SENIOR FIVE TERM 2

TOPIC 6/6: Permutations and Combinations

Competency: The learner applies the permutations and combinations concepts to solve mathematical problems and model real-world situations.

Contents

Permutations and combinations

Permutations and combinations are powerful mathematical tools used to solve problems involving arrangement and selection. Permutations deal with ordered arrangements, while combinations focus on selections where order doesn't matter. They have wide applications in daily life, probability, and decision-making.

Permutation

A permutation is an **arrangement of objects in a specific order**.

Consider digits 1, 2 and 3; find the possible arrangements of the digits

123, 132, 321
231, 321, 321 } The total of six

This problem may also be solved as follows:

Given the three digits above, the first position can take up three digits, the second position can take up two digits and the third position can take up 1 digit only

1^{st} position	2^{nd} position	3^{rd} position
3	2	1

The total is thus 3 x 2 x 1 = 6

If the digits were four say 1, 2, 3, 4 the arrangement would be

1^{st}	2^{nd}	3^{rd}	4^{th}
4	3	2	1

The total is thus 4 x 3 x 2 x 1 = 24

In summary the number of ways of arranging n different items in a row is given by n(n – 1)(n – 2)(n -3) x ……….2 x 1 and can be expressed as n!

If the total number of books is 6

The total number of arrangements = 6!

= 6 x 5 x 4 x 3 x 2 x 1 = 720 ways

Example 1

Find the values of the following expression

(a) 5!
Solution
5!= 5 x 4 x 3 x 2 x 1 = 120

(b) $\frac{8!}{5!}$
Solution
$\frac{8!}{5!} = \frac{8\ x\ 7\ x\ 6\ x\ 5!}{5!} = 336$

(c) $\frac{10!}{6!x\ 5!\ x\ 2!}$
Solution
$\frac{10!}{6!x\ 5!\ x\ 2!} = \frac{10\ x\ 9\ x\ 8\ x\ 7\ x\ 6!}{6!x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1\ x\ 2x\ 1} = 21$

(d) Four different pens and 5 different books are to be arranged on a row. Find
   (i) The number of possible arrangements of items
   Solution
   Total number of items = 4 + 5 = 9
   Total number of arrangements = 9!
   = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1
   = 362,880 ways
   (ii) The number of possible arrangements if three of books must be kept together
   Solution
   The pens are taken to be one since they are to be kept together. So we consider total number of items to six. The number

of arrangements of six items = 6! = 6 x 5 x 4 x 3 x 2 x 1

= 720 ways

The arrangement of 4 pens = 4!

= 4 x 3 x 2 x 1 =24

Total number arrangements of all the items = 720 x 24 = 17, 280

Multiplication principle of permutation

If one operation can be performed independently in a different ways and the second in b different ways, then either of the two events can be performed in (a + b) ways

Example 2

There are 6 roads joining P to Q and 3 roads joining Q to R. Find how many possible routes are from P to R

From P to Q = 6 ways

From Q to R = 3 ways

Number of routes from P to R = 6 x 3 = 18

Example 3

Peter can eat either matooke, rice or posh on any of the seven days of the week. In how many ways can he arrange his meals in a week?

Solution

For each of the 7 days, there are 3 choices

Total number of arrangements

= 3 x 3 x 3 x 3 x 3 x 3 x 3 = 3^7 = 2187 ways

Example 4

There are four routes from Nairobi to Mombasa. In how many different ways can a taxi go from Nairobi to Mombasa and returning if for returning:

(a) any of the route is taken
= 4 x 4 = 16 ways
(b) the same route is taken
= 4 x 1 = 4 ways
(c) the same route is not taken
= 4 x 3 = 12 ways

Example 5

David can arrange a set of items in 5 ways and John can arrange the same set of items in 3 ways. In how many ways can either David or John arrange the items?

Solution

Number of ways in which David arranges = 5

Number of ways in which John arranges = 3

Number of ways in which either David or John arrange the items = 5 + 3 = 8 ways

The number of permutation of r objects taken from n unlike objects

The permutation of n unlike objects taking r at a time is denoted by ^nP_r which is defined as

$^nP_r = \frac{n!}{(n-r)!}$, where r ≤ n.

