direction are equal to the sum of forces acting in opposite direction.

(ii) sum of clockwise moments about a point are equal to the sum of anticlockwise moment about the same point

Example 1

Given the diagram below. Find the value of y

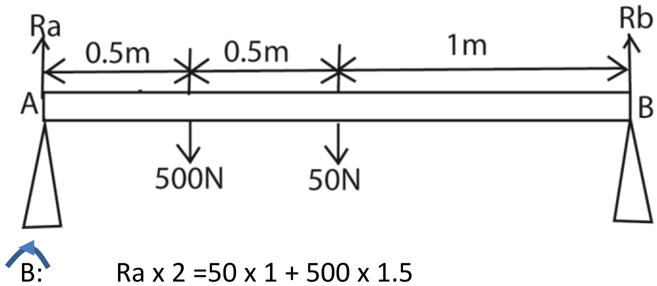


500 x y = (300 x 3) + (50 x 1)

y = $\frac{950}{500} = 1.9m$

Example 2

A uniform beam of weight 50N and length 2m rests horizontally on two supports pivoted at each end. A load of weight 500N is placed 0.5m from one end. Find the reaction on each support.



B: Ra x 2 =50 x 1 + 500 x 1.5

2Ra = 50 + 750

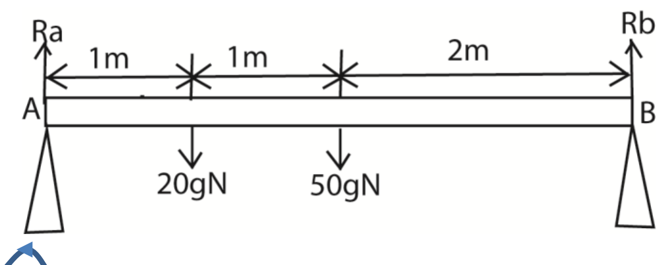
Ra = 400N

Also Ra + Rb = 500N + 50 N

Rb = 550N – 400N = 150N

Example 3

A uniform beam of mass 50kg and length 4m rests horizontally on two supports pivoted at each end. A load of 20kg is placed 1m from one end. Find the reaction on each support



B: Ra x 4 =50g x 2 + 20g x 3

4Ra = 100g + 60g = 160 x 9.8

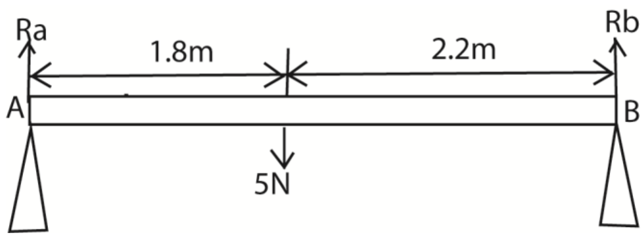
Ra = 392N

Also Ra + Rb = 500N + 50 N

Rb = 20gN + 50gN – 392N = 294N

Example 4

A non- uniform beam AB of length 4m has its weight 5N acting at a point 1.8m from end A. The beam rests horizontally on two supports pivoted at each end. Find the reaction on each support.



B: Ra x 4 =5 x 2.2 =11

Ra = 2.75N

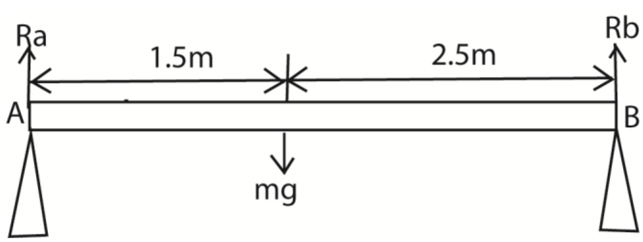
Also Ra + Rb = 5N

Rb = 5N – 2.75N = 2.25N

Example 5

A non- uniform beam AB of length 4m rests in horizontal position on vertical support at A and B. The centre of gravity is at 1.5m from end A. The reaction at B is 37.5N fine the

(a) mass of the beam (b) reaction at B



A: 37.5 x 4 = mg x 2.5

m = 10.2kg

Also Ra + 37.5 = 10.2 x 9.8

Ra = 62.5N

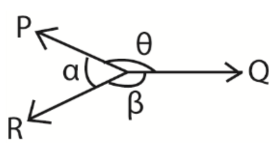
Revision exercise on parallel forces

1. A uniform beam AB of length 10m rests horizontally on two supports A and B. If the beam has a mass of 20g, find the reaction on each support. [98N, 9N]
2. A uniform beam of length 14m and mass 20kg rests horizontally on two supports, one at A and another at C which is 4m from B. find the reactions at each support [58.8N, 137.2N]
3. A uniform beam AB of length 10m and mass 20kg rests horizontally on two supports, one at A and another at C which is 2m from B. If a weight of mass 20kg is attached to the beam at a point 6m from A. Find the reaction on the supports. [392N, 196.2N]
4. A uniform beam AB of length 4m and mass 10kg rests horizontally on two supports at A and the other at C which is 1m from B. Where must a body of mass 50kg stand on the beam so that the reaction on each support is equal? [1.4m]

5. A uniform beam AB of length 12m and mass 12kg rests on two supports A and B. At what distance must a particle of mass 4g be tied so that the reaction of each support is equal. [9m from A]
6. A playground sea saw consists of a uniform beam of length 4m supported at its mid-point. If a girl of mass 25kg sits at one end of the sea saw, find where her brother of mass 40kg must sit if the sea saw is to balance horizontally. [75cm from other end]
7. A broom consists of a uniform broom stick of length 120cm and mass 4kg and a broom head of mass 6kg attached at the other end. Find where a support should be placed so that the broom balances horizontally. [24cm from the head]
8. A non-uniform beam AB of length 4m rests horizontally on two supports, one at A and the other at B. The reaction at the supports are 5gN and 3gN respectively. If instead the rod the rod were to rest horizontally on one support, find how far from A this support would have to be placed. [1.5m from A]
9. A uniform beam AB of mass 80g and of length 100cm is pivoted at 30cm from A, a force of 10N is placed on the beam at the 80cm from end A and a string is tied at the 40cm from end B so that the beam rests horizontally. Find the tension in the string. [17.2N]
10. A uniform beam AB of length 100cm is pivoted at 60cm from end B. The beam rests horizontally when a mass at A is 35g. Calculate the mass 9m) of the beam. [0.14kg]
11. A uniform meter rule pivoted at 10cm mark balances when a mass of 400N is suspended at the 0cm mark. If the system is in equilibrium. Find the mass of the ruler [10kg]
12. Two boys are carrying a uniform ladder of weight 800N, if the boys hold the ladder at 2m and 3m respectively from the centre of gravity, calculate the weight that each boy support. [480N, 320N]

Equilibrium of three forces Lami's theorem

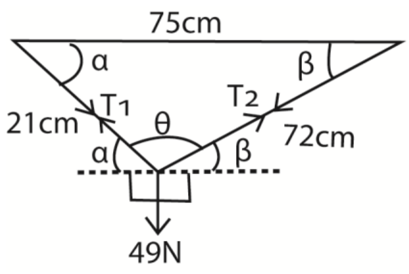
For any three forces acting on a particle in equilibrium where none of them is parallel to each other, Lami's theorem is applicable



$$\frac{P}{sin\beta} = \frac{Q}{sin\alpha} = \frac{R}{sin\theta}$$

Example 1

A weight of 49N is suspended by two strings of length 21 cm and 72cm attached to 2 points in a horizontal line a distance of 75cm apart. Find the tension in the strings so that the particle remain in equilibrium



By cosine rule:

$75^2 = 21^2 + 72^2 - 2 \times 21 \times 72\cos\theta$

$\theta = 90^0$

Similarly, $\beta = 16.26^0$ and $\alpha = 73.74^0$

$\frac{T_1}{\sin(16.26+90)} = \frac{49}{\sin 90}$;

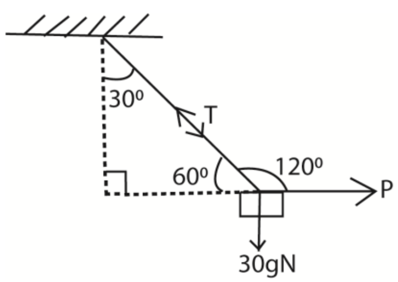
$\therefore T_1 = 47N$

$\frac{T_2}{\sin(73.74+90)} = \frac{49}{\sin 90}$;

$\therefore T_2 = 13.72N$

Example 2

Mass of 30kg hangs vertically at the end of a light string. If the mass is pulled by a horizontal force P so that the string makes 30^0 with the vertical. Find the magnitude of the force and the tension in the string so that the particle remain in equilibrium.

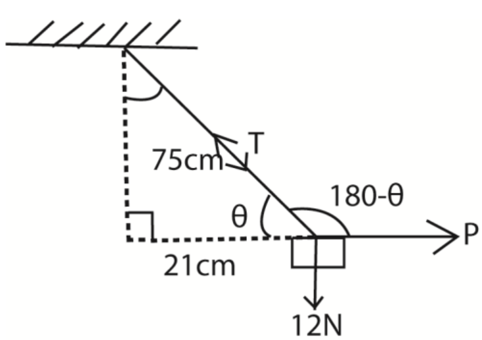


$\frac{T}{\sin 90} = \frac{30 \times 9.8}{\sin 120}$; T = 339.48N

$\frac{P}{\sin(60+90)} = \frac{30 \times 9.8}{\sin 120}$; P = 169.74N

Example 3

One end of a light inextensible string of length 75cm is fixed to a point on a rigid pole. The particle of weight 12N is attached to the other end of the string. The particle is held 21cm away from the pole by a horizontal force, P. Find the magnitude of the force, P and the tension of the string so that the particle remain in equilibrium



$\theta = \cos^{-1}\left(\frac{21}{75}\right) = 73.74^0$

$\frac{T}{\sin 90} = \frac{12}{\sin(180-73.74)}$

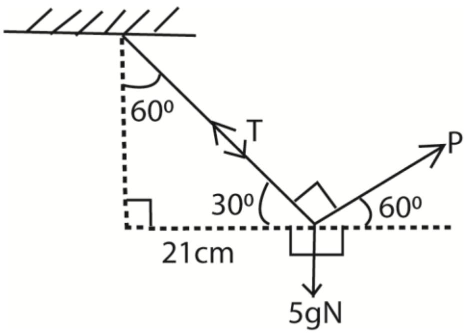
T = 12.5N

$\frac{P}{\sin(90+73.4)} = \frac{12}{\sin(180-73.74)}$

P = 3.5N

Example 4

A light inextensible string AB whose end A is fixed has end B attached to a particle of mass 5kg. A force P acting perpendicular to the string is applied on the particle keeping it in equilibrium with the string inclined at 60^0 to the vertical. Find the value of P and the tension in the string



$$\frac{T}{\sin(90+60)} = \frac{5 \, x \, 9.8}{\sin 90}$$

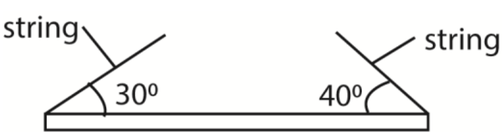
T = 24.5N

$$\frac{5 \, x \, 98}{\sin 90} = \frac{P}{\sin \ (90+30)}$$

P = 42.44N

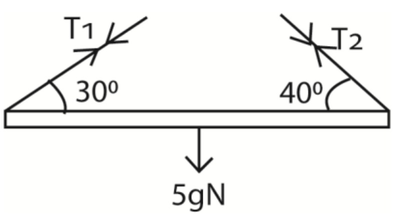
Example 5

A non-uniform beam of mass 5kg rests horizontally in equilibrium supported by two strings attached to the ends of the beam.



The strings make 30^0 and 40^0 with the horizontal beam as shown above. Find the tension in the strings.

Solution



(→) $T_1\cos30 = T_2\cos40$; $T_1 = 0.8846T_2$

(↑)$T_1\sin30 + T_2\sin40 = 5g$

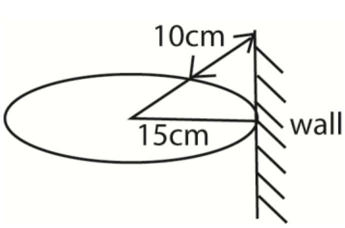
$0.8846T_2\sin30 + T_2\sin40 = 5 \text{ x } 9.8$

$T_2 = 45.159$N

$T_1 = 0.8846 \text{ x } 45.159 = 39.94$N

Example 6

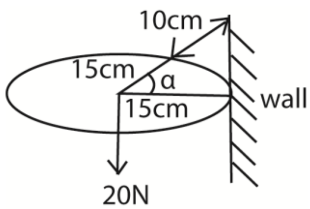
A sphere of weight 20N and radius 15cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 10cm attached to a point on the sphere and to a point on the wall as shown.



(i) calculate the reaction on the sphere due to the wall

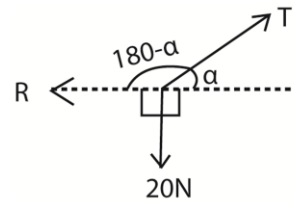
(ii) Find the tension in the string

Solution



α = $\cos^{-1}\left(\frac{15}{25}\right)$ = 53.13^0

Using Lami's theory



$\frac{T}{sin90} = \frac{20}{\sin(180-53.13)}$; T =25N

$\frac{R}{\sin(90+53.13)} = \frac{20}{\sin(180-53.13)}$; R = 15N

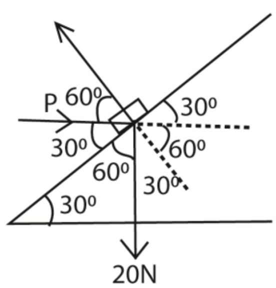
Example 7

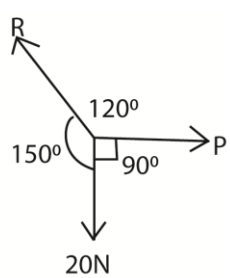
A particle of weight 20N is held at equilibrium on a smooth plane inclined at 30^0 to the horizontal by a horizontal force P.

(i) Find the value of P and the reaction between the particle and the plane.

(ii) If the force P is removed and a string parallel to the plane is used to hold the particle, find the tension in the string and the new value of the reaction.

