



Dr. Bhasa Science

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## SENIOR SIX TERM 2

### TOPIC 2/5: Trapezium Rule

**Competency:** The learner applies the trapezium rule to estimate definite integrals and areas under a curve to solve problems involving numerical integration.

#### Trapezium rule

It is used for estimating an integral area under a curve of continuous function over a given interval [a, b]

if  $y = f(x)$

$$A = \int_a^b y dx$$

Using several strips between  $x = a$  and  $x = b$  of equal width, trapezium rule can be used to determine the area.

$$A \approx \frac{1}{2}h[(\text{first} + \text{last ordinates}) + 2(\text{sum of the middle ordinates})]$$

$$\text{where } h = \frac{b-a}{\text{subintervals}}$$

Note

- (i) sub-intervals, subdivision and strips are the same
- (ii) subinterval = ordinates
- (iii) when dealing with a trigonometric function, calculators must be in radian mode
- (iv) when the final answer is required to a specific number of d.p.'s, the working's should be done at least a d.p higher but the final answer rounded to the required d.p's

### Example 1

Use the trapezium rule with four-intervals to estimate  $\int_{0.2}^{1.0} \left(\frac{2x+1}{x^2+x}\right) dx$ . Correct to two decimal places.

$$\text{Let } y = \left(\frac{2x+1}{x^2+x}\right)$$

$$h = \frac{1.0-0.2}{4} = 0.2$$

x	$y = \frac{2x+1}{x^2+x}$	
0.2	5.8333	
0.4		3.2143
0.6		2.2917
0.8		1.8056
1.0	1.5000	
Sum	7.3333	7.3116

$$\begin{aligned}\int_{0.2}^{1.0} \left(\frac{2x+1}{x^2+x}\right) dx &= \frac{1}{2} \times 0.2(7.3333 + 7.3116) \\ &= 2.1955 \\ &= 2.20 \text{ (2D)}\end{aligned}$$

### Example 2

Use the trapezium rule with seven coordinates to estimate

$$\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}} dx \text{ correct to 2 decimal places (05marks)}$$

#### Solution

For 7 ordinates, there are 6 subintervals

$$\text{Width, } h = \frac{b-a}{\text{subinterval}} = \frac{3-0}{6} = 0.5$$

$$\text{Let } y = \sqrt{(1.2)^x - 1}$$

x	y	
0	0	
0.5		0.309
1		0.447
1.5		0.561
2		0.663
2.5		0.760
3	0.853	
Sum	0.853	2.74

Using the trapezium rule

$$\int_0^3 [(1.2)^x - 1]^{\frac{1}{2}} dx = \frac{0.5}{2} [0.853 + 2(2.74)] = 1.58$$

### Example 3

(a) Use the trapezium rule with 6-ordinates to estimate the value of  $\int_0^{\frac{1}{2}}(x + \sin x)dx$ , correct to three decimal places.

$$h = \frac{\frac{1}{2}-0}{5} = \frac{\pi}{10}$$

x	y	
0	0	
$\frac{\pi}{10}$		0.6232
$\frac{2\pi}{10}$		1.2161
$\frac{3\pi}{10}$		1.7515
$\frac{4\pi}{10}$		2.2077
$\frac{\pi}{2}$	2.5708	
Sum	2.5708	5.7985

$$\begin{aligned}\int_0^{\frac{1}{2}}(x + \sin x)dx &= \frac{1}{2} \times \frac{\pi}{10} (2.5708 + 2 \times 5.7985) \\ &= 2.225\end{aligned}$$

(b)(i) Evaluate  $\int_0^{\frac{1}{2}}(x + \sin x)dx$ , correct to three decimal places

$$\begin{aligned}\int_0^{\frac{1}{2}}(x + \sin x)dx &= \left| \frac{x^2}{2} - \cos x \right|_0^{\frac{1}{2}} \\ &= \frac{1}{2} \left( \frac{\pi^2}{4} - 0 \right) - (\cos \frac{\pi}{2} - \cos 0) \\ &= \frac{\pi^2}{8} + 1 \\ &= 2.234\end{aligned}$$

(ii) Calculate the error in your estimation in (a) above

$$\text{Error} = |2.234 - 1.225| = 0.009$$

(iii) Suggest how the error may be reduced (06marks)