In case r = n, we have ^nP_n which is interpreted as the number of arranging n chosen objects from n objects denoted by n!

$^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n! => 0! = 1$

Example 6

How many three letter words can be formed from the sample space {a, b, c, d, e, f}

Solution

Total number of letters = 6 and r = 3

Total number of worms = $^6P_3$

$= \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6\ x\ 5\ x\ 4\ x\ 3!}{3!} = 120$ways

Example 7

Find the possible number of ways of arranging 3 letters from the word MANGOES

Solution

Total number of letter in the word = 7

and r = 3

Number of ways $^7P^3 = \frac{7!}{(7-3)!}$

$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$ways

Find number of ways of arranging six boys from a group of 13

Solution

Number of arrangements = $^{13}P_6$

$= \frac{13!}{(13-6)!} = \frac{13!}{7!}$

$= \frac{13 x 12 x 11 x 10 \; x 9 \; x \; 8 \; x \; 7!}{7!}$

= 1235520 ways

Example 8

The number of permutations of n objects of which r are alike

The number of permutations of n objects of which r are alike is given by $\frac{n!}{r!}$

Example 9

Find the number of arranging in a line the letters B, C, C, C, C, C, C

The number of ways of arranging the seven letters of which of which 6 are alike $= \frac{7!}{6!} = \frac{7 \; x \; 6!}{6!} = 7$ ways

The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike, and so on

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The number of ways of permutations on n objects of which p of one type are alike, q of the second type are alike, r of the third type are alike given by $\frac{n!}{p! \; x \; q! \; x \; r!}$

Example 10

Find the possible number of ways of arranging the letter of the word MATHEMATICS in line

Solution

The word MATHEMATICS has 11 letters and contains 2 M, 2A and 2T repeated

The number of ways = $\frac{11!}{2! x \; 2! \; x \; 2!}$

$= \frac{11 \; x \; 10 \; x \; 9 \;\; x \; 8 \; x \; 7 \; x \; 6 \; x \; 5 \; x \; 4 x 3 \; x \; 2 \; x \; 1}{2 \; x \; 2 \; x \; 2}$

= 4,989,600

Example 11

Find the possible number of ways of arranging the letter of the word 'MISSISSIPPI' in line

Solution

The word 'MISSISSIPPI' has 10 letters with 4I, 4S, and 2P

The number of ways = $\frac{11!}{4! x \; 4! \; x \; 2!} = 34650$

The number of permutations of the like and unlike objects with restrictions

One should be cautious when handling these problems

Example 12

Find the possible number of ways of arranging

The letters of the word MINIMUM if the arrangement begins with MMM?

Solution

There is only one way of arranging MMM

The remaining contain four letters with 2I can be arranged in

$\frac{1 \ x\ 4!}{2!} = \frac{1\ x\ 4\ x\ 3\ x\ 2!}{2!} = 12$ ways

Example 13

(a) How many 4 digit number greater than 6000 can be formed from 4, 5, 6, 7, 8 and 9 if:

(i) Repetitions are allowed

Solution

The first digit can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd, 3rd and 4th are chosen from any of the six digits since repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	6	6	6

Number of ways

= 4 x 6 x 6 x 6 = 864 ways

(ii) Repetition are not allowed

The first can be chosen from 6, 7, 8 and 9, hence 4 possible ways, the 2nd from 5, 3rd from 4 and 4th from 3 since no repetitions are allowed

position	1st	2nd	3rd	4th
selections	4	5	4	3

Number of ways

= 4 x 5 x 4 x 3 = 240 ways

(b) Find how many four digit numbers can be formed from the six digits 2, 3, 5, 7, 8 and 9 without repeating any digit.