Solution

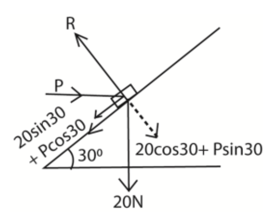




$\frac{P}{sin150} = \frac{R}{sin90} = \frac{20}{sin120}$

R = 23.09N and P = 11.55N

Alternatively: by resolving forces



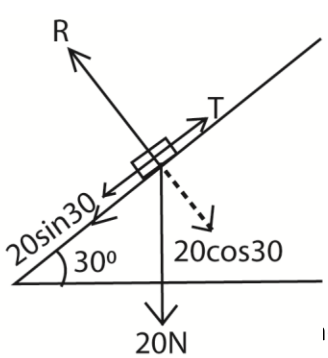
At equilibrium parallel to plane forces = 0

Pcos30 + 20sin30 = 0; P = 11.55N

R = 20cos30 + Psin30

R = 20cos30 + 11.55sin30 = 23.09N

(ii)



Alternatively by Lami's theory

$\frac{T}{sin150} = \frac{R}{sin120} = \frac{20}{sin90}$

T = 10N

R = 1.3N

Parallel to the plane T = 20sin30 = 10N

Perpendicular to the plane R = 20cos30 = 13N

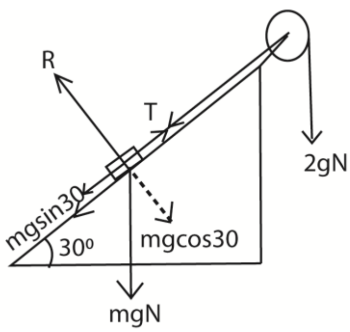
Example 8

A light inextensible string passes over a smooth fixed pulley at the top of a smooth plane inclined at 300 to the horizontal. A particle of mass 2kg is attached to one end of the string and rests vertically in equilibrium when the particle of ma resting on the surface of the plane is attached to the other end of the string. Find

(i) the normal reaction between m and the plane
(ii) tension in the string and the value of m.

Solution

By resolving forces



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

Parallel to the plane

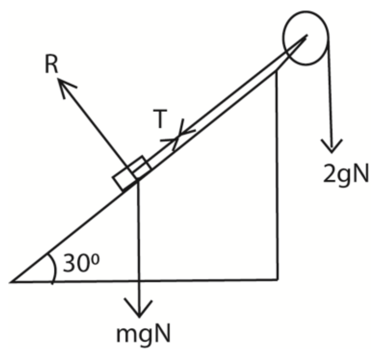
T – mgsin30 = 0; m = 4kg

Perpendicular to the plane

R = mgco30

R = 4 x 98cos30 = 33.98

Alternatively by using Lami's theorem



For 2kg mass: T – 2 x 9.8 = 0; T = 19.62N

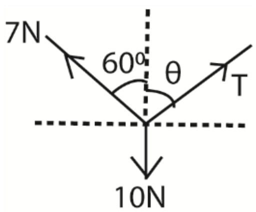
$$\frac{T}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$$

$$\frac{19.62}{sin150} = \frac{mg}{sin90} = \frac{R}{sin120}$$

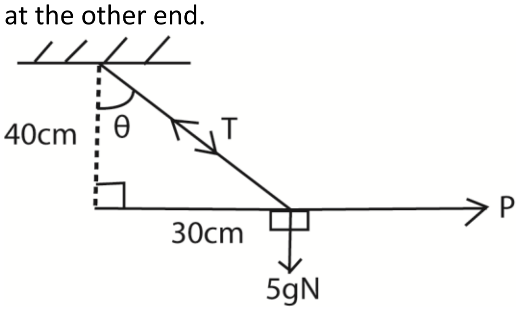
m = 4kg and R = 33.98N

Revision exercise on three forces in equilibrium

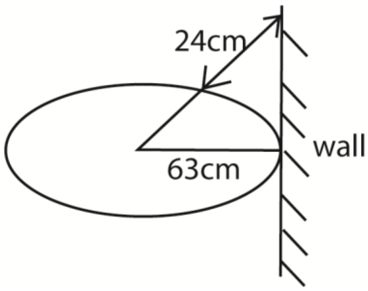
1. A particle P of mass 2kg is suspended from a fixed point O by means of a light inextensible string. The string is taut and makes an angle of 300 with the downward vertical through O and a particle is held in equilibrium by means of a horizontal force of magnitude F acting on the particle. Find the value of F and the tension in the string [F = 11.3161, T = 22.6321N]
2. A particle of mass 3kg lies on a smooth plane inclined at angle θ to the horizontal, where tanθ = $\frac{3}{4}$. The particle is held in equilibrium by horizontal force of magnitude FN. The line of action of this force is the same vertical plane as a line of greatest slope of inclined plane. Find the value of F. [22.05N]
3. The diagram below shows a body of weight 10N supported in equilibrium by two light inextensible strings. The tension in the strings are 7N and T and the angle the string makes with the upward vertical are 60^0 and θ respectively.



   Find T and θ. [T = 8.9N, θ = 43^0]
4. A particle of weight 8N is attached to a point B by a light inextensible string AB. It hangs in equilibrium with point A fixed and AB at an angle of 30^0 to the downward vertical. A force F at B acting at right angles to AB, keeps the particle in equilibrium. Find the magnitude of force F and the tension in the string. [4N, $4\sqrt{3}N$]
5. The diagram shows a light inextensible string with one end fixed at A and a mass of 5kg suspended at the other end.

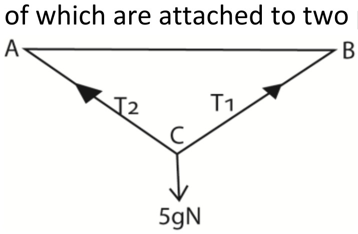


   The mass is held in equilibrium at an angle θ to the downward vertical by a horizontal force P. Find the value of θ, P and the tension in the string [θ = 36.9^0, P = 36.75N, T = 61.25N)
6. A sphere of mass 5kg and radius 63cm rests against a smooth vertical wall. A sphere is supported in its position by a string of length 24cm attached to a point on the sphere an to a point on the wall as shown.



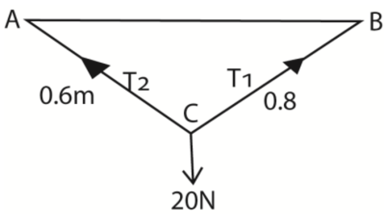
Find the tension in the string. [71.05N]

7. A particle whose weight is 50N is suspended by a light string which is 35^0 to the vertical under the action of a horizontal force F. Find the force F and the tension in the string. [35.0N, 61.0N]
8. A particle of weight w rests on a smooth plane which inclined at 40^0 to horizontal. The particle is prevented from slipping by a force of 50.0N acting parallel to the plane and up a line of greatest slope. Calculate w and reaction due to the plane. [77.8N, 59.6N]
9. A mass of 2kg is suspended by two light inextensible strings. One making an angle of 600 with the upward vertical and the other 300with the upward vertical. Find the tension in each string. [9.8N, 17.0N]
10. A heavy uniform rod of weight W is hung from a point by two equal strings, one attached to each end of the rod. A body of weight w is hang half-way between A and the center of the rod. Prove that the ratio of tension in the string is$\frac{2W+3w}{2W+w}$.
11. A non-uniform beam AB of length 8m and its weight 10N acts from a point G between A and B such that AG = 6m. The beam is supported horizontally by strings attached to A and B. The string attached to A makes an angle of 30^0 with AB. Find the angle that the string attached to B makes with AB and find the tension in the strings. [60^0, 5N, 8.66N]
12. A light inextensible string of length 40cm has its upper end fixed to a point A and carries a mass of 2kg at its lower end. A horizontal force applied to the mass keeps it in equilibrium, 20cm from the vertical through A. Find the magnitude of this horizontal force and the tension in the string. [11.3N, 22.6N]
13. The diagram shows a body of mass 5kg supported by two light inextensible strings, the other ends of which are attached to two points A and B on same level as each other end 7m apart.



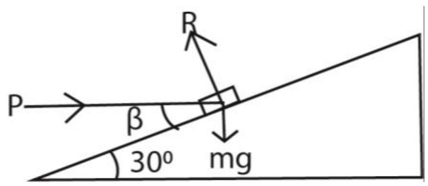
The body rests in equilibrium at 3m vertically below AB. If angle CBA = 45^0, find T_1 and T_2 the tensions in the strings. [35N, $28\sqrt{2}N$]

14. The diagram shows a body of weight 20N supported by two light inextensible strings of length 0.6m and 0.8m from two points 1m apart on a horizontal beam.



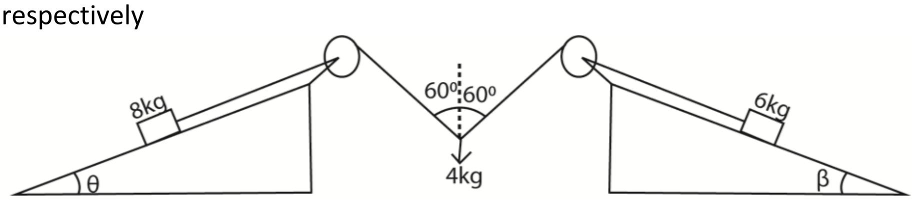
The body rests in equilibrium, find T_1 and T_2 the tensions in the strings. [16N, 12N]

15. A light inextensible string of length 50cm has its upper end fixed at point A and carries a particle of 8kg at its lower end. A horizontal force P applied to the particle in equilibrium 30cm from the vertical through A, find the magnitude of P and the tension in the string. [58.8N, 98N}
16. A article is in equilibrium under the action of forces 4N due north, 8N due west, $5\sqrt{2}$N south east and P, find the magnitude and direction of P. [3.16N, N71.6^{0}E]
17. A force P holds a particle of mass mkg in equilibrium on a smooth plane which is inclined at 300 to the horizontal.



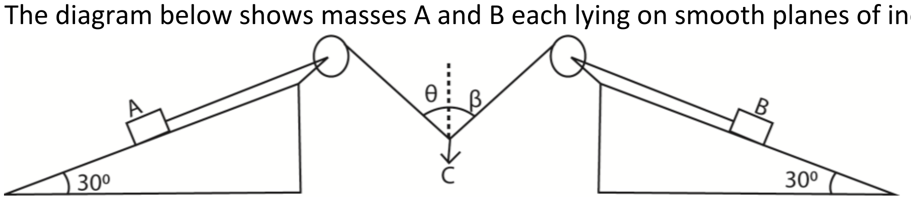
If P makes an angle β with the plane, find β when R the normal reaction between the particle and the plane is 15mg [51.7^0]

18. The diagram below shows masses of 8kg and 6kg lying on smooth planes of inclination θ and β respectively



Light inextensible strings attached to these masses pass along the line of greatest slopes over smooth pulleys and are connected to 4kg mass hanging freely. The strings both make an angle of 600 with the upward vertical as shown above. If the system rest in equilibrium find θ and β. [θ = 30^0 and β 41.8^0]

19. The diagram below shows masses A and B each lying on smooth planes of inclination30^0.



Light inextensible strings attached to A and B pass along the lines of greatest slopes, over smooth pulleys and are connected to a third mass C hanging freely. The strings make angles of θ and β with the upward vertical as shown above. If A, B and C have masses 2m, m, and m respectively and the system rests in equilibrium show that sinθ = 2sinβ and cosβ +2cosθ =2. Hence find θ and β. [29.0^0, 75.5^0]

Finding Equilibrium of force using vectors

Several forces acting on a particle are said to be in equilibrium when the resultant force is equal to zero

i.e. $F_R = \binom{0}{0}$

Example 1

For the following set of forces in equilibrium find the values of a and b in each case

(i) (6i + 4j)N, (-2i -5j)N, (ai + bj)N

$\binom{6}{4} + \binom{-2}{-5} + \binom{a}{b} = \binom{0}{0}$

6 – 2 + a = 0 => a = -4

4 – 5 + b = 0 => b = 1

(ii) (5i + aj + ck)N, (bi – 6j – k)N, and (-3i + 2j + ck)

$\begin{pmatrix}5\\a\\c\end{pmatrix} + \begin{pmatrix}b\\-6\\-1\end{pmatrix} + \begin{pmatrix}-3\\2\\c\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

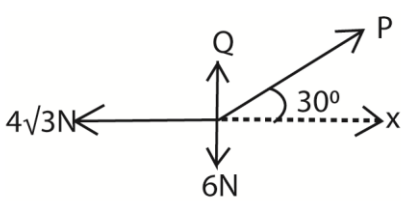
5 + b – 3 = 0 => b = -2

a – 6 +2 = 0 => a = 4

2c – 1 = 0 => c = 0.5

Example 2

In the diagram below, the particle is in equilibrium, find the values of P and Q.



Solution

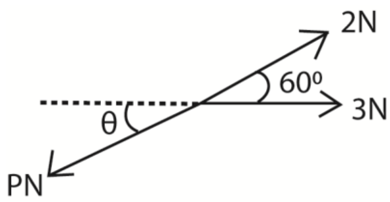
$\binom{0}{Q} + \binom{Pcos30}{Qcos30} + \binom{0}{-6} + \binom{-4\sqrt{3}}{0} = \binom{0}{0}$

0 + Pcos30 + 0 -4$\sqrt{3}$ = 0; => P = 8N

Q + Qcos 30 – 6 + 0 = 0; => Q = 2N

Example 3

Diagram below shows three coplanar forces of magnitude 2N, 3N and PN all acting at point O in the direction shown. Given that the forces are in equilibrium, Find the value of P



Solution

-Pcos θ + 2cos 60 + 3 = 0 …. (i)

cos θ = $\frac{2cos60+3}{P}$

-Psinθ + 2sin60 = 0

sin θ = $\frac{2sin60}{P}$ ……………… (ii)

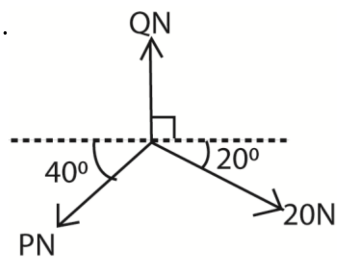
Eqn. (i) and (ii)

θ = $\tan^{-1}\left(\frac{2sin60}{2cos60+3}\right)$= 23.413

from (ii) P = $\frac{2\sin 60}{\sin 23.413}$ =4.3589N

Revision Exercise on forces in equilibrium

1 .



(i) The diagram above shows three coplanar forces in equilibrium. Find the value of P and Q.