Increasing on number of strips or subintervals

### Example 4

A student used the trapezium rule with five sub-intervals to estimate

$$\int_2^3 \frac{x}{(x^2-3)} dx \text{ correct to **three** decimal places}$$

Determine;

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(a) The value the student obtained (06marks)

$$h = \frac{3-2}{5} = 0.2$$

X	Y <sub>1</sub> , Y <sub>6</sub>	Y <sub>2</sub> .....Y <sub>5</sub>
2.0	2.0	
2.2		1.1956
2.4		0.8696
2.6		0.6915
2.8		0.5785
3	0.5	
Sum	2.5	3.3352

$$\int_2^3 \frac{x}{(x^2-3)} dx = \frac{1}{2} x 0.2 [2.5 + 2(3.3352)]$$

$$= 0.91704 = 0.917 \text{ (3D)}$$

(b) The actual value of the integral (03marks)

$$\int_2^3 \frac{x}{(x^2-3)} dx = \left[ \frac{1}{2} \ln x^2 - 3 \right]_2^3$$

$$= \frac{1}{2} (\ln 6 - \ln 1)$$

$$= 0.896$$

(c) (i) the error the student made in the estimate

$$\text{Error} = |0.896 - 0.917| = 0.021$$

(ii) how the student can reduce the error(03marks)

Increasing on the number of sub-intervals or ordinates or reducing the width of h

### Example 5

Use trapezium rule with 4 subintervals to estimate to 3 decimal places  $\int_0^{\frac{\pi}{2}} \cos x dx$

Solution

$$h = \frac{\frac{\pi}{2} - 0}{4} = \frac{\pi}{8}$$

x	f(x) = cos x	
0	1.0000	
$\frac{\pi}{8}$		0.9239
$\frac{2\pi}{8}$		0.7071
$\frac{3\pi}{8}$		0.3827
$\frac{4\pi}{8}$	0.0000	
sum	1.0000	2.0137

$$\int_0^{\frac{\pi}{2}} \cos x dx = \frac{1}{2} \times \frac{\pi}{8} [1 + 2 \times 2.0137]$$

$$= 0.987$$

### Example 6

Use trapezium rule with 7 ordinates to estimate  $\int_0^3 \frac{1}{1+x} dx$  correct to 3dp

Solution

$$h = \frac{3-0}{7-1} = 0.5$$

x	f(x) = cos x	
0	1.0000	
0.5		0.6667
1.0		0.5000
1.5		0.4000
2.0		0.3333
2.5		0.2757
3.0	0.2500	
sum	1.2500	2.157

$$\int_0^3 \frac{1}{1+x} dx = \frac{1}{2} \times 0.5 [1.25 + 2 \times 2.1857]$$

$$= 1.405$$

### Example 7

(a) Use the trapezium rule to estimate the integral value of  $\int_2^3 \frac{x}{1+x^2} dx$  using five subinterval and correct to 3d.p.

(b) (i) find the exact value of  $\int_2^3 \frac{x}{1+x^2} dx$

(ii) suggest how the error may be reduced.

(a)  $h = \frac{3-2}{5} = 0.2$

x	$f(x) = \frac{x}{1+x^2}$	
2.0	0.40000	
2.2		0.37671
2.4		0.35503
2.6		0.33505
2.8		0.31674
3.0	0.30000	
sum	0.70000	1.3353

$$\int_2^3 \frac{x}{1+x^2} dx = \frac{1}{2} x \cdot 0.2 [0.7 + 2 \times 1.38353]$$

$$= 0.3467$$

(b)(i)  $\int_2^3 \frac{x}{1+x^2} dx = \left[ \frac{1}{2} \ln(1+x^2) \right]_2^3 = \frac{1}{2} (\ln 10 - \ln 5) = 0.3466$

(ii) error =  $|\text{exact value} - \text{approximate value}| = |0.3466 - 0.3467| = 0.0001$

(iii) the error can be reduced by reducing h or increasing the number of sub-intervals.