Find also how many of these numbers

(i) Are less than 7000

(ii) Are odd

Solution

position	1st	2nd	3rd	4th
selections	6	5	4	3

Total number of ways = 6 x 5 x 4 x 3 = 360

(i) The 1st number is selected from three (2, 3, 5), the 2nd number from 5, the 3rd from 4 and the 4th from 3 digits

position	1st	2nd	3rd	4th
selections	6	5	4	3

Total number of less than 7000

= 3 x 5 x 4 x 3 = 180

(ii) The last number is selected from four odd digits (3, 5, 7, and 9), the 1st number selected from five remaining, 2nd from 4 and 3rd from 3

position	1st	2nd	3rd	4th
selections	5	4	3	4

Total number of odd numbers formed

= 5 x 4 x 3 x 4 = 240

(c) How many different 6 digit number greater than 500000 can be formed by using the digits 1, 5, 7, 7, 7, 8

Solution

The 1st digit is selected from five (5, 7, 7, 7, 8), the 2nd from remaining five, 3rd from four, 4th from three, 5th from two and 6th from one

1st	2nd	3rd	4th	5th	6th
5	5	4	3	2	1

Total number = $\frac{5\ x\ 5\ x\ 4\ x\ 3\ x\ 2\ x\ 1}{3!}$ = 100

NB. The number is divided by 3! Because 7 appears three times

(d) How many odd numbers greater than 60000 can be formed from 0, 5, 6, 7, 8, 9, if no number contains any digit more than once

Solution

Considering six digits

Taking the first digit to be odd, the first digit is selected from 3 digits (5, 7, 9) and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
3	4	3	2	1	2

Number of ways = 3 x 4 x 3 x 2 x 1 x 2

= 144

Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 2 odd digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6th
2	4	3	2	1	3

Number of ways = 3 x 4 x 3 x 2 x 1 x 2

= 144

Considering five digits

Taking the first digit to be odd, the digit greater than 6 are 7 and 9 so first digit is selected from 2 digits and the last is selected from 2 digits

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	2

Number of ways = 2 x 4 x 3 x 2 x 2

= 96

Taking the first digit to be even, the first digit is selected from 2 digits (6, 8) since the number should be greater than 60000 and the last is selected from 3 odd digits (5, 7, 9)

1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
2	4	3	2	3

Number of ways = 2 x 4 x 3 x 2 x 3

= 144

The total number of selections

= 144 + 144 + 96 + 144 = 528

Example 14

The six letter of the word LONDON are each written on a card and the six cards are shuffled and placed in a line. Find the number of possible arrangements if

(a) The middle two cards both have the letter N on them

Solution

If the middle letter are NN, the we need to find the number of different arrangements of the letter LODO.

With the 2O's, the number of arrangements = $\frac{4!}{2!} = 12$

(b) The two cards with letter O are not adjacent and the two cards with letter N are also not adjacent

Solution

If the two cards are not adjacent, the number of arrangements = Total number of arrangements of the word LONDON – number of arrangements when the two letters are adjacent

= = $\frac{6!}{2!2!} - 24 = 156$

Example 15

In how many different ways can letters of the word MISCHIEVERS be arranged if the S's cannot be together

Solution

There are 11 letters in the word MISCHIEVERS with 2S's, 2I's and 2E's

Total number of arrangements

= $\frac{11!}{2!2!2!}$ = 4989600

If S's are together, we consider them as one, so the number of arrangements

$= \frac{10!}{2!2!} = 907200$

∴ the number of possible arrangements of the word MISCHIEVERS when S's are not together

= 4989600 – 907200 = 4082400

The number of permutation of n different objects taken r at a time, if repetition are permitted

Example 16

How many four digit numbers can be formed from the sample space {1, 2, 3, 4, 5} if repetitions are permissible

Solution

The 1st position has five possibilities, the 2nd five, the 3rd five, the 4th five

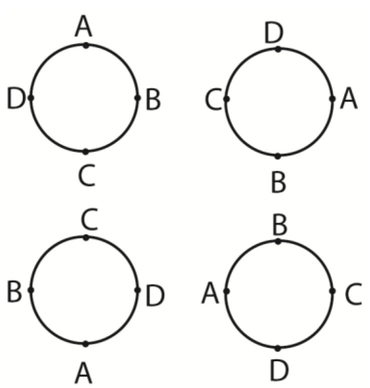
Number of permutations = 5 x 5 x 5 x 5 = 625

Circular permutations

Here objects are arrange in a circle

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise are different.

Consider four people A, B, C and D seated at a round table. The possible arrangements are as shown below



With circular arrangements of this type, it is the relative positions of the objects being arranged which is important. The arrangements of the people above is the same. However, if the people were seated in a line the arrangements would not be the same, i.e. A, B, C, D is not the same as D, A, B, C. When finding the number of different arrangements, we fix one person say A and find the number of ways of arranging B, C and D.