(ii) If the direction of Q s now reversed, find the magnitude and direction of the resultant [(i) 24.5N, 22.6N; (ii) 45.2N]

2. Forces F_1 = (-3i + 7j)N, F_2= (i – j)N and F_3 =(pi +qj) act on a particle.

(i) If the particle is in equilibrium, find the vlues of p and q. [p = 2, q= -6]

(ii) Find the magnitude and direction of the resultant of F1 and F2 . [6.3246N, 71.57^0]

3. Forces of 6N, 5N, 8N, 5N and 9N act on a particles in the direction N30^{0}E, N30^{0}W, S50^{0}E, N60^{0}W, N80^{0}E and S40^{0}W respectively. Find the additional force that will keep the system of force in equilibrium. [5.358N at 68.920 above the positive axis]

4. Forces of 7N, 2N, 4N and 5N act on a particle in directions of 0600, 1600 2000 and 3150 respectively. Find the additional force that will keep the system of forces in equilibrium. [2.3125N at 37.18^0 below the negative axis]

5. Forces of 2N, 1N, 3N and 4N act on a particle in the direction 0^0, 90^0, 270^0 and 330^0 respectively. Find the additional force that will keep the system of forces in equilibrium. [6.8N at 36^0 above the negative axis]

6. Forces of 6N, 5N, 7N, 4N, $3\sqrt{2}\,N$ and $7\sqrt{2}$N act in direction AB, CB, CD, DA, CA and DB respectively on a square ABCD. Find the additional force that will keep the system of forces in equilibrium. [19.2N at 81^0 above the negative axis]

7. Forces 8N, 7N, 6N, 4N, 7N and 6N act along the sides of a regular hexagon ABCDEF in direction AB, CB, CD, DE, EF and FA respectively. Find the additional force that will keep the system of forces in equilibrium. [12.49N at 76^0 above AB]

Resolutions of forces acting on a polygon

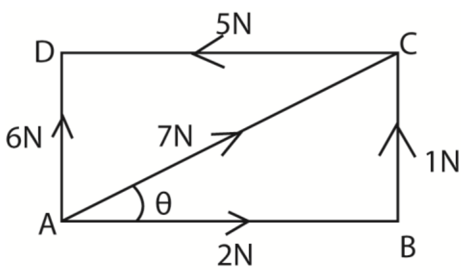
For any regular polygon

- all sides are equal
- all angles are equal
- an exterior angle $=\frac{360}{n}$ where n is the number of sides

Example 1

ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 2N, 1N, 5N,6N and 7N act along AB, BC, CD, AD and AC respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine

(i) the magnitude of the resultant force

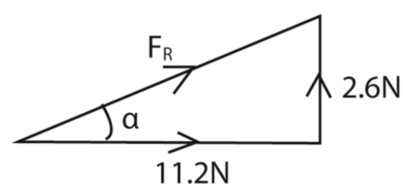


$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87$

R = $\binom{2}{0} + \binom{0}{1} + \binom{-5}{0} + \binom{0}{6} + \binom{7cos36.87}{7sin36.87} = \binom{2.6}{11.2}$

$R = \sqrt{2.6^2 + 11.2^2}$ = 11.498N

(ii) direction of the resultant with AB



Direction,$\alpha = \tan^{-1}\left(\frac{2.6}{11.2}\right)$

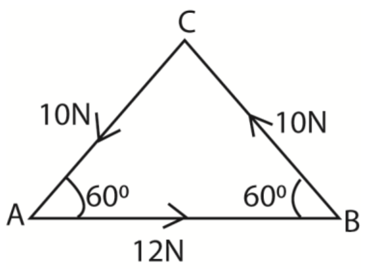
$=13.069^0$

Direction is 13.069^0 above AB

Example 2

ABC is an equilateral triangle. Forces of magnitude 12N, 10N and 10N act along AB, BC and CA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine

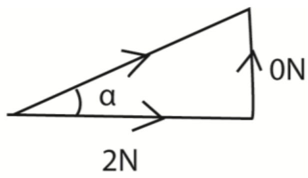
(i) the magnitude of the resultant force



$R = \begin{pmatrix}12\\0\end{pmatrix} + \begin{pmatrix}-10cos60\\10sin60\end{pmatrix} + \begin{pmatrix}-10cos60\\-10sin60\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

$R = \sqrt{2^2 + 0^2}$ = 2N

(ii) Direction of the resultant with AB

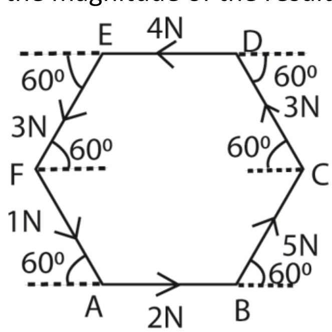


Direction $\alpha = \tan^{-1}\left(\frac{0}{2}\right) = 0$

Example 3

ABCDEF is a regular hexagon. Force of magnitude 2N, 5N, 3N, 4N, 3N and 1N act along the line AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine

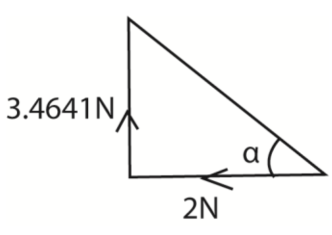
(i) the magnitude of the resultant force and



$R = \begin{pmatrix}2\\0\end{pmatrix} + \begin{pmatrix}5cos60\\5sin60\end{pmatrix} + \begin{pmatrix}-3cos60\\3sin60\end{pmatrix} + \begin{pmatrix}-4\\0\end{pmatrix} + \begin{pmatrix}-3cos60\\-3sin60\end{pmatrix} + \begin{pmatrix}1\cos 60\\-1sin60\end{pmatrix} = \begin{pmatrix}-2\\3.4641\end{pmatrix}$

$R = \sqrt{(-2)^2 + 3.4641^2} = 4N$

(ii) direction of the resultant with AB.

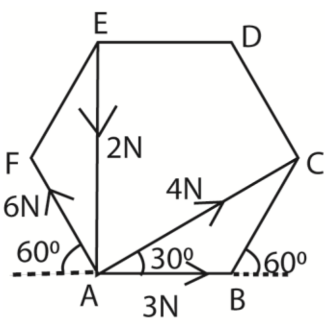


$\alpha = \tan^{-1}\left(\frac{3.461}{2}\right)$ = 60^0 to AB

Example 4

ABCDEF is a regular hexagon. Forces of magnitude 3N, 4N, 2N and 6N act along the line AB, AC, EA and AF respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine

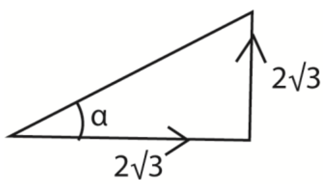
(i) the magnitude of the resultant force



$$R = \binom{3}{0} + \binom{4cos30}{4sin30} + \binom{-6cos60}{6sin60} + \binom{0}{-2} = \binom{2\sqrt{3}}{3\sqrt{3}}$$

$$R = \sqrt{(2\sqrt{3})^2 + (3\sqrt{3})^2} = 6.245\text{N}$$

(ii) direction of the resultant force



$$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{2\sqrt{3}}\right) = 56.3^0$$

Revision questions on resolution of forces acting on a polygon

1. ABCD is a square. Forces of magnitude 6N, 4N and $2\sqrt{2}N$ act along AD, AB and AC respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [10N at 53.1^0 with AB]
2. ABCD is a square. Forces of magnitude 2N, 1N, $\sqrt{2}N$ and 4N act along AB, BC and AC and DA respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [5.13N at 33.7^0 with AB]
3. ABCD is a square. Three forces of magnitude 4N, 10N and $7N$ act along AB, AD and CA respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude [5.1388N]
4. In equilateral triangle PQR, three forces of magnitude 5N, 10N and 8N act along the side PQ, QR and PR respectively. Their direction are the order the letters. Find the magnitude of the resultant force. [16.1N]
5. ABCD is a square. Forces of magnitude $6\sqrt{3}N$ N, 2N and $4\sqrt{3}N$ act along AB, CB and CD respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [4N at 30^0 to AB]
6. ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 3N, 1N, and 10N act along AB, DC and AC respectively. In each case the direction of the force being given by the order of

the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force. [13.4N at 26.6^0 with AB]

7. ABCD is a rectangle. Forces of magnitude 8N, 4N, 10N and 2N act along AB, CB, CD and AD respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force. [283N at 45^0 at AB]
8. In equilateral triangle ABC, forces of magnitude 10N each act along the side AB, BC and AC respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [20N at $60^0$to AB]
9. In equilateral triangle ABC, forces of magnitude 5N, 9N and 7N act along the side AB, BC and CA respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [$2\sqrt{3}$N at $30^0$to AB]
10. In equilateral triangle ABC, forces of magnitude 4N, 4N and 6N act along the side AB, BC and AC respectively. Their direction are the order the letters. Find the magnitude of the resultant force and the angle it makes with AB. [10N at 60^0 to AB]
11. ABCDEF is a regular hexagon. Forces of magnitude 2N, 5N, 3N, 4N, 3N and 1N act along the line AB, BC, CD,DE, EF and AF respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [6N at 60^0 to AB]
12. ABCDEF is a regular hexagon. Forces of magnitude 8N, 7N, 6N, 4N, 7N, and 6N act along the line AB, BC, CD, DE, EF and FA respectively, in each case the direction of the force being given by the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [12.5N at 76^0 to AB]
13. PQRSTU is a regular hexagon. Forces of magnitude 4N, 5N, 2N, and 6N act along the line PQ, PR, PT and PU respectively, in each case the direction of the force being given by the order of the letters. Given that PQ is horizontal, determine the magnitude and direction of the resultant force. [11.065N at 61.2^0 to PQ]
14. ABCD is a square. Forces of magnitude $10N$ N, 9N, 8N and $5N$ act along AB, BC, CD and AD respectively in each case the direction of force being the order of the letters. Given that AB is horizontal, determine the magnitude and direction of the resultant force. [$2\sqrt{5}$N at 63.43^0 to AB]

ABCD is a rectangle with AB= 4cm and BC = 3cm. Forces of magnitude 3N, 10N, 4N, 6N and 5N act along AB, BC, CD, DA, and AC respectively. In each case the direction of the force being given by the order of the letters. Given that AB is horizontal determine the magnitude and direction of the resultant force. [7.62N at 66.8^0 with AB]

Friction

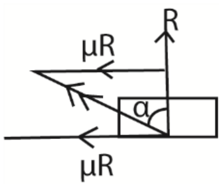
Friction is a force that opposes relative motion or attempted motion between two bodies in contact.

Friction force F = μR where R = normal reaction and μ = coefficient of friction

At limiting equilibrium, the body is at the point of moving (slip or slide) and friction force is maximum.

Angle of friction

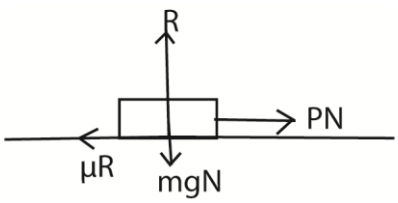
This is the angle between the resultant force and the normal reaction



tanα = $\frac{\mu R}{R} = \mu$

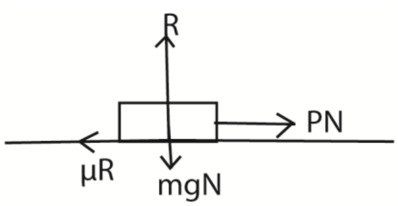
A horizontal plane

(i) at limiting equilibrium (about to slip or slid)



P = μR

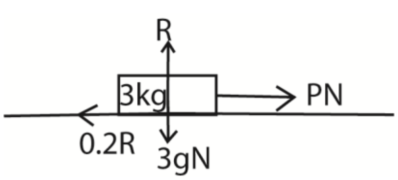
(ii) In motion



P – μR = ma

Example 1

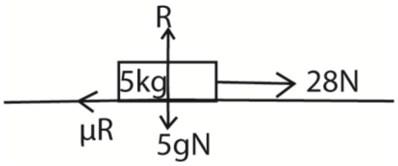
Calculate the maximum frictional force which can act when a block of mass 3kg rests on a rough horizontal surface, the coefficient of friction between the surface being 0.2



F = μR = 0.2 x 3 x 9.8 = 5.88N

Example 2

When a horizontal force of 28N is applied to a body of mass 5kg which is resting on a rough horizontal surface, the body is found to in limiting equilibrium. Find the coefficient of friction between the body and the plane



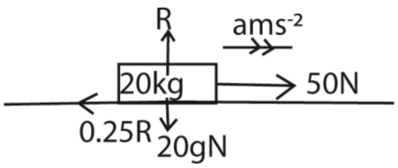
28 = μR

28 = μ x 5 x 9.8

$\mu = 0.57$

Example 3

A block of mass 20kg rests on a rough horizontal plane. The coefficient of friction between the block and the plane is 0.25. If a horizontal force of 50N acts no the body, find the acceleration of the body.



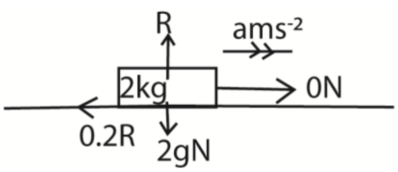
50 - μR = 20a

50 - $(0.25 \; x \; 20 \; x \; 9.8)$ = 20a

a = 0.05ms^{-2}

Example 4

A block of mass 2kg sliding along a smooth surface at a constant speed of 2ms-1. When the mass encounter a rough surface of coefficient of friction 0.2, it comes to rest. Find the distance the body will move across the rough surface before it comes to rest.