### Example 8

(a) Use trapezium rule to estimate the integral value of  $\int_0^1 x^2 e^x dx$

(b) (i) find exact value of  $\int_0^1 x^2 e^x dx$

(ii) determine the percentage error in your estimation

$$(a) h = \frac{1-0}{5} = 0.2$$

x	f(x) = x <sup>2</sup> e <sup>x</sup>	
0	0	
0.2		0.0489
0.4		0.2387
0.6		0.6560
0.8		1.4243
1.0	2.7183	
sum	2.7183	2.3679

$$\int_0^1 x^2 e^x dx = \frac{1}{2} x \cdot 0.2 [2.713 + 2 x \cdot 2.3679]$$

$$= 0.74541 \approx 0.745$$

$$(b)(i) \int_0^1 x^2 e^x dx = [x^2 e^x - 2x e^x + 2e^x]_0^1$$

$$= 0.718$$

$$(ii) \text{error} = |0.718 - 0.745| = 0.027$$

$$\text{Percentage error} = \frac{\text{error}}{\text{exact value}} \times 100\%$$

$$= \frac{0.027}{0.718} \times 100 = 3.8\%$$

### Revision Exercise

- Use trapezium rule with six strips to estimate  $\int_0^\pi x \sin x dx$  [3.069]
  - Determine the percentage error in your determination. [2.3%]
- Use the trapezium rule to estimate the approximate value of  $\int_0^1 \frac{1}{1+x^2} dx$  using 6 ordinates and correct to 3 decimal places. [0.784]
- Use trapezium rule with six strips to estimate  $\int_2^4 \frac{10}{2x+1} dx$  correct 4dp. [2.9418]
  - Determine the percentage error in your estimation and suggest how this error may be reduced. [0.098%]
- Use trapezium rule to estimate the area of  $y = 3x$  between x-axis,  $x = 1$  and  $x = 2$ , using five subintervals. Give your answer correct to four significant figures. [5.483]
  - Find the exact value of  $\int_1^2 3^x dx$  [5.461]
  - Find the exact percentage error in calculations (a) and (b) above. [0.4028%]
- Use trapezium rule with 7 ordinates to estimate  $\int_0^3 \frac{1}{1+x} dx$ , correct to 3 decimal places [1.405]
- Use the trapezium rule with 6 ordinates to evaluate  $\int_0^1 e^{-x^2} dx$  correct to 2 decimal places. [0.74]
- Use the trapezium rule with 6 ordinates to estimate  $\int_1^2 \frac{\ln x}{x} dx$ . Give your answer correct to 3 decimal places [0.237]
- Find the approximate value to one decimal place of  $\int_0^1 \frac{dx}{1+x}$ , using the trapezium rule with five strips. [0.7]
- Use trapezium rule with five subintervals to estimate  $\int_0^{\frac{\pi}{3}} \tan x dx$  correct to 3dp. [0.704]
  - (i) Find the exact value of  $\int_0^{\frac{\pi}{3}} \tan x dx$  to 3 d.p. [0.693]

- (ii) Calculate the percentage error in your estimation in (a) above [1.587%]  
 (iii) Suggest how the percentage error in (b)(ii) may be reduced.

10. Use the trapezium rule with four subdivisions to estimate  $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin x} dx$ . Give your answer correct to three decimal places. [1.013]
11. Find the approximate value of  $\int_0^2 \frac{1}{1+x^2} dx$  using trapezium rule with 6 ordinates. Give your answer to 3 decimal places (05marks)[1.105]
12. Use the trapezium rule with five subintervals to estimate  $\int_2^4 \frac{5}{(x+1)} dx$ . Give your answer correct to 3 decimal places (05marks)[2.559]
13. A student used the trapezium rule with five sub-intervals to estimate  $\int_2^3 \frac{x}{(x^2-3)} dx$  correct to **three** decimal places
- Determine;
- (a) The value the student obtained (06marks) [0.917]  
 (b) The actual value of the integral (03marks) [0.896]  
 (c) (i) the error the student made in the estimate [0.021]  
 (ii) how the student can reduce the error(03marks)
14. (a) Use the trapezium rule with 6-ordinates to estimate the value of  $\int_0^{\frac{1}{2}} (x + \sin x) dx$ , correct to three decimal places, [2.225]
- (b)(i) Evaluate  $\int_0^{\frac{1}{2}} (x + \sin x) dx$ , correct to three decimal places [2.234]  
 (ii) Calculate the error in your estimation in (a) above [0.009]  
 (iii) suggest how the error may be reduced (06marks)

**Thank you**  
**Dr. Bbosa Science**