Therefore, the number of different arrangements of four people around the table is 3!

Hence the number of different arrangements of n people seated around a table is (n – 1)!

Example 17

(a) Seven people are to be seated around a table, in how many ways can this be done
Solution
The number of ways = (7 – 1)! = 6!
= 720

(b) In how many ways can five people A, B, C, D and E be seated at a round table if
(i) A must be seated next to B

Solution

If A and B are seated together, they are taken as bound together. So four people are considered

The number of ways = (4 – 1)! = 3! = 6

The number of ways in which A and B can be arranged = 2

The total number of arrangements

= 6 x 2 = 12 ways

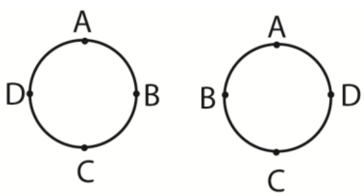
(ii) A must not seat next to B

If A and B are not seated together, then the number of arrangements = total number of arrangements – number of arrangements when A and B are seated together

= (5 – 1)! – 12 = 12 ways

The number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same

Consider the four people above, if the arrangement is as shown below



Then the above arrangements are the same since one is the other viewed from the opposite side

The number of arrangements = $\frac{3!}{2}$ = 3ways

Hence the number of ways of arranging n unlike objects in a ring when clockwise and anticlockwise arrangements are the same = $\frac{(n-1)!}{2}$

Example 18

A white, a blue, a red and two yellow cards at arranged on a circle. Find the number of arrangements if red and white cards are next to each other.

Solution

If red and white cards are next to each other, they are considered as bound together. So we have four cards. Since anticlockwise and clockwise arrangements are the same and there are two yellow cards, the number of arrangements = $\frac{(4-1)!}{2\ x\ 2!} = \frac{3!}{2\ x\ 2!}$

The number of ways of arranging red and white cards = 2

Total number of ways of arrangements

=$\frac{3!}{2\ x\ 2!}\ x\ 2$ = 3

Revision exercise 1

1. In how many ways can the letters of the words below be arranged
   (a) Bbosa (5!]
   (b) Precious [8!]
2. How many different arrangements of the letters of the word PARALLELOGRAM can be made with A's separate [83160000]
3. How many different arrangements of the letters of the word CONTACT can be made with vowels separated? [900]
4. How many odd numbers greater than 6000 can be formed using digits 2, 3, 4, 5 and 6 if each digit is used only once in each number [12]
5. Three boys and five girls are to be seated on a bench such that the eldest girl and eldest boy sit next to each other. In how many ways can this be done [2 x 7!]
6. A round table conference is to be held between delegates of 12 countries. In how many ways can they be seated if two particular delegates wish to sit together [2 x 10!]

7. In how many ways can 4 boys and 4 girls be seated at a circular table such that no two boys are adjacent [144]

8. How many words beginning or ending with a consonant can be formed by using the letters of the word EQUATION? [4320]

Combinations

A combination is **a way of selecting items from a set where the *order does not matter***

Consider the letters A, B, C, D

The possible arrangements of two letters chosen from the above letters are

AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. AS seen earlier, the total number of arrangements of the above letters is expressed as $\frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$

However, when considering combinations, the grouping such as AB and BA are said to be the same groupings such as CA and AC, AD and DA, etc.

So the possible combinations are AB, AC, AD, BC, BD, CD which is six ways.

Thus the number of possible combinations of n items taken r at a time is expressed as ^nC_r or $\binom{n}{r}$ which is defined as $^nC_r = \frac{n!}{(n-r)!r!}$ where $r \leq n$

Hence the number of combinations of the above letters taken two at a time is

$\binom{4}{2} = \frac{4!}{2!2!} = 6$

Example 19

A committee of four people is chosen at random from a set of seven men and three women

How many different groups can be chosen if there is at least one

(i) Woman on the committee
Solution

Possible combinations

7 men	3 women
3	1
2	2
1	3

The number of ways of choosing at least one woman

$=\binom{7}{3} x \binom{3}{1} + \binom{7}{2} x \binom{3}{2} + \binom{7}{1} x \binom{3}{3}$