F = ma

0 – μR = 20a

-0.2 x 2 x 9.8 = 20a

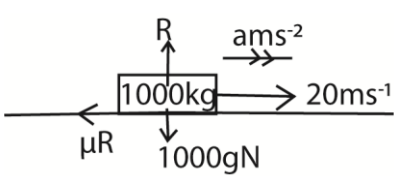
a = -1.96ms^{-2}

s = $\frac{v^2 - u^2}{2a} = \frac{0^2 - 20^2}{2x\,(-1.96)}$ = 1.02m

Example 5

A car of mass 1000kg moving along a straight road with speed of 72kmh^{-1} is brought to rest by a speedy application of brakes in a distance of 5m. Find the coefficient of kinetic friction between the tyres and the road.

u = $\frac{72 \; x \; 1000}{2600}$ = 20ms^{-1}



a = $\frac{v^2 - u^2}{2s} = \frac{0^2 - 20^2}{2x\;5}$ = -4ms^{-2}

ma = μR

4 x 1000 = 1000 x 9.8 x μ

μ = 0.41

Alternatively

Work done against friction = loss in kinetic energy

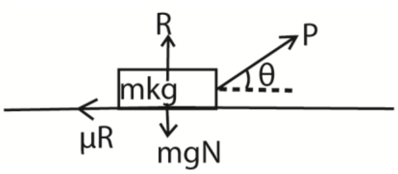
μ(mg)x s = $\frac{1}{2}mv^2$

$\mu \times 9.8 \times 50 = \frac{1}{2} x\ 20^2$

$\mu = 0.408$

Revision exercise on friction

1. When a horizontal force of 0.245N is applied to a body of mass 250g which is resting on a rough horizontal plane, the body is found to be in limiting equilibrium. Find the coefficient of friction between the body and the plane. [0.1]
2. A body of mass 40kg is resting on a rough horizontal plane and can just move by a force of 98N acting horizontally. Find the coefficient of friction. [0.25]
3. A block of mass 0.5kg rests on a rough horizontal plane. The coefficient of friction between the block and the table is 0.1. When a horizontal force of 1N acts on the block, find
   (i) friction force experienced by the block. [0.49N]
   (ii) acceleration with which the block will move. [1.02ms^{-2}]
4. When a horizontal force of 37N is applied to the body of mass 10kg which is resting on a rough horizontal surface, the body moves along the surface with acceleration 1.25ms^{-2}. Find the coefficient of friction between the body and the surface. [0.25]
5. **A force inclined at an angle θ to the horizontal**



At limiting equilibrium

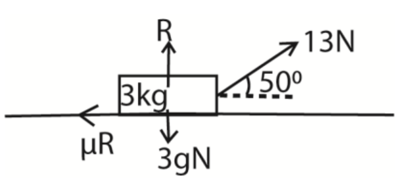
(→): Pcosθ = μR

(↑): R + Psinθ = mg

Example 6

A particle of mass 3kg resting on a rough horizontal plane is pulled by a force of magnitude 13N inclined at an angle 50^0 to the horizontal, if the particle does not move find the

(i) Normal reaction (ii) coefficient of friction

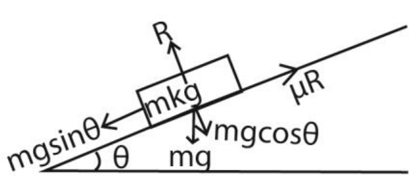


(↑):R = 3 x 9.8 – 13sin 50^0 = 19.4414N

(→): 13cos50^0 = μ x 19.4414

μ = 0.4298

Friction and inclined planes



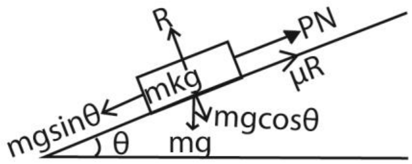
At limiting equilibrium

mgsinθ = μR

mgsinθ = μmgcosθ

μ = tanθ

(ii) A force P applied parallel to and up the plane to just move the particle upwards

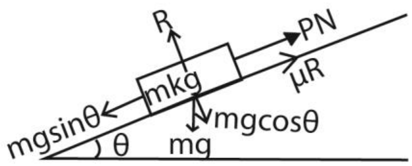


Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ + μR = P

P = mgsinθ + μmgcosθ

(iii)
(iv) A force P applied parallel to and up the plane so that the particle is on the point of moving downwards (prevent moving downwards)

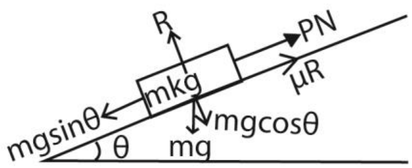


Normal to the plane: mgcosθ = R

Parallel to the plane; mgsinθ = P+ μR

P = mgsinθ - μmgcosθ

(v)
(vi) A force P applied parallel to and up the plane to move the particle upwards



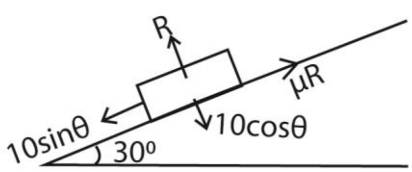
Normal to the plane: mgcosθ = R

Parallel to the plane; P- (mgsinθ + μR) = ma

P – (mgsinθ + μmgcosθ) = ma

Example 7

A particle of weight 10N rests on a rough plane inclined at $30^0$to the horizontal and is just about to slip. Find the value of coefficient of friction between the plane and the particle.



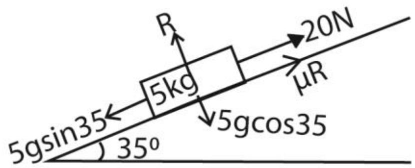
R = 10cos 30 and μR = 10sin30

μ(10cos30) = 10sin30

μ = 0.5774

Example 8

A body of mass 5kg lies on a rough plane which is inclined at 35^0 to the horizontal. When a force of 20N is applied to the body parallel to and up the plane, the body is on the point of moving down the plane. Find the coefficient of friction between the body and the plane.



At limiting equilibrium

R = 5gcos 35

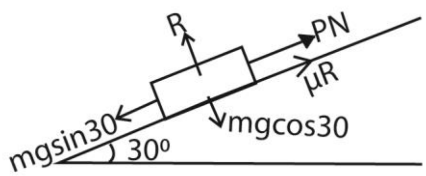
20 +μR = 5gsin35

20 +μ(5gcos35) =5gsin35

μ = 0.2

Example 9

A block of wood of mass 150g rest on an inclined plane. If the coefficient of friction between the surface of contact is 0.3. Find the force parallel to the plane necessary to prevent slipping when the angle of the plane to the horizontal is 30^0.



At limiting equilibrium

R =0.15 x 9.8cos 30

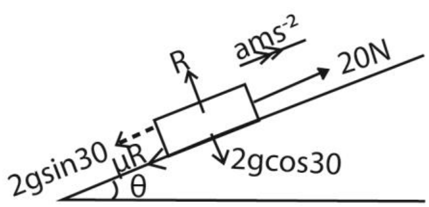
P +μR = 0.15 x 9.8sin30

P +0.3(0.15 x 9.8cos 30) =5gsin30;

P= 0.353N

Example 10

A body of mass 2kg lies on a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$ to the horizontal. A force of 20N is applied to the body, parallel to and up the plane. If the body accelerates up the plane at $1.5ms^{-2}$, find the coefficient of friction between the body and the plane.



Sin θ =$\frac{5}{13}$, cos θ = $\frac{12}{13}$

R = 2 x 9.8 cosθ = 2 x 9.8 x $\frac{12}{13}$ =18.09N

F = ma

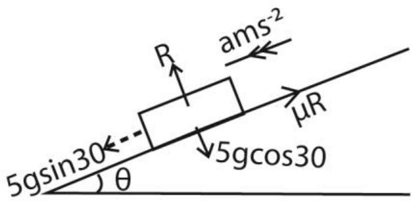
20 – (2gsinθ + μR) = 2a

20 –(2 x 9.8 x $\frac{5}{13}$ + 2 x 9.8 x $\frac{12}{13}$ x μ) = 2 x 1.5

μ =0.523

Example 11

A body of mass 5kg is released from rest on a rough surface of a plane inclined at 30^0 to the horizontal. If the body takes2.5s to acquire a speed of $4ms^{-2}$ from rest, find the frictional force and coefficient of friction.



v = u + at

4 = 0 + 2.5a

a = 1.6ms^{-2}

F = ma

5 x 9.8sin30 – μR = 5 x 1.6;

Frictional force, μR = 16.5N

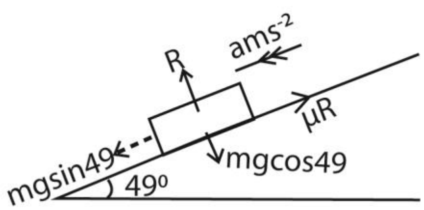
μR = 16.5N

μ = $\frac{16.6}{5 \: x \: 9.8cos30}$ = 0.243

Example 12

A car of mass 500kg moves from rest with engine switched off down a road which is inclined at an angle 49^0 to the horizontal.

(a) calculate the normal reaction
(b) if the coefficient of friction between the tyres and the surface of the road is 0.32. Find the acceleration of the car.



(a) R =mgcos49 = 500 x 9.8cos49 = 3217.97N

(b) F = ma

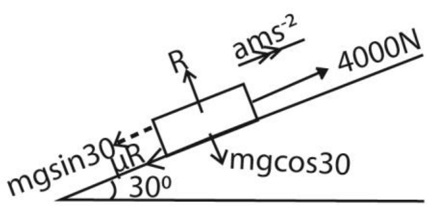
mgsin49 – μR = 500a

500 x 9.8sin49 – 0.32 x 3217.97 = 500a

a = 5.34ms^{-2}

Example 13

A car of mass 1000kg climbs a plane which is inclined at 30^0 to the horizontal. The speed of the car at the bottom of the incline is 36kmh^{-1}. If the coefficient of friction between the plane and the car tyres is 0.3 and the engine exerts a force of 4000N, how far up the incline does the car move in 5s



u = 36kh^{-1} = $\frac{36 \: x \: 1000}{3600}$ = 10ms^{-1}

F = ma

4000 – (mgsin30 + μR) = ma

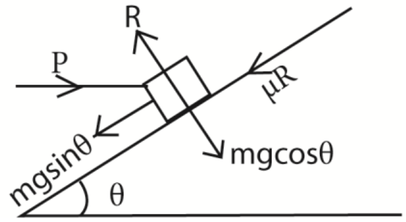
4000 – (1000 x 9.8sin30 + 0.3 x 1000 x 9.8cos30) = 1000a

a = -3.45ms^{-2}

s = ut + $\frac{1}{2}at^2$ = 10 x 5 + $\frac{1}{2} \: x \: (-3.45) x \: 5^2$ = 6.9m

Horizontal force on inclined planes

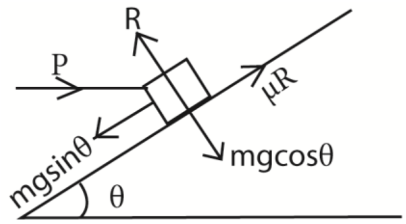
(i) A horizontal force P required to just move the particle upwards



Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ + μR = Pcosθ

(ii) a horizontal force O required to prevent the particle from moving downwards

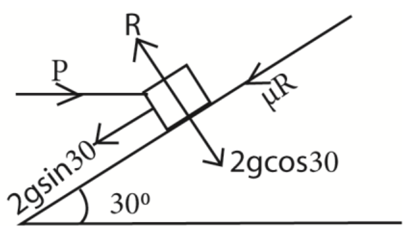


Normal to the plane: mgcosθ + Psinθ = R

Parallel to the plane: mgsinθ - μR = Pcosθ

Example 14

A body of mass 2kg lies on a rough plane inclined at 300 to the horizontal. When a horizontal force of 20N is applied to the body in an attempt to push it up the plane, the body is found to be on the point of moving up the plane. Find the coefficient of friction between the body and the plane.



At limiting
Normal to the plane: 2gcos30 + 20sin30 = R……(i)

Parallel to the plane: 2gsin30 + μR = 20cos30 …… (ii)

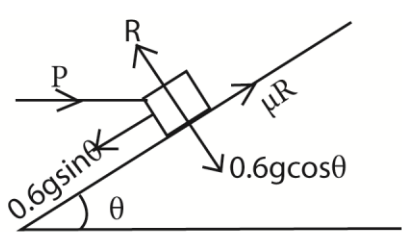
(i) and (ii)

2gsin30 +μ(2gcos30 + 20sin30) = 20cos30

μ =0.279

Example 15

A horizontal force of 1N is just sufficient to prevent a brick of mass 600g sliding down a rough plane which is inclined at $\sin^{-1}\left(\frac{5}{13}\right)$to horizontal. Find the coefficient of friction between the brick and the plane.



$\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$

At limiting
Normal to the plane: 0.6gcosθ + 1xsinθ = R……(i)

Parallel to the plane: 0.6gsinθ - μR = 1 x cosθ …… (ii)

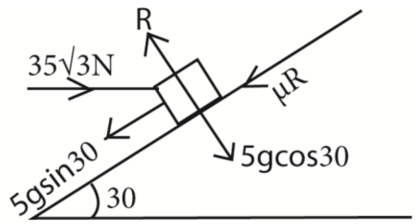
(i) and (ii)

$0.6 \times 9.8 \times \frac{5}{13} - \mu(0.6 \times 9.8 \times \frac{12}{13} + 1 \times \frac{5}{13}) = 1 \times \frac{12}{13}$

μ =0.23

Example 16

A body of mass 5kg is initially at the bottom of a rough inclined plane of length 6.3m. The plane is inclined to the horizontal and the coefficient of friction between the body and the plane is $\frac{1}{2}\sqrt{3}$. A constant horizontal force of $35\sqrt{3}N$is applied to the body causing it to accelerate up the plane. Find the time taken for the body to reach the top and its speed on arrival.



R = 5gcos 30 + $35\sqrt{3}$sin30

$35\sqrt{3}$cos 30 – (μR + 5gsin30) = 5a

a = 1.39ms^{-2}

s = ut + $\frac{1}{2}at^2$

6.3 = 0 x t + $\frac{1}{2}x1.39\ x\ t^2$; t = 3s

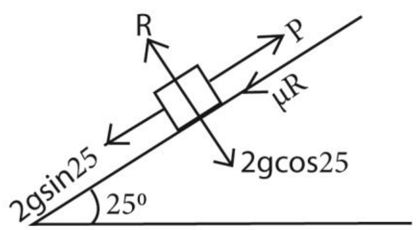
v = u + at

v = 0 + 1.39 x 3 =4.17ms^{-1}

Example 17

A box of mass 2kg at rest on a plane inclined at 25^0 to the horizontal. The coefficient of friction between the box and the plane is 0.4. What minimum force applied parallel to the plane would move the box up the plane?

Let the minimum force required be P



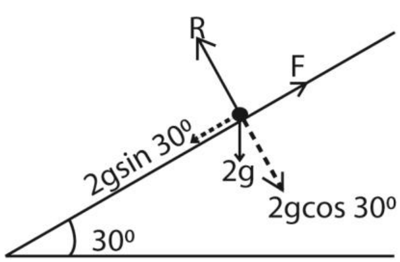
Perpendicular to the plane: R = 2gcos25 …. (i)
Along the plane: P = 2gsin25 + μR …………….(ii)

(i) and (i)

P = 2gsin25 + 0.4(2gcos25) = 8.3 + 7.1 = 15.388N

Example 18

A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.



R = 2gcos 30^0
F = 2gsin 3^0

$\mu R = 2g\sin 30^0$

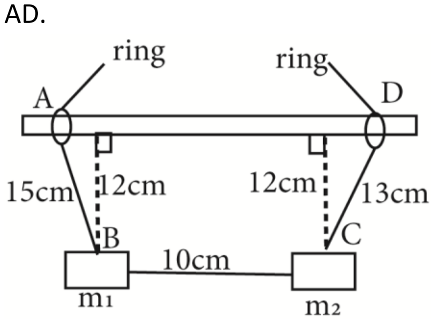
$\mu[2g\cos 30^0] = 2g\sin 30^0$

$\mu = \frac{2g\sin 30^0}{2g\cos 30^0} = \tan 30^0 = 0.57735$

Revision exercise on friction of inclined planes

1. The resistance to motion of a lorry of mass mkg is 1/200 of its weight. When travelling at $108kmh^{-1}$ on a level road and ascends a hill inclined at 1 in 100. Its engine fails to work. Find how far up the hill (in km) the lorry moves before it comes to rest. [36.12m]
2. A vehicle of mass 2.5metric tonnes is drawn up on a slope of 1 in 10 from rest with an acceleration of 1.2 ms-2 against a constant frictional resistance of $\frac{1}{100}$ of the weight of the vehicle, using a cable. Find the tension in the cable. [T = 5695N]
3. (a) A particle of mass, m kg is projected with a velocity of 10ms-1 up a rough plane of inclination 300 to the horizontal. If the coefficient of friction between the particle and the plane is $\frac{1}{4}$. Calculate how far up the plane the particle travels.[s = 7.121m]

   (b) A car is working at 5kW and is travelling at a constant speed of $72knh^{-1}$. Find the resistance to motion. [250N]
4. A body of mass 8kg rests on a rough plane inclined at θ to horizontal. If the coefficient of friction is μ, find the least horizontal force in terms of μ, θ and g which will hold the body in equilibrium. $[\frac{8g(\sin\theta-\mu\cos\theta)}{(\cos\theta+\mu\sin\theta)}]$
5. A carton of 3kg rests on a rough plane inclined at an angle 30^0 to the horizontal. The coefficient of friction between the carton and the plane is $\frac{1}{3}$ > find a horizontal force that should be applied to make the carton just about to move up the plane. [33.155N]
6. A particle of weight 20N is placed on a rough plane inclined at an angle of 400 to the horizontal. the coefficient of friction between the plane and the particle is 0.25. When a horizontal force P is applied on the particle it rests in equilibrium. Calculate the value of P. [9.739N]
7. The diagram below shows the three strings AB = 15cm, BC = 10cm and CD = 13cm, A and D are fixed to small rings each of mass 2kg which can slide on a rough horizontal rail AD. Masses m_1 and m_2 are attached at B and C respectively. The system rests in equilibrium with BC at a distance 12cm below AD.



   (a) Show that $9m_1 = 5m_2$.

   (b) If the coefficient of friction between each ring and the rail is 0.25 and the ring A is on the pint of slipping, determine the value of m_1. [m_1 = 1kg]

8. A 2kg body lies on a plane of inclination 600. The coefficient of friction between the body and the plane is 0.25. Find the least horizontal force which prevents the body from sliding down the plane. [20.27N]
9. A particle od mass 12kg slides from rest down a plane inclined at 50^0 to the horizontal. If the coefficient of friction between the particle and the plane is 0.4, calculate the acceleration of the particle. [$4.99ms^{-2}$]
10. A body of mass 3kg is released from a rough surface which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. If after 2.5s the body has acquired a velocity of 4.9ms-1 down the surface. Find the coefficient of friction between the body and the surface.

Force and Newton's laws of motion

Law I: A body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force

Law II: The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction of the force.

$F = m\left(\frac{v-u}{t}\right)$ but $\left(\frac{v-u}{t}\right)$ =a

= ma

NB: F must be the resultant force

Example 1

Find the acceleration produced when a body of mass 5kg experiences a resultant force of 10N

F = ma | 10 = 5a | $a = 2ms^{-2}$

Example 2

A car of mass 600kg travels a distance of 24m while uniformly accelerated from rest to $12ms^{-1}$

(i) Find the acceleration of the car
$v^2 = u^2 + 2as$
$12^2 = 0^2 + 2a \times 24$
$a = 3ms^{-2}$
(ii) determine the accelerating force
F = ma = 600 x 3 = 1800N

Example 3

A body of mass 500g experiences a resultant force 3N. Find

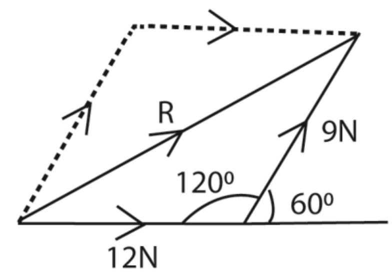
(i) Acceleration produced
F = ma | 3 = 0.5 x a | $a = 6ms^{-2}$
(ii) Distance travelled by the body while increasing speed from $1ms^{-1}$ to $7ms^{-1}$
$v^2 = u^2 + 2as$

$7^2 = 1^2 + 2x\ 6\ x\ s$

$s = 4m$

Example 4

Two forces of magnitude 12N and 9N act on a particle producing an acceleration of 3.65ms^{-2}. The two forces act at an angle of 600 to each other. Find the mass of the particle.



$R^2 = 12^2 + 9^2 - 2 \times 12 \times 9 \cos 120^0$

R = 18.25N

F = ma

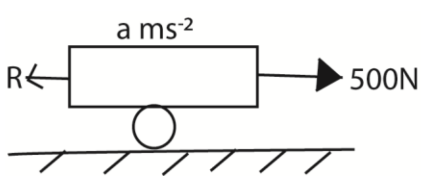
18.25N = 3.65m

m = 5kg

When resistance or friction is involved

Example 5

A car moves along a level road at constant velocity of 22ms^{-2}. If its engine is exerting a forward force of 500N, what resistance is the car experiencing.



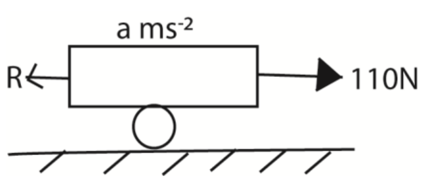
F = ma

500 – R = m x 0

R = 500N

Example 6

A car of mass 500kg moves along a level road with acceleration of 2ms^{-2}. Its Engine is exerting a forward force of 110N. What is the resistance a car is experiencing?



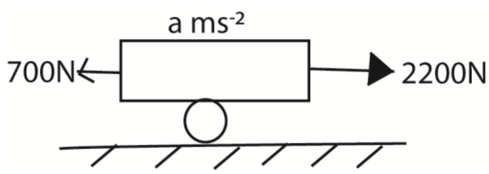
F = ma

500 – R = 500 x 2

R = 100N

Example 7

A van of mass 2tonnes moves along a level road against resistance of 700N. If its engine is exerting a forward force of 2200N. Find the acceleration of the van



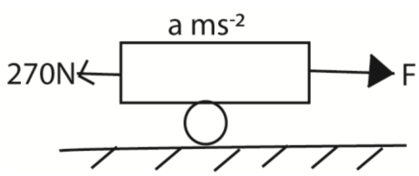
F = ma

2200 – 700 = 2000 x a

a = 0.75ms^{-2}

Example 8

Find the constant force necessary to accelerate a car of mass 1000kg from 15ms^{-1} to 20ms^{-1} in 10s against a resistance of 270N



v = u + at

20 = 15 + 10a

a = 0.5ms^{-2}

F = ma

F – 270 = 1000 x 0.5

F = 770N

Calculations involving vector form

Find the resultant force required to make a body of mass 2kg at (5i + 2j)ms^{-2}.

F = ma

F = $2\binom{5}{2} = \binom{10}{4} N$

Example 9

F = ma $\binom{4}{2} + \binom{1}{1} = 0.5a$ a = $\binom{6}{6} ms^2$

Find the acceleration produced in a body of mass 500N is subjected to forces of (4i + 2j)N and (-i + j)N

F = ma $\binom{4}{2} + \binom{1}{1} = 0.5a$ a = $\binom{6}{6} ms^2$

Example 10

Find the magnitude of the acceleration produced in a body of mass 2kg subjected to forces of (2i – 3j +4k)N and (i + 5j + 2k)N

F = ma $\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} = 2a$ a = $\begin{pmatrix} 1.5 \\ 1 \\ 3 \end{pmatrix} ms^2$ $|a| = \sqrt{1.5^2 + 1^2 + 3^2} = 2.3ms^{-2}$

Example 10

A particle of mass 2.5kg is acted on by a resultant force of 15N acting in the direction (2i – j – 2k). find the magnitude of the acceleration

$F = 15 \times \frac{2i-j-2k}{\sqrt{2^2+(-1)^2+(-2)^2}}$

$= 15 \times \frac{2i-j-2k}{3}$

F = 10i – 5j -10k

F = ma

$\begin{pmatrix} 10 \\ -3 \\ -10 \end{pmatrix} = 2a$

$a = \begin{pmatrix} 4 \\ -2 \\ -4 \end{pmatrix}$

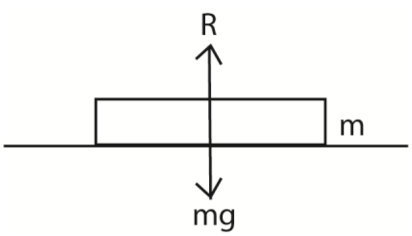
$|a| = \sqrt{10^2 + (-3)^2 + (-10)^2}$

=6ms-2

Law III: To every action there is an equal but opposite reaction

Consider

1. a body of mass m place on a smooth horizontal surface

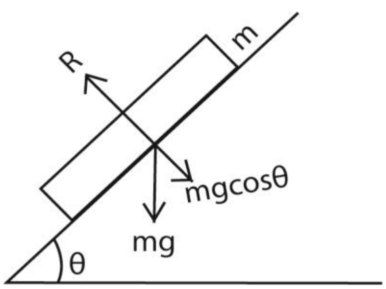


R = mg

R = normal reaction

mg gravitational pull (weight)

2. Mass m laced on a smooth inclined lane of angle of inclination θ



R = mgcosθ

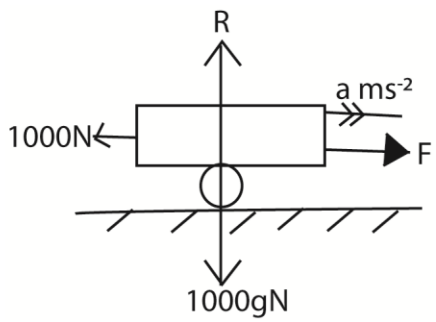
- All objects placed on or moving on an inclined plane experience a force mgsinθ **down** the plane no matter the direction of movement.
- If the plane is rough, the body experience a frictional force whose direction is opposite of the direction of motion

Motion on horizontal plane

Example 11

A car of 1000kg is accelerating at $2ms^{-2}$. If the resistance to motion is 100N

(i) Find the normal reaction of the car on the road surface



R = 1000gN

= 1000 x 9.8 = 9800N

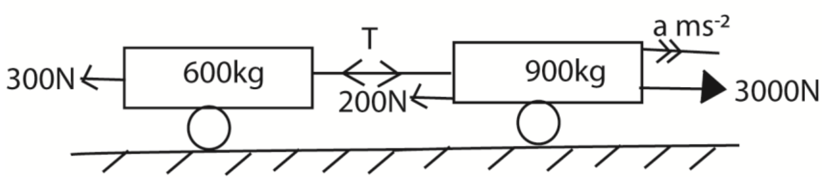
(ii) What accelerating force acts on the car?

F = ma

F – 1000 = 1000 x 2; F = 3000N

Example 12

A car of mass 900kg tows a trailer of mass 600kg along a level road by means of a rigid bar. The car experiences a resistance of 200N and the trailer a resistance of 300N, if the car engine exerts a force of 3kN, find the acceleration produced and the tension in the tow bar



For 900kg: 3000 – (T + 200) = 900a … (i)

For 600kg: T – 300 = 600a ……………. (ii)

(i) and (ii) a= 1.6667ms^{-2}

Alternatively

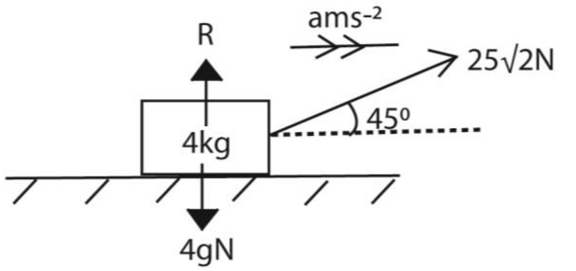
3000 – (200 + 300) = (900 + 600)a

a = 1.6667ms^{-2}

Force inclined at an angle to the horizontal

Example 13

A body of mass 4kg is acted on by force of $25\sqrt{2}$N which is inclined at 450 to a smooth horizontal surface. Find the acceleration of the body and the normal reaction between the body and the surface.



(→) $25\sqrt{2}$Ncos45 = 4a

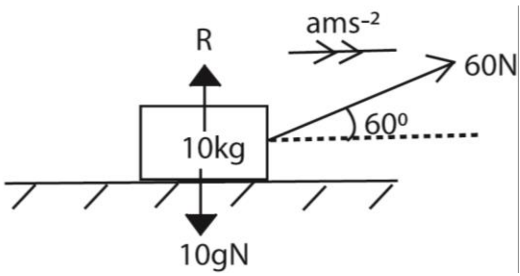
a = 6.25ms^{-2}

(↑) R + $25\sqrt{2}$Nsin 45 - = 0

R = 14.2N

Example 14

A body of mass 10kg is initially at rest on a rough horizontal surface. It is pulled along the surface by constant force of 60N inclined at 600 above the horizontal. If the resistance to motion totals 10n, find the acceleration of the body and the distance travelled in the first 3s.



(→) 60sos60 - 10 = 10a

a = 2ms^{-2}

s = ut +$\frac{1}{2}at^2$

s = 0 x 3 + $\frac{1}{2}$ x 2 x 3^2 =9m

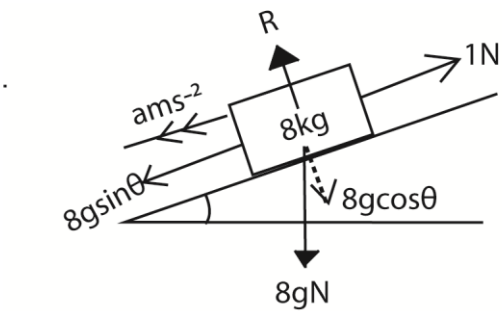
Revision exercise on forces and Neuton's law

1. A railway engine of mass 100tonnes is attached to a line of trucks of total mass 80 tonnes. Assuming there is no resistance to motion, find the tension in the coupling between the engine and the leading truck when the train has acceleration of 0.020ms^{-2} [25.6kN]
2. A body of mass 5kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at 450 above the horizontal. In the first 5 seconds of motion, the body moves a distance of 10m along the surface. Find the
   (i) acceleration of the body [0.8ms^{-2}]
   (ii) magnitude of P [$4\sqrt{2}$N]
   (iii) normal reaction between the body and the surface. [45N]
3. A body of mass m kg, initially at rest on a smooth horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. Show that the body moves a distance in time t along the surface given by $\frac{Pt^2cos\theta}{2m}$.
4. A body of mass m kg, initially at rest on a rough horizontal surface is pulled along the surface by a constant force P inclined at θ above the horizontal. If the mass acquire velocity v in a distance d. Show that the resistance to motion is given by Pcosθ = $\frac{mv^2}{2d}$

Motion on an inclined plane

Example 15

A body of mass 8kg is released from on the surface of a plane at 1 in 40. If the resistance to motion is 1N, find the acceleration of the body and the speed it acquired after 6s.