$=\frac{7!}{3!4!} x \frac{3!}{1!2!} + \frac{7!}{2!5!} x \frac{3!}{2!1!} + \frac{7!}{1!6!} x \frac{3!}{3!0!} = 175$

(ii) Man on the committee

7 men	3 women
1	3
2	2
3	1
4	0

The number of ways of choosing at least one man

$\binom{7}{1} x \binom{3}{3} + \binom{7}{2} x \binom{3}{2} + \binom{7}{3} x \binom{3}{1} + \binom{7}{4} x \binom{3}{0}$

$=\frac{7!}{1!3!} x \frac{3!}{0!3!} + \frac{7!}{5!2!} x \frac{3!}{1!2!} + \frac{7!}{4!3!} x \frac{3!}{2!1!} + \frac{7!}{3!4!} x \frac{3!}{3!0!}$

=210

Example 20

A group of nine has to be selected from ten men and eight women. It can consist of either five men and four women or four men and five women. How many different groups can be chosen?

Solution

Possible combination

10 men	8 women
5 4	4 5

Number of groups = $\binom{10}{5} x \binom{8}{4} + \binom{10}{4} x \binom{8}{5}$

= $\frac{10!}{5!5!}$ x $\frac{8!}{4!4!}$ + $\frac{10!}{6!4!}$ x $\frac{8!}{5!3!}$ = 29400

Example 21

A team of six is to be formed from 13 boys and 7 girls. In how many ways can the team be selected if it must consist of

(a) 4 boy and 2 girls

13 boys	7 girls
4	2

$\binom{13}{4}.\binom{7}{2}$= $\frac{13!}{9!4!} x \frac{7!}{5!2!}$ = 15015

(b) At least one member of each sex

Possible combinations

13 boys	7 girls
5 4 3 2 1	1 2 3 4 5

=$\binom{13}{5}.\binom{7}{1} + \binom{13}{4}.\binom{7}{2} + \binom{13}{3}.\binom{7}{3} +$
$\binom{13}{2}.\binom{7}{4} + \binom{13}{1}.\binom{7}{5}$

=$\frac{13!}{8!5!}.\frac{7!}{6!1!} + \frac{13!}{9!4!}.\frac{7!}{5!2!} + \frac{13!}{10!3!}.\frac{7!}{4!3!} + \frac{13!}{11!2!}.\frac{7!}{3!4!} +$
$\frac{13!}{12!1!}.\frac{7!}{2!5!}$

= 37037

Example 22

A team of 11 players is to be chosen from a group of 15 players. Two of the 11 are to be randomly elected a captain and vice-captain respectively. In how many ways can this be done?

Number of ways of choosing 11 players from 15 = $\binom{15}{11}$

A captain will be elected from 11 players and a vice-captain from 10 players

Total number of selection =$\binom{15}{11}$ x 11 x 10

=$\frac{15!}{4!11!}$ x 11 x 10 = 150150

Example 23

(a) Find the number of different selections of 4 letters that can be made from the word UNDERMATCH.
Solution
There are 10 letters which are all different
Number of selections of 4 letters from 10 is given by $\binom{10}{4} = \frac{10!}{(10-4)!4!} = \frac{10!}{6!4!} = 210$

(b) How many selections do not contain a vowel?
Solution
Number of vowels in the word = 2
Number of letters not vowels = 8
Number of selections of 4 letters from 10 without containing a vowel = selecting 4 letters from 8 consonants =
$\binom{8}{4} = \frac{8!}{(8-4)!4!} = \frac{8!}{4!4!} = 70$

Example 24

In how many ways can three letters be selected at random from the word BIOLOGY is selection

(a) Does not contain the letter O
Solution

Number of selections without the letter O = number of ways of choosing three letters from B, I, L, G, y

$= \binom{5}{3} = \frac{5!}{2!3!} = 10$

(b) Contain only the letter O

Solution

Number of selections with one letter O = number of ways of choosing two letters from B, I, L, G, y

$= \binom{5}{2} = \frac{5!}{3!2!} = 10$

(c) Contains both of the letters O

Solution

Number of selections with two letter O = number of ways of choosing one letter from B, I, L, G, y

$= \binom{5}{1} = \frac{5!}{4!1!} = 5$

Example 25

In how many ways can four letters be selected at random from the word BREAKDOWN if the letters contain at least one vowel?