$\sin\theta = \frac{1}{40}$

F = ma

8gsinθ – 1 = 8a

a = 0.12ms^{-2}

v = u + at

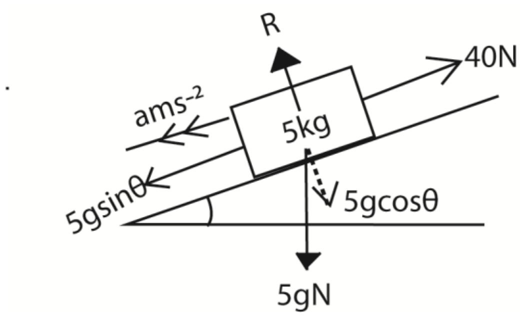
= 0 + 0.12 x 6 =0.72ms^{-2}

Example 16

A body of mass 5kg is pulled u a smooth plane inclined at 30^0 to the horizontal by a force of 40N acting parallel to the plane. Find

(i) acceleration of the body

(ii) force exerted on the body by the plane R



F = ma

40 -5gsin30^0 = 5a

a = 3.095ms^{-2}

(ii) R = 5gcos30

= 5 x 9.8cos30 = 42.4N

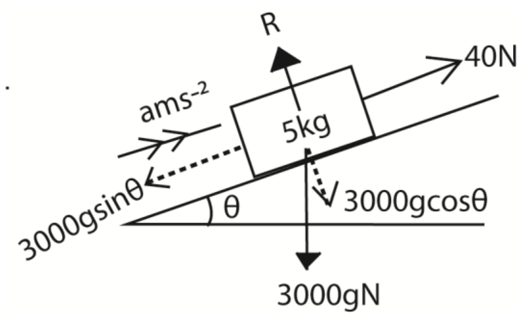
Example 17

A lorry of mass 3tonnes travelling at 90k/h starts to climb an incline of 1 in 5. Assuming the attractive pull between its tyres and the road remains constant and that its velocity reduces to 54kmh in a distance of 500m. Find the attractive pull.

u = 90kmh = $\frac{90 \times 1000}{3600}$ = 25ms^{-1}

V = 54kmh = $\frac{54 \times 1000}{3600}$ = 15ms^{-1}

a = $\frac{v^2 - u^2}{2s} = \frac{15^2 - 25^2}{2 \times 500}$ = -0.4ms^{-2}



sinθ = $\frac{1}{40}$

F = ma

F - 3000gsinθ = 3000a

F – 3000 x 9.8 x $\frac{1}{5}$ = 3000 x 0.4

F = 4686N

Revision exercise 2 on forces and Newton's law

1. A particle of mass 5kg resting on smooth plane inclined at $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ to the horizontal. Find the magnitude of the horizontal force required to keep the particle in equilibrium and the normal reaction. [28.29N, 56.58N]
2. The engine of a train exerts a force of 35,000N on a train of mass 240 tonnes and draws up a slope of 1 in 120 against resistance totaling to 60N per tonne. Find the acceleration of the train. [0.004167ms^{-2}]
3. A car of mass 2.5 metric tonnes is drawn up a slope of 1 in 10 from rest with acceleration of 1.2ms^{-2} against a constant frictional force of $\frac{1}{100}$ of the weight of the vehicle using a cable. Find the tension in the cable. [5695N]
4. A mass 5kg is initially at the bottom of a smooth slope which is inclined at $\sin^{-1}\left(\frac{3}{5}\right)$ to the horizontal. The mass is pushed up the slope by horizontal force 50N, find
   (i) the normal reaction between the mass and the plane [69.2N]
   (ii) calculate the acceleration up the slope [2.12ms^{-2}]
   (iii) how far up the slope the mass travels in the first 4s [16.96m]
5. A body of mass 100kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. Find
   (i) velocity of the body when it has travelled 20m down the slope. [14ms^{-1}]
   (ii) velocity, if the mass of the body was 50kg. [14ms^{-1}]
6. A body of mass 20kg is released from rest at the top of a smooth slope which is inclined at 30^0 to the horizontal. If the body accelerates down the slope at 3ms^{-2}, find the constant resistance to motion experienced by the body. [38N]
7. A body of mass 20kg is released from rest at the top of a rough slope which is inclined at 30^0 to the horizontal. 6s later the body has a velocity of 21ms^{-1} down the slope, find the constant resistance to motion experienced by the body. [28N]

8. A car of 1 tonne accelerated from $36kmh^{-1}$ to $72kmh^{-1}$ while moving 0.5km up a road inclined at an angle of α to the horizontal where sinα = $\frac{1}{20}$. If the total resistive force to its motion is 0,3kN, find the driving force of the car engine. [1090N]
9. A railway truck of mass 6.0 tonnes moves with an acceleration of $0.050ms^{-2}$ down a track which is inclined to the horizontal at an angle α where sinα = $\frac{1}{120}$. find the resistance to motion. [190N]
10. A body of mass 5.0kg is pulled along a smooth horizontal ground by means of 40N acting at 60^0 above the horizontal. find
    (i) Accelerating force [$4ms^{-2}$]
    (ii) Force the body exerts on the ground [14.4N]
11. A body of mass 3.0kgslides down a plane which is inclined at 300 to the horizontal. Find the acceleration of the body if
    (i) the plane is smooth [$4.9ms^{-2}$]
    (ii) there is frictional resistance of 9.0N [$1.9ms^{-2}$]
12. A car of mass 1000kg tows a caravan of mass 600kg u a road which rises 1m vertically for every 20m of its length. There is a constant frictional resistance of 200N and 100N to the motion of the car and caravan respectively. The combination has an acceleration of $1.2ms^{-2}$ with the engine exerting a constant driving force. (Take g = $10ms^{-2}$). Find
    (a) driving force[3020N]
    (b) Tension in the tow-bar [1120N]

Connected particles

Simple connections

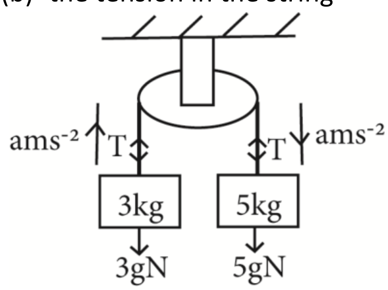
When two particles are connected by a light inextensible string passing over a smooth pulley and allowed to move freely, then as long as the string is tout, the following must be observed.

- acceleration of the particles is the same
- tension in the uninterrupted string is constant
- tensions in interrupted strings are different.

Example 1

Two particles of masses 5kg and 3kg are connected by a light inextensible string passing over a smooth fixed pulley. Find

(a) acceleration of the particle
(b) the tension in the string



For 5kg mass: 5g – T = 5a …. (i)
For 3kg mass: T – 3g = 3a …. (ii)

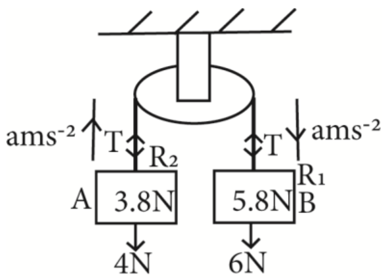
(i) and (ii)
2g = 8a
a = $\frac{2 \; x \; 9.8}{8}$ = $2.45ms^{-2}$
(ii) tension in the string
T -3g = 3a
T = 3 x 2.45 + 3 x 9.8 = 36.78N
(iii) Force on the pulley
R = 2T = 3 x 36.78 = 73.56N

Example 2

An inextensible string attached to two scale A and B each of weight 20g passes over a smooth fixed pulley. Particles of weight 3.8N and 5.8N are placed in pans A and B respectively. If the system s released from rest (take g = $10ms^{-2}$). Find the

(a) Tension in the string
(b) Reaction of the scale pan holding the 3.8N weight

(c)



Weight of the scale pan = $\frac{20}{1000} x\ 10 = 0.2N$

Total weight of A = 3.8 + 0.2 = 4N

Total weight of B = 5.8 + 0.2 = 6N

For 6N: 6 – T = 0.6a …….. (i)

For 4N: T – 4 = 0.4a …….. (i)

Adding (i) and (ii)

a = 2ms^{-2}

T = 4 + 0.4 x 2 = 4.8N

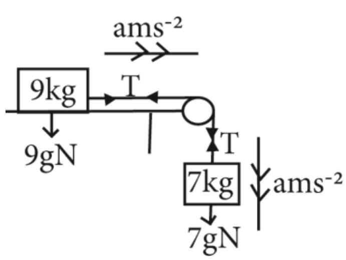
For scale pan A R_2 – 3.8 = 0.38a

R_2 = 3.8 + 2 x 0.38 x 2 = 4.56N

Example 3

A mass of 9kg resting on a smooth horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table to the pulley is a 7kg mass hanging freely 1.5m above the ground. Find

(a) common acceleration
(b) tension in the string
(c) force on the pulley when the system is allowed to move freely
(d) time taken for the 7kg mass to hit the ground



F = ma

For 7kg mass: 7g – T = 7a

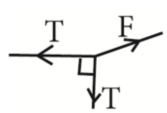
For 9kg mass: T = 9a

(i) + (ii): 7g = 16a

a = $\frac{7\ x\ 9.8}{16} = 4.29ms^{-2}$

(b) Tension: T = 9a = 9 x 4.29 = 38.61N

(c) The force on the pulley, F:



F =$\sqrt{T^2 + T^2} = T\sqrt{2}$ = 38.61$\sqrt{2}$ = 54.603N

(d) s = ut + $\frac{1}{2}at^2$

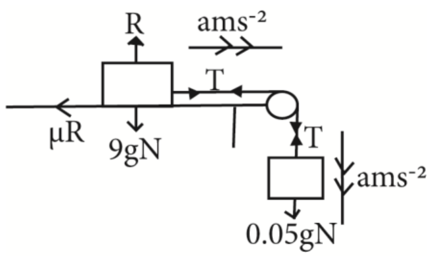
1.5 = 0 x t + $\frac{1}{2} x\ 4.29\ x\ t^2$

t = 0.84s

Example 4

A mass of 90g resting on a rough horizontal table is connected by a light inextensible string passing over a smooth pulley at the edge of the table attached to a 50g mass hanging freely. The coefficient of friction between the 90g mass and the table is $\frac{1}{3}$ and the system is released from rest, find

(a) common acceleration



For 50g mass: 0.05g – T = 0.05a ….(i)

For 90g mass: T – μR = 0.09a

(b) the tension in the string

T - $\frac{1}{3}$ x 0.09 x 9.8 = 0.09a …. (ii)

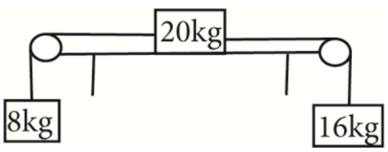
(i) + (ii): 0.05g - $\frac{1}{3}$ x 0.09 x 9.8 = 0.14a

a = $\frac{0.02g}{0.14}$ = 1.4ms^{-2}

(b) 0.05g – T = 0.05a

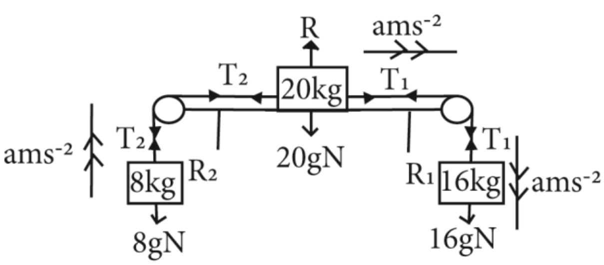
T = 0.05 x 9.8 – 0.05 x 1.4 = 0.42N

Example 5



The figure shows a block of mass 20kg resting on a smooth horizontal table. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) Acceleration of 16kg mass
(b) tension in the string
(c) reaction on each pulley



For 16kg mass: 16g – T_1 = 16a …… (i)

For 20kg mass: T_1 – T_2 = 20a ……. (ii)

For 8kg mass: T_2 – 8g = 8a ……… (iii)

(i) + (ii) + (iii): 8g = 44a

a = $\frac{8\,x\,9.8}{44} = 1.782ms^{-2}$

(b) Tension in the string

16g – T_1 = 16a

T_1 = 16 x 9.8 – 16 x 1.782 = 128.288N

T_2 – 8g = 8a

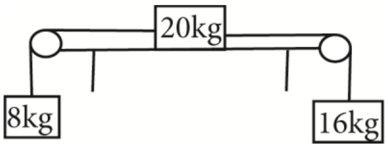
T_2 = 8 x 9.8 + 8 x 1.782 = 92.656N

(c) Reaction on each pulley

R_1 = $\sqrt{{T_1}^2 + {T_1}^2}$ = $T_1\sqrt{2}$ = $\sqrt{2}$ x 128.288 =181.427N

R_2 = $\sqrt{{T_2}^2 + {T_2}^2}$ = $T_2\sqrt{2}$ = $\sqrt{2}$ x 92.626 = 131 N

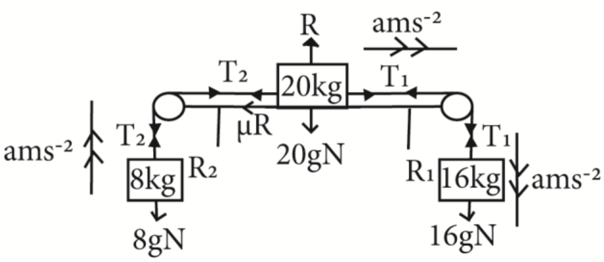
Example 6



The figure shows a block of mass 20kg resting on a rough horizontal table of coefficient of friction 0.21. It is connected by light inextensible string which pass over fixed pulleys at the edges of the table to two loads of masses 8kg and 16kg which hang freely vertically. When the system is released freely calculate:

(a) acceleration of the 16kg mass
(b) Tension in each string
(c) reaction on each pulley

Solution



For 16kg mass: $16g - T_1 = 16a$ …… (i)

For 20kg mass: $T_1 - T_2 - 20g\mu = 20a$ ……. (ii)

For 8kg mass: $T_2 - 8g = 8a$ ……… (iii)

(i) + (ii) + (iii): $8g - 20g\mu = 44a$

$a = \frac{8 \times 9.8 - 20 \times 9.8 \times 0.21}{44} = 0.846ms^{-2}$

(b) Tension in the string

$16g - T_1 = 16a$

$T_1 = 16 \times 9.8 - 16 \times 0.846 = 143.264N$

$T_2 - 8g = 8a$

$T_2 = 8 \times 9.8 + 8 \times 0.846 = 85.168N$

(c) Reaction on each pulley

$R_1 = \sqrt{T_1^2 + T_1^2} = T_1\sqrt{2} = 2 \times 128.291 = 202.606N$

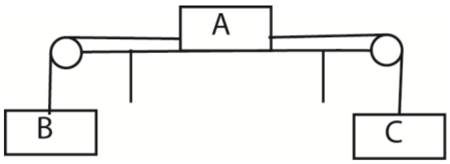
$R_2 = \sqrt{T_2^2 + T_2^2} = T_2\sqrt{2} = \sqrt{2} \times 85.168 = 120.446N$

Revision exercise 1

1. Two particles of masses 7kg and 3kg are connected by a light inelastic string passing over a smooth fixed pulley. Find
   (a) acceleration of the particles [3.92ms^{-2}]
   (b) the tension in the string [41.16N]
   (c) the force on the pulley [82.32N]
2. Two particles of masses 6kg and 2kg are connected by a light inextensible string passing over a smooth fixed pulley. With the masses hanging vertically, the system is released from rest. Find
   (a) acceleration of the particles [4.9ms^{-2}]
   (b) the tension in the string [29.4N]
   (c) distance moved by the 6kg mass in the first 2s of motion [9.8m]
3. A man of mass 70kg and a bucket of bricks of mass 100kg are tied to the opposite ends of a rope which passes over a frictionless pulley so that they hang vertically downwards.
   (a) what is the tension in the section of the rope supporting the man [807.06N]

(b) what is the acceleration of the bucket [1.73ms^{-2}]

4. Two particles of masses 200g and 300g are connected to a light inelastic string passing over a smooth pulley; when released freely find
   (i) common acceleration [1.96ms-2]
   (ii) the tension in the string [2.352N]
   (iii) the force on the pulley [4.704N]
5. The diagram below shows a particles of mass 8kg connected to a light scale pan by a light inextensible string which passes over a smooth fixed pulley. The scale pan holds two blocks A and B of mass 3kg and 4kg, with B resting on top of A. If the system is released from rest find
   (a) acceleration of the system [0.653ms^{-2}]
   (b) the reaction between A and B[41.813N]
6. A mass of 5kg is placed on a smooth horizontal table and connected by a light string to a 3kg mass passing over a smooth pulley at the edge of the table and hanging freely. If the system is allowed to move, calculate
   (a) the common acceleration of the masses [3.675ms^{-2}]
   (b) the tension in the string [18.375N]
   (c) the force acting on the pulley[26N]
7. A mass of 3kg on a smooth horizontal table is attached by a light inextensible sting passing over a smooth pulley at the edge of the table, to another mass of 2kg hanging freely 2.1m above the ground; find
   (a) common acceleration[3.92ms^{-2}]
   (b) the tension in the string [11.76N]
   (c) The force on the pulley in the system if it's allowed to move freely. [16.63N]
   (d) the velocity with which the 2kg mass hits the ground [4.06ms^{-1}]
8. A mass of 5kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to a 3kg mass hanging freely. the coefficient of friction between the 5kg mass and the table is 0.25 and the system is released from rest find
   (a) common acceleration[2.144ms^{-2}]
   (b) tension in the string [22.97N]
9. A mass of 11kg rests on a rough horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table to 500g mass hanging freely. The coefficient of friction between the 1kg mass and the table is 0.1 and the system is released from rest find
   (a) common acceleration [2.61ms^{-2}]
   (b) the tension in the string [3.593N]
10. The objects of mass 3kg and 5kg are attached to ends of a cord which passes over a fixed frictionless pulley placed at 4.5m above the floor. The objects are held at rest with 3kg mass touching the floor and the 5kg mass at 4m above the floor and then release, what is
    (a) the acceleration of the system[2.45ms^{-2}]
    (b) tension in the chord[36.75N]
    (c) the time that will elapse before the 5kg object hits the floor [1.81s]
11.

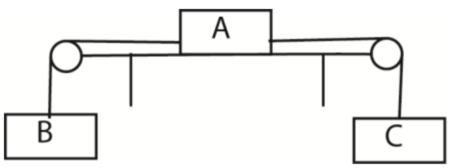


The diagram shows a particle A of mass 2kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 5kg and C of mass 3kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(a) common acceleration [0.98ms^{-2}]

(b) the tension of each string [12.37N, 44.15N]

12.

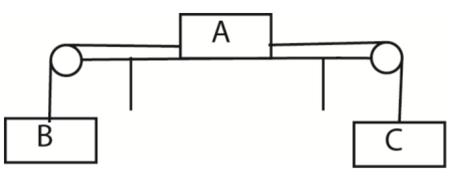


The diagram shows a particle A of mass 3kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 4kg and C of mass 6kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(a) common acceleration [0.75ms^{-2}]

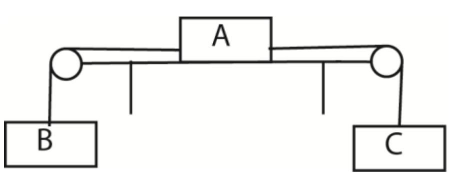
(b) the tension of each string [54.277N, 31.662N]

13.



The diagram shows a particle A of mass 5kg resting on a rough horizontal table of coefficient of friction 0.5. It is attached to the particle B of mass 3kg and C of mass 2kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest body B descend with an acceleration of 0.28ms-2, find the coefficient of friction between the body A and the surface of the table. [0.143]

14.



The diagram shows a particle A of mass 10kg resting on a smooth horizontal table. It is attached to the particle B of mass 4kg and C of mass 7kgby light inextensible strings hanging over smooth pulleys. If the system is released from rest find the

(c) common acceleration [1.4ms^{-2}]

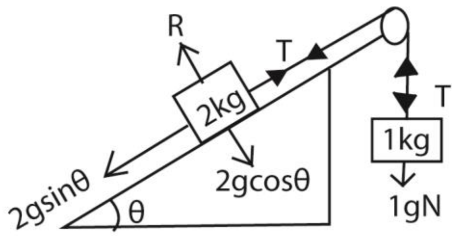
(d) the tension of each string [44.8N, 58.8N]

Connected particles on inclined planes

Example 7

A mass of 2kg lies on a smooth plane of inclination 1 in3. One end of a light inextensible sting is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its end. If the system is released from rest , find the

(i) acceleration of the masses
(ii) tension in the string
(iii) distance each particle travels in the first 2s.



sinθ =$\frac{1}{3}$ F =ma

For 2kg mass: T – 2gsinθ = 2a….. (i)

For 1kg mass: 1g – T = 1a ……….. (ii)

(ii) + (i): = 1g – 2gsinθ = 3a

a = $\frac{9.8 - 2\ x\ 9.8\ x\ \frac{1}{3}}{3}$ = 1.089ms^{-2}

(ii) Tension: 1g – T = 1a

T = 9.8 – 1.089 = 8.71N

(iii) s = ut + $\frac{1}{2}at^2$

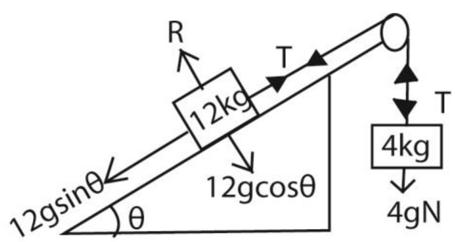
s = 0 x 2 + $\frac{1}{2}x\ 1.089\ x\ 2^2\ = 2.178m$

Example 8

A mass of 12kg lies over a smooth incline plane 6m long and 1m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane to freely suspended mass of 4kg at its other end. If the system is released from rest, find the

(a) acceleration of the system
(b) velocity with which the 4kg mass hits the ground
(c) time the 4kg mass takes to hit the ground.

Solution



sinθ =$\frac{1}{6}$ F =ma

For 12kg mass: T – 12gsinθ = 2a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 12gsinθ = 16a

a = $\frac{4\ x9.8 - 12\ x\ 9.8\ x\ \frac{1}{6}}{16}$ = 1.225ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 1.225 = 34.3N

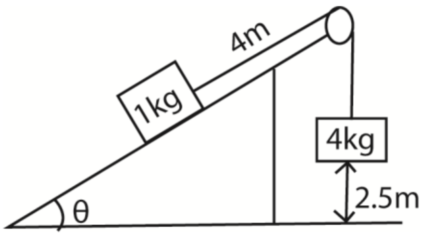
(iii) s = ut + $\frac{1}{2}at^2$

1 = 0 x t + $\frac{1}{2}x\ 1.225\ x\ t^2$

t = 1.28s

Example 9

A mass of 1kg lies on a rough plane with coefficient of friction 0.25. One end of a light inextensible string is attached to 1kgmass and passes up the line of greatest slope over a smooth fixed pulley at the top of the plane and the other end of a string is tied to a mass of 4kg hanging freely.



The plane makes an angle θ with the horizontal where sinθ = $\frac{3}{5}$. When the system is released from rest, find:

(i) the acceleration of the system
(ii) tension in the string
(iii) velocity with which the 4kg mass hits the floor
(iv) velocity with which the 1kg mass hits the pulley

Solution

sinθ = $\frac{3}{5}$; cosθ = $\frac{4}{5}$ F =ma

For 12kg mass: T – 1gsinθ- 0.25R = 1a….. (i)

For 4kg mass: 4g – T = 4a ……….. (ii)

(ii) + (i): = 4g – 1gsinθ - 0.25R = 5a

a = $\frac{4\ x 9.8 - 1\ x\ 9.8\ x \frac{3}{5} - 0.25\ x\ 1\ x\ 9.8\ \ x\frac{4}{5}}{5}$ =6.272ms^{-2}

(ii) Tension: 4g – T = 4a

T = 4 x 9.8 – 4 x 6.272 = 14 .112N

Example 10

A particle of mass 2kg on a rough plane inclined at 300 to the horizontal is attached by means of light inextensible string passing over a smooth pulley at the top edge of the plane to a particle of mass 3kg which hangs freely. If the system is released from rest with above parts of the strings taut, the 3kg mass travels a distance of 0.75m before attains a speed of 2.25ms^{-1}. Calculate

(a) acceleration
(b) coefficient of friction between the plane and 2kgmass
(c) reaction of the pulley on the string

Solution

(i) $v^2 = u^2 + 2as$

$a = \frac{2.25^2 - 0^2}{2 \, x \, 0.75} = 3.375ms^{-2}$

(ii) F = ma

For 2kg mass: $T - 2g\sin\theta - \mu R = 2x\ a$ ….. (i)

For 4kg mass: $3g - T = 3\ a$ ……….. (ii)

(ii) + (i): $3g - 2g\sin\theta - \mu(2g\cos\theta) = 5a$

$$\mu = \frac{(3\,x\,9.8) - (2\,x\,9.8 sin30 + 5\,x\,3.375)}{2\,x\,9.8\,x\,cos30} = 0.161$$

Double inclined plane

Example 11

The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ = 0.6. Two particles of mass 3kg and mkg, where m< 3kg are connected by a light inextensible string passing over a smooth fixed pulley.

The particles are released from rest with a string taut. After travelling a distance of 1.08m, the speed of the particle is 1.8ms-1. Calculate

(i) acceleration
(ii) tension in the string
(iii) value of m

(i) $v^2 = u^2 + 2as$

Example 12

Two rough planes inclined at 300 and 600 to the horizontal and of the same height are placed back to back. Masses of 4kg and 12 kg are placed on the faces and connected by a light string passing over smooth pulley on the top of the planes.

If the coefficient of friction is 0.5 on both faces, find

(a) acceleration
(b) Tension in the strings

Solution

For 4kg mass: T -4gsin30 – 0.5 x 4gcos30 = 4a .... (i)

For 12kg mass: 12gsin60 –T – 0.5 x 12gcos 60 = 12a ..... (ii)

(i) + (ii)

12gsin60 – 4gsin30 - 0.5(4gcos30 + 12gcos 60) = 16a

a = 2.25ms^{-2}

Example 13

1. The diagram below shows a 12kg mass on a horizontal rough plane. The 6kg and 4kg masses are on rough planes inclined at angles 600 and 300 respectively. The masses are connected to each other by light inextensible strings over light smooth pulleys A and B.

   The planes are equally rough with coefficient of friction$\frac{1}{12}$. If the system is released from rest find the;

   (a) Acceleration of the system (08marks)

   For 6kg mass

   6gsin60-(T$_1$ + $\frac{1}{12}$ $x\ 6gcos60) = 6a$

   6gsin60 – T$_1$ - $\frac{1}{2}cos60 = 6a$ .... (i)

   For 4kg mass

   T$_2$ – ($\frac{1}{12}$ $x\ 4gcos30 + 4gsin30) = 4a$

   T$_2$ - $\frac{1}{3}gcos30 - 4gsin30$ = 4a .... (ii)

   For 12kg mass

   T$_1$ – (T$_2$ +$\frac{1}{12}$ R_3) = 12a

   T$_1$ – (T$_2$ +$\frac{1}{12}$ $x\ 12g$) = 12a

   T$_1$ –T$_2$ - g = 12a... (iii)

   Eqn. (i) + Eqn. (ii) + Eqn. (iii)

   6gsin60 - $\frac{1}{2}gcos60 - \frac{1}{3}gcos30 - 4gsin\ 30 - g = 22a$

   16.24327742 = 22a

   a = $\frac{16.24327742}{22} = 0.73833ms^{-2}$

(b) Tensions in the strings. (04marks)

From equation (i)

$T_1 = 6gsin60 - \frac{1}{2}cos60 - 6a$

$= 6gsin60 - \frac{1}{2}cos60 - 6 \, x \, 0.73833$

= 44.0423N

From eqn. (ii)

$T_2 = \frac{1}{3}gcos30 + 4gsin30 + 4a$

$= \frac{1}{3}gcos30 + 4gsin30 + 4 \, x \, 0.73833$

= 25.3823N

Revision exercise 2

1. A mass of 2kg lies on a smooth inclined plane 9m long and 3m high. One end of a light inextensible string is attached to this mass and the string passes up the line of greatest slope over a smooth pulley fixed at the top of the plane is freely suspended mass of 1kg at its other end. If the system is released from rest, find
   (i) acceleration of the system[1.089ms^{-2}]
   (ii) tension in the string. [8.711N]
   (iii) velocity with which the 1kg mass will hit the ground[2.556ms^{-1}]
   (iv) time the 1kg mass will hit the ground[2.347s]
2. A mass of 15kg lies on a smooth plane of inclination in 49. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 10kg at its other end. If the system is released from rest, find the acceleration of the masses and the distance each travel in the first 2s. [3.8ms^{-2}, 7.6m]
3. A mass of 2kg lies on a rough plane which is inclined at 300 to the horizontal. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 5kg at its other end. The system is released from rest as the 2kg mass accelerates up the slope, it experiences a constant resistance to motion of 14N down the slope due to friction. Find the tension of the string. [31N]
4. A mass of 10kg lies on a smooth plane which is inclined at θ to the horizontal. The mass is 5m from the top, measured along the plane. One end of a light inextensible string is attached to this mass and the string passes up a line of greatest slope, over a smooth pulley fixed at the top of the plane is freely suspended mass of 15kg at its other end. The 15kg mass is 4m above the floor. The system is released from rest and the string first goes slack $1\frac{3}{7}s$ latter. Find the value of θ. [30^0]
5. One of two identical masses lies on a smooth plane, which is inclines at $\sin^{-1}\left(\frac{1}{4}\right)$ to the horizontal and is 2m from the top. A light inextensible string attached to this mass passes along the line of greatest slope over a smooth pulley fixed at the top of the incline, the other end carries the other mass hanging freely 1m above the floor. If the system is released from rest, find the time taken for the hanging mass to reach the floor. [0.663s]
6. A particle of mass 2kg on a smooth plane inclined at 300 to the horizontal is attached by means of a light inextensible string passing over a smooth pulley at the edge of the plane to a particle of mass 4kg which hangs freely.