Solution

Vowels: E, A, O (3)

Consonants: B, R, K, D, W, N (6)

Consonants (6)	Vowels (3)
3	1
2	2
1	3

Number of selection of four letters with at least one vowel

$= \binom{6}{3}.\binom{3}{1} + \binom{6}{2}.\binom{3}{2} + \binom{6}{1}.\binom{3}{3} = 111$

Example 26

How many different selections can be made from the six digits 1, 2, 3, 4, 5, 6

Solution

Note: this an open questions because selections can consist of only one digit, two digits, three digits, four digits, five digits or six digits

Number of selection of 1 digit = $^6C_1$ = 6

Number of selection of 2 digits = $^6C_2$ = 15

Number of selection of 3 digits = $^6C_3$ = 20

Number of selection of 4 digits = $^6C_4$ = 15

Number of selection of 5 digits = $^6C_5$ = 6

Number of selection of 6 digits = $^6C_1$ = 1

Total number of selections

= 6 + 15 + 20 + 15 +6 + 1 = 63

This approach is tedious for a large group of objects.

The general formula for selection from n unlike objects is given by $2^n - 1$.

For the above problems, number of selections = $2^6 - 1 = 63$

Example 26

How many different selections can be made from 26 different letters of the alphabet?

Number of selection = $2^{26} - 1$

= 67,108,863

Combination cases involving repetitions

Suppose we need to find the number of possible selections of letters from a word containing repeated letters, we take the selections mutually exclusive

Example 27

How many different selections can be made from the letters of the word CANADIAN?

Solution

There are 3A's, 2N's and 3 other letters

The A's can be dealt with in 4 ways (either no A, 1A's, 2A's or 3A's)

The N's can be dealt in 3 ways (no N, 1N, or 2N's)

The C can be dealt with in 2 ways (no C, 1C)

The D can be dealt with in 2 ways (no D, 1D)

The I can be dealt with in 2 ways (no I, 1I)

The number of selections

= 4 x 3 x 2 x 2 x 2 – 1 = 95

Example 28

How many different selections can be made from the letters of the word POSSESS?

Solution

There are 4S's and 3 other letters

The S's can be dealt in 5 ways (no S, 1S, 2N's, 3S's, 4S's, or 5S's)

The P can be dealt with in 2 ways (no P, 1P)

The O can be dealt with in 2 ways (no O, 1O)

The E can be dealt with in 2 ways (no E, 1E)

Total number of selections = 5 x 2 x 2 x 2 – 1

= 39

Combination cases involving division into groups

The number of ways of dividing n unlike objects into say two groups of p and q where p + q = n is given by $\frac{n!}{p!q!}$

For three groups of p, q and r provided p + q + r = n

Number of ways of division = $\frac{n!}{p!q!r!}$

However, for the two groups above, if p = q then the number of ways of division = $\frac{n!}{p!p!2!}$

For three groups where p = q = r

then the number of ways of division = $\frac{n!}{p!p!p!3!}$

Example 29

The following letters a, b, c, d, e, f, g, h, I, j, k, l are to be divided into groups containing

(a) 3, 4, 5
(b) 5, 7
(c) 6, 6
(d) 4, 4, 4 letters. In how many ways can this be done?

Solution

(a) Number of ways = $\frac{12!}{3!4!5!}$ = 27720
(b) Number of ways = $\frac{12!}{5!7!}$ = 792
(c) Number of ways = $\frac{12!}{6!6!2!}$ = 462
(d) Number of ways = $\frac{12!}{4!4!4!3!}$ =5775

Example 30

Find the number of ways that 18 objects can be arranged into groups if there are to be

(a) Two groups of 9 objects each
(b) Three groups of 6 objects each
(c) 6 groups of 3 objects each
(d) Three groups of 5, 6 and 7 objects each

Solution

(a) Number of ways = $\frac{18!}{9!9!2!}$ = 24310
(b) Number of ways = $\frac{18!}{6!6!6!3!}$ = 2858856
(c) Number of ways = $\frac{18!}{3!3!3!3!3!3!6!}$ =190590400
(d) Number of ways = $\frac{18!}{5!6!7!}$ = 14702688

Example 31

(a) Find how many words can be formed using all letters in the word MINIMUM.