If the system is released from rest with above parts of the string taut, find the speed acquired by the particles when both have moved a distance of 1m [2.8ms^{-1}}

7. A body A of mass 13kg lying on a rough inclined plane, coefficient of friction, μ. From A, a light inextensible string passes up the line of greatest slope and over a smooth fixed pulley to a body B of mass mkg hanging freely, the plane makes an angle θ with the horizontal where sinθ = $\frac{5}{13}$.

   When m = 1kgand the system is released from rest, B has upward acceleration of a ms^{-2}. When m = 11kg and the system released from rest, B has downward acceleration of ams^{-2}. Find a and μ. [1.96ms^{-2}, 0.1]

8. A particle A of mass 2kg and B of mass 1.5kg are connected by light inextensible string passing over a smooth pulley. The system is released from rest with A at height of 3.6m above the horizontal ground and B at the foot of a smooth slope inclined at an angle θ to the horizontal where sinθ = $\frac{1}{6}$. Take g = 10ms^{-2}.

   Calculate

   (i) the magnitude of the acceleration of particles [6ms^{-1}]

   (ii) the speed with which A reaches the ground[5ms^{-2}]

   (iii) the distance B moves up the slope before coming to instantaneous rest. [14.4m]

9. A mass A of 4kg and a mass B of 3kg are connected by a light inextensible string passing over a smooth pulley. The system is released from rest and mass accelerates up along a smooth slope inclined at an angle θ to horizontal where θ = 30^0.

   If y = 3m and x = 2.8m, calculate the velocity with which A hits the pulley [2.42ms^{-1}]

10. The diagram below shows particles A, B and C of masses 10kg, 8kg and 2kg respectively connected by a light inextensible strings. The string connecting B and C passes over a smooth light pulley fixed at the top of the plane.

    If the coefficient of friction between the plane and particles A and B are 0.22 and 0.25 respectively, calculate

    (i) acceleration of the system [1.6477ms^{-2}]

    (ii) tension in the strings[22.89N, 13.851N]

11. The diagram below shows two smooth fixed slopes each inclined at an angle θ to the horizontal where sinθ =$\frac{3}{5}$. Two particles of mass 5kg and 15kg are connected by a light inextensible string over a smooth fixed pulley.

The particles are releases from rest with a string taut calculate

(i) Acceleration of the particles

(ii) Tension in the string

12. The diagram below shows two smooth plane and a rough plane both inclined at 450 to the horizontal. Two particles of mass of mass 1kg and 3kg are connected by light inextensible string passing over a smooth fixed pulley.

    The particle are released from rest with a string taut. Calculate
    (i) acceleration of the particle [$1.4ms^{-2}$]
    (ii) tension in the string [.48N
    (iii) coefficient of friction [0.4]
13. Two equally rough planes inclined at 30^0 and 60^0 to the horizontal and of the same height are placed back to back. Masses of 5kg and 2.5kg are placed on the faces and connected by a light string passing over a smooth pulley at the top of the planes.

    If the string is taut and 5kg is just about to slip downwards find the
    (i) coefficient of friction[0.06]
    (ii) tension in the string [21.9538N]
14. In the diagram, particle A and particle B are masses of 10kg and 8kg respectively and rest on planes as shown below. They are connected by a light inextensible string passing over a smooth pulley C.

    Find the acceleration in the system and the tension in the string if
    (i) the particles are in contact with smooth planes [$3.08ms^{-2}$, 30.N]
    (ii) the particles are in contact with rough planes with coefficient of friction 0.25. [$0.95ms^{-2}$, 33.98N]
15. In the diagram particles A and B are of masses mkg and 5mkg respectively and rest on the planes as shown below. They are connected by a light inextensible string passing over a smooth fixed pulley at C

    Find the acceleration of the system and the tension in the string if sinα = $\frac{4}{5}$ when;

    (i) the particles are in contact with smooth plane[6.533ms-2, 6.533N]
    (ii) the particles are in contact with rough plane of coefficient of friction $\frac{1}{3}$. [$4.356ms^{-2}$, 7.622N]

16. In the diagram particles A and B of masses 2.4kg and 3.6kg respectively. A rests on a rough horizontal plane (coefficient of friction 0.5), it is connected by a light inextensible string passing over a smooth pulley C to particle B resting on smooth plane inclined at 30^0 to the horizontal.

    When the system is released from rest find
    (i) acceleration of the system and tension in the string [$0.98ms^{-2}$, 14.112N]
    (ii) the force on the pulley C [7.3049N]
    (iii) the velocity of A mass after 2 seconds[$1.96ms^{-2}$]
17. The diagram below shows a 4kg mass on a horizontal rough plane with coefficient of friction 0.25. The $4\sqrt{3}kg$ mass rests on a smooth plane inclined at angle 600 to the horizontal while the 3kg mass

rests on a rough plane inclined at an angle 300 to the horizontal and coefficient of friction $\frac{1}{\sqrt{3}}$. the masses are connected to each other by a light inextensible strings over light smooth fixed pulleys B and C.

Find the

(i) acceleration of the system[1.407ms-2]

(ii) tension in the string [49.051N, 33.622N]

(iii) work done against frictional force when the particles each moved 0.5m [12.25J]

Multiple connections

- Acceleration of a particle moving between to portions of the string is equal to half the net acceleration of the particle (s) attached to the end of the string
- The tension in uninterrupted string is constant
- The tensions in interrupted strings are different.

Case I: A pulley moving between two portions of a string

Example 15

The diagram below shows particle A of mass 0.5kg attached to one end of alight inextensible string passing over a fixed pulley and under a movable light pulley B. The other end of the string is fixed as shown

Solution

(i) Let T = tension in string

m = mass at B

(ii) Let a_1 = acceleration of A

a_2 = acceleration of B

Example 16

A particle of mass 3kg on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass 5kg, its other end being fixed so that the string beyond the table are vertical.

Solution

F = ma

For 3kg: T = 3 x 2a ……… (i)

For 5kg: 5g – 2T = 5a …… (ii)

Example 17

A particle of mass m_1 on a smooth horizontal table is tied to one end of the string which passes over a fixed pulley at the edge and then under a movable pulley of mass m_2, its other end being fixed so that the parts of the string beyond the table is vertical.

Solution

Example 18

A string has a load of mass 2kg attached at one end. The string passes over a smooth fixed pulley then under a movable pulley of mass 6kg and over another fixed pulley and has a load of mass 3kg attached to its end.

Solution

For 2kg mass: $2g - T = 2a_1$ …………………..… (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For 6kg mass: $2T - 6g = 6 x \frac{1}{2}(a_1 + a_2)$…. (iii)

(ii) – (i): $g = (3a_2 - 2a_1)$ ……………………………. (iv)

2 x (ii) + (iii): $0 = 9a_2 + 3a_1$ …………………….. (v)

Example 19

In the pulley system below, A is a heavy pulley which is free to move

Solution

For 2kg mass: $6g - T = 6a_1$ …………………..… (i)

For 3kg mass: $3g - T = 3a_2$ …………………….. (ii)

For mkg mass: $2T - mg = 0$……………………… (iii)

Case 2: A pulley moving on one portion of a string

Example 20

The diagram below shows two pulley to pulleys of masses 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.

The hanging portions of the strings are vertical. Given that the string of the fixed pulley remains stationary, find the

(i) tensions in the string
(ii) value of m

Solution.

For 3kg mass: $T_2 - 3g = 3a_2$ ………… (i)

For 6kg mass: $6g - T_2 = 6a_2$ ………… (ii)

For 4kg mass: $T_1 - 4g = 4a_1$ ………… (iii)

For mkg mass: $mg - T_1 = ma_1$ ……….(iv)

For 8kg mass: $2T_1 + 8g = T$ ……………. (v)

For 12kg mass: $2T_2 + 12g = T$ ……………. (vi)

eqn. (i) + eqn. (ii): $3g = 3a_2$

Example 21

The diagram shows a particle of mass 2kg on a smooth plane inclined at 450 to the horizontal and attached by means of a light inextensible string over a smooth pulley, A at the top of the plane to pulley B of mass 0.5kg which hangs freely. Pulley B carries to particles of mass 0.5kg and 1kg on either side

Find

(a) acceleration of 2kg, 0.5kg and 1kg mass
(b) the tension in the strings

Solution

For 2kg mass: T_1 - 2gsin45 = $2a_1$ ……….. (i)

For 0.5kg mass: T_2 – 0.5g =(a_2 – a_1) …… (ii)

For 1kg mass: gN – T_2 = 1(a_1 + a_2) ……… (iii)

For pulley B: $2T_2$ + 0.5g – T_1 = $0.5a_1$ ……. (iv)

eqn. (ii) + eqn (iii): 0.5g = 1.5a2 + 0.5a1

$9.8 = 3a_2 + a_1$ ……….. (v)

eqn. (i) + eqn. (iv): $2T_2$ – 2gsin45 + 0.5g =$2.5a_1$

$2T_2$ -8.9593 = $2.5a_1$ ………… (vi)

2 x eqn. (iii) + eqn. (vi): 10.6407 = $4.5a_1 + 2a_2$

Example 22

The diagram shows a fixed pulley carrying a string which has a mass of 3kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 6kg at one and a mass of 2kg at the other end.

Solution

eqn. (i) + eqn. (iv): $2T_2$ – 3g = $3a_1$…… (vi)

2 x eqn. (iii) - eqn. (vi)

$-g = 4a_2 - 7a_1$ ……………………………. (vii)

2eqn. (vii) – eqn. (v)

$-18a_1 = -6g$

$a_1 = \frac{6 \, x \, 9.8}{18} = 3.27ms^{-2}$

$4g = 8a_2 + 4a_1$

$a_2 = \frac{9.8 - 3.27}{2} = 3.27ms^{-2}$

Acceleration of pulley A =$3.27ms^{-2}$

Example 23

The diagram shows a fixed pulley carrying a string which has mass of 4kg attached at one end and a light pulley A at the other end. Another string passes over pulley A and carries a mass of 3kg at one end and a mass of 1kg at the other end.

Solution

For 4kg mass: $4g - T_1 = 4a_1$ ………………. (i)

For 3kg mass: $3g – T_2 = 3(a_2 - a_1)$ ……… (ii)

For 1kg mass: $T_2 – g = (a_2 + a_1)$ ……… (iii)

For pulley A: $T_1 – 2T_2 = 0 \times a_1$ ………….. (iv)

eqn. (ii) and eqn. (iii): $g = 2a_2 - a_1$ ….. (v)

$T_1 = 4 \times 9.8 – 4 \times 1.4 = 33.6N$

$T_1 – 2T_2 = 0 \times a_1$

Example 24

The diagram below shows a fixed pulley carrying a movable pulley of mass 8kg at one end and a light pulley A attached at the other end. A string passes over pulley A and carries a mass of 4kg at one end and a mass of 6kg at the other end.

Solution

For 8kg mass: $2T_2 - 8g = 8a_1$ ………………. (i)

For 4kg mass: $T_1 – 4g = 4(a_2 - 2a_1)$ ……… (ii)

For 6kg mass: $6g– T_1 = 6(2a_1 + a_2)$ ……… (iii)

For pulley A: $2T_1 – T_2 = 0 \times a_1$ ………….. (iv)

eqn. (ii) and eqn. (iii): $2g = 10a_2 + 4a_1$

$4.9 = 2.5a_2 + a_1$ ….. (v)

eqn. (i) + 2 x eqn. (iv): $4T_1 – 8g = 8a_1$…. (vi)

4 x eqn. (iii) + eqn. (vi): $16g = 56a_1 + 24a_2$

$2g = 7a_1 + 3a2$ ………………………………. (vii)

Example 25

The diagram below shows two pulleys of mass 8kg and 12kg connected by a light inextensible string hanging over a fixed pulley.

(a) tensions in the strings (09marks)
Let T = tension n the string joining masses 8kg and 12kg
T_1 = tension in the string joining masses 4kg and 12kg
T_2 = tension in string joining masses 3kg and mkg

Since the string of the fixed pulley remains stationary, this means the pulleys of the 8kg and 12kg are stationary or fixed

(b) value of m. (03marks)
For the m kg mass
Resultant force = mg $-T_2$

$ma_2 = mg - T_2$

$m(\frac{g}{3}) = mg - 4g$

$\frac{2}{3}mg = 4g$

$m = \frac{12}{2} = 6kg$

Revision exercise 3

1. A string with one end fixed passes under a movable pulley of mass 2kg, over a fixed pulley and carries a 5kg mass at its end

2. a string with one end fixed passes under a movable pulley of mass 5kg, over a fixed pulley and carries a mass of7kg at its other end.

3. In the pulley system below, A is a heavy pulley which is free to move.

4. Two particles of mass 3kg and 6kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 2kg, the portions of the string not in contact are vertical

5. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over the pulley A and caries a mass of 4kg at one end and a mass of 2kg at the other end.

6. Two particles of mass 3kg and 5kg are connected by a light inextensible string passing over two fixed smooth pulleys and under a heavy smooth movable pulley of mass 6kg, the portions of the string not in contact are vertical

7. The diagram shows a fixed pulley carrying a string which has a mass of 7kg attached at one end and a light pulley A attached at the other end. Another string passes over pulley A and carries a mass of 4kg at one end and a mass of 2kg at the other end.

8. The diagram shows a system of masses and pulleys.

9. For each of the systems below: all the strings are light and inextensible, all pulleys are light and smooth and all surface are smooth. In each case find the acceleration of A and the tension in the string.

(i) [7.127ms^{-2}, 13.364N] (ii) [1.547ms^{-2}, 24.758N]

(iii) [3.564ms^{-2}, 10.691N] (iv) [4.731ms^{-2}, 12.166N, 24.331N]

10. Two particles A and B of mass 4kg and 2kg respectively and a movable pulley c of mass 6kg are connected by a light inextensible string as shown below

Given that the coefficient of friction between A and the plane is 0.2 and the system is released from rest, find the acceleration of A, B, C and the tension in the string. [A = -0.25ms^{-2}, B = 2.9ms^{-2}, C = 1.325ms^{-2}]

Thank you

Dr. Bbosa Science