Solution

Number of ways of arranging the letters = 7!

There are 3M's and 2I's

Number of words formed = $\frac{7!}{3!2!} = 420$

(b) Compute the sum of four-digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement

Solution

Number of ways of arranging a four digit number = 4!

Sum of any four digit number formed = 2 + 5 + 3 + 8 = 18

Total sum of four digit numbers formed

= 18 x 4! = 432

(c) A committee consisting of 2 men and 3 women is to be formed from a group of 5 men and 7 women. Find the number of different committees that can be formed. If two of the women refuse to serve on the same committee, how many committees can be formed?

Solution

The committees formed = ${}^5C_2.{}^7C_3$

= 10 x 35 = 350

Suppose two women are to serve together, we take them as glued together, so the number of committees =${}^5C_2.{}^6C_3$ = 200

Number of committees in which two women refuse to serve together = 350 – 200 = 150

Revision exercise 2

1. (a) Find the number of different selection of 3 letters that can be made from the word PHOTOGRAPH. [53]
   (b) How many of these selections contain no vowel [18]
   (c) How many of these selections contain at least one vowel? [35]
2. (a) find the number of different selections of 3 letters that can be made from the letters of the word SUCCESSFUL.[36]
   (c) How many of these selections contain only consonants [11]
   (d) How many of these selections contain at least one vowel [25]
3. (a) Find the value of n if ${}^nP_4 = 30{}^nC_5$ [8]
   (b) How many arrangement can be made from the letters of the name MISSISSIPPI
      (i) when all the letters are taken at a time [34650]
      (ii) If the two letters PP begin every word [630 ways]
   (c) Find the number of ways in which a one can chose one or more of the four girls to join a discussion group [15 ways]
4. Find in how many ways 11 people can be divided into three groups containing 3, 4, 4 people each. [5775]
5. A group of 5 boys and 8 girls. In how many ways can a team of four be chosen, if the team contains
   (a) No girl [5]
   (b) No more than one girl [85]
   (c) At least two boys [365]
6. Calculate the number of 7 – letter arrangements which can be made with the letters of the word MAXIMUM. In how

many of these do all the 4 consonants appear next to each other? [840, 96]

7. In how many ways can a club of 5 be selected from 7 boys and 3 girls if it must contain
   (a) 3 boys and 3 girls [105]
   (b) 2 men and 3 girls [21]
   (c) At least one girl [231]
8. How many different 6 digit numbers greater than 400,000 can be formed form the following digits 1, 4, 6, 6, 6 7? [100]

Applications of Permutation and combinations

Applications of Permutations

(i) **Arranging Books on a Shelf**

If you have 5 books and want to arrange 3 of them, the number of ways is:

$P(5,3)=\frac{5!}{(5-3)!}=\frac{120}{2}=60$ ways

(ii) **Seating Arrangements**

In a classroom, arranging 10 students in 10 seats involves

10!=3,628,800 ways

(iii) **Password/Lock Combinations (Order Matters)**

A 4-digit PIN using digits 0–9 has 10^4=10,000 possible permutations.

(iv) **Assigning Roles**

Choosing a President, Vice President, and Secretary from 5 people:

$P(5,3)=\frac{5!}{(5-3)!}=\frac{120}{2}=60$ ways.

Applications of Combinations

(i) **Lottery Tickets**

Choosing 6 numbers from 49 (order doesn't matter):

$C(49,6)=\frac{49!}{6!.43!}$ =13,983,816 possible outcomes.

(ii) **Committee Selection**

Selecting 3 members from a group of 10:

$C(10,3)=\frac{10!}{3!7!}=$ = 120 ways.

(iii) **Food Choices**

Picking 2 ice cream flavors out of 5 available:

$C(5,2)=\frac{5!}{3!2!}$ == 10 possible combinations.

(iv) **Card Games**

Choosing 5 cards from a standard 52-card deck: $C(52,5)=\frac{52!}{5!47!}$ == 2,598,960 possible hands.

Thank you